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SETS & OPERATION ON SETS

A set is a collection of **well – defined distinct** objects. "Objects" may be things, people or symbols. The objects in a given set are called the elements or members of that set and are said to belong to or contained in the set. "Well – defined" means that there must be no doubt whatsoever about whether or not a given item belongs to the set under consideration. "Distinct" is used in the sense that no two identical objects should be contained in the same set.

Membership

If x is a member of a set A, we write $x \in A$, which is read as "x belongs to A". The symbol $\in$ stands for "is a member of" or "is an element of" or "belongs to" and $\notin$ means it is not a member of.

Sets are generally denoted by capital letters A, B, C, … , X, Y, Z and the members or elements are denoted by lowercase letters $a, b, c, \ldots, x, y, z$.
e.g. $\mathrm{A} = \{a, b, c, d\}$

clearly b is a member of or "an element" of A and we write b $\in$ A. Clearly m is not a member of or "not an element" of A and we write $m \notin$ A.

Exercise 1 **Date:……………….**

List five elements of each of the following.

(i) The set of prefects in your school.
(ii) The set of students in your class.
(iii) The set of integers greater than 4 but less than 20.
(iv) The set of prime numbers greater than 2 but less than 32.

Exercise 2 **Date:……………….**

Complete the following statements by inserting $\in$, $\notin$, M or N.
If $\mathrm{M} = \{1, 3, 5, 7, 9, 11, 13, 15\}$ and $\mathrm{N} = \{2, 4, 6, 8, 10, 12, 14, 16\}$.

(i) 7 … … … M
(ii) 8 … … … . N
(iii) 2 … … … . . M
(iv) 9 … … … . . N
(v) 8 $\notin$ …………
(vi) 13 $\notin$ …………

Exercise 3 **Date:……………….**

List the elements of each of the following sets.

(i) $\mathrm{A} = \{$whole numbers from 5 to 15$\}$
(ii) $\mathrm{B} = \{$multiples of 3 between 9 and 51$\}$
(iii) $\mathrm{P} = \{$prime numbers between 1 and 20$\}$
(iv) $\mathrm{Q} = \{$natural numbers less than 17$\}$
(v) $\mathrm{N} = \{$composite numbers between 1 and 17$\}$

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SET – BUILDER NOTATION
Set – builder notation is a mathematical notation for describing a set by stating the property that its members must satisfy.
For example,
$A = \{x: x \text{ is a prime number}, 1 < x < 20\}$. In the above expression for A, the colon ':' means 'such that'. Some use "|" instead of ":" so A could be written as
$A = \{x|x \text{ is a prime number}, 1 < x < 20\}$

Example 1
Use set – builder notation to describe the following.
(i) The set of integers from 1 to 1000
(ii) The set of even integers.

Solution…
(i) $\{x: x \text{ is an integer, and } 1 \leq x \leq 1000\}$
(ii) $\{x: x \text{ is an even integer}\}$

Exercise 4 **Date:………………..**
Rewrite the following sets using set – builder notation.
1. {days of the week}
2. {triangles}
3. {rivers}
4. {students}

Exercise 5 **Date:………………..**
List the elements in the following sets.
1. $\{x: x \text{ is a month with 31 days}\}$
2. $\{x: x \text{ is a factor of 80}\}$
3. $\{x: 4x = 16\}$
4. $\{x: x \text{ is a multiple of 4 but less than 48}\}$
5. $\{x: x \text{ is a square number greater than 3 and less than 47}\}$

TYPES OF SETS
1. **Finite Set**
   A set is said to be finite if it is empty or it consists of exactly n distinct elements where n is a positive integer.
   e.g. $A = \{1, 2, 3, 4, 5, 6\}$
2. **Infinite Set**
   A set is said to be infinite if it has unlimited number of elements.
   e.g. $A = \{2, 4, 6, 8, \ldots\}$

3. **Unit Set (Singleton Set)**
   A unit set is a set with exactly one member.
   e.g. $\{2\}, \{5\}, \{\emptyset\}$

4. **Empty (Null) Set**
   A set with no element or member is called an **empty set** or a null set. It is denoted by the Scandinavian symbol ∅ or { }.

   NOTE:
   (i) $\{\emptyset\} \neq \emptyset$ but $\{\ \} = \emptyset$
   (ii) An empty set is a finite set.

5. **Universal Sets**
   The universal set is the set of all objects under consideration. It is usually denoted by U or ξ.

Exercise 6 **Date:……………….**

Classify the following sets as finite or infinite.

1. $\{1, 3, 5, \ldots, 100\}$
2. $\{0, 5, 10, \ldots\}$
3. {even numbers greater than 3}
4. $\{x: x \text{ is an integer}, x > 100\}$
5. $\{x: x \text{ is an integer}, 0 < x < 80\}$

Exercise 7 **Date:……………….**

Find the possible Universal sets for each of the following.

1. {Togo, Zambia, Niger, Nigeria}
2. {lion, tiger, leopard, fox}
3. {February, March, June, December}
4. {1, 3, 5, …, 101}
5. {2, 4, 6, … }

Exercise 8 **Date:……………….**

Identify unit sets from the following

1. $\{x: x - 3 = 0\}$
2. $\{x: x \text{ is a quadrilateral}\}$
3. $\{x: x \text{ is a factor of } 6\}$
4. $\{x: x \text{ is an even factor of } 30\}$
5. $\left\{x: \frac{x-4}{2} = 3\right\}$

CARDINALITY OF A SET

The cardinality of any set **A** is the number of elements in the set **A**.
A synonym for cardinality is the cardinal number. The cardinality of, say, set A, is denoted by the symbol n(**A**), or |**A**|, or sometimes #**A**, and read as "the number of elements in set **A**".
i.e. $\mathrm{A} = \{a, b, c, d, e\}$ $\therefore \mathrm{n(A)} = 5$

EQUAL SETS

Two sets A and B are said to be equal if A and B have the same elements, not necessarily listed in order.
i.e. if $A = \{a, b, c\}$ and $B = \{b, a, c\}$
Clearly $A = B$ since A and B have the same elements.

EQUIVALENT SETS

Two sets **A** and **B** are said to be equivalent if they contain the same number of elements. Equivalent sets are denoted by "⟺", that is, if set **A** is equivalent to set **B**,
we write **A** ⟺ **B**.
For example, if $\mathrm{A} = \{1, 2, 3\}$ and
$\mathrm{B} = \{a, b, c\}$, then A is equivalent to B since $\mathrm{n(A)} = \mathrm{n(B)} = 3$.

NOTE:

All equal sets are equivalent sets but not all equivalent sets are equal sets.

SUBSETS

Set A is called a subset of a set B **if every member of set A is a member of set B**. This relationship is written as $\mathbf{A} \subseteq \mathbf{B}$ and may also be read as "**A** is contained in **B**". We may also write $\mathbf{A} \subseteq \mathbf{B}$ as $\mathbf{B} \supseteq \mathbf{A}$ and read as "**B** contains **A**", or "**B** is a superset of **A**". If set **A** is not a subset of **B**, we write $\mathbf{A} \not\subseteq \mathbf{B}$.

If $\mathbf{A} \subseteq \mathbf{B}$ and $\mathbf{A} \neq \mathbf{B}$, we sometimes write $\mathbf{A} \subset \mathbf{B}$ and say **A is a proper subset of B**.

In other words, if every element in **A** is an element in **B**, and also **B** has at least 1 other element which is not in **A**, then **A** is called "the proper subset" of **B**.

If **A** is a proper subset of **B**, we write $\mathbf{A} \subset \mathbf{B}$ or $\mathbf{B} \supset \mathbf{A}$.

NOTE:

(i) $\mathrm{A} = \{a, b, c, d\}$ and $\mathrm{B} = \{a, b, c, d\}$ clearly $\mathrm{A} = \mathrm{B}$ and we say A is a subset of B written $\mathrm{A} \subseteq \mathrm{B}$.
(ii) If $\mathrm{A} = \{a, b, c\}$ and $\mathrm{B} = \{a, b, c, d, e\}$ clearly $\mathrm{A} \neq \mathrm{B}$ and we say A is a proper subset of B written $\mathrm{A} \subset \mathrm{B}$.
(iii) Any set is a subset of itself.
(iv) $\emptyset$ i.e. the empty set is a subset of any set.
(v) A finite set with n elements has 2^n subsets.

Exercise 9 Date:………………

Let $P = \{2, 3, 4, 5, 6, 7\}$, $Q = \{2, 4, 7, 8\}$ and $R = \{2, 4\}$. Fill in the blanks by $\subset$ and $\not\subset$ to make the resulting statements true.

(i) $R \ldots\ldots\ldots P$ (iv) $\emptyset \ldots\ldots..Q$
(ii) $Q \ldots\ldots\ldots.R$ (v) $Q \ldots\ldots\ldots Q$
(iii) $Q \ldots\ldots\ldots.P$ (vi) $R \ldots\ldots\ldots Q$

Exercise 10 Date:………………

$A = \{a, b, c, d, e\}$
Which of the following are true?

1. $\{a, b\} \subset A$
2. $\{a, b\} \in A$
3. $\{a, b\} \subset A$
4. $A \supset \{e, a, b\}$
5. $\{1, a, b\} \subset A$
6. $\{\ \} \in A$

Exercise 11 Date:………………

If $\mathrm{U} = \{1, 2, 3, 4, 5, \ldots, 12\}$, $\mathrm{A} = \{2, 3, 4, 7, 9, 11\}$ and $\mathrm{B} = \{4, 11\}$.

(a) Find
(i) $\mathrm{n(U)}$ (ii) $\mathrm{n(A)}$ (iii) $\mathrm{n(B)}$

(b) True or False?
(i) $\mathrm{A} \subseteq \mathrm{U}$ (ii) $\mathrm{A} \subseteq \mathrm{B}$ (iii) $\mathrm{B} \subset \mathrm{A}$

(c) True or False?
(i) $5 \in A$ (ii) $5 \notin B$

(d) If $\mathrm{C} = \left\{4, 7, 11, 9, \frac{x-1}{2}\right\}$ and $\mathrm{A} = \mathrm{C}$, find x.

Exercise 12 **Date:……………….**

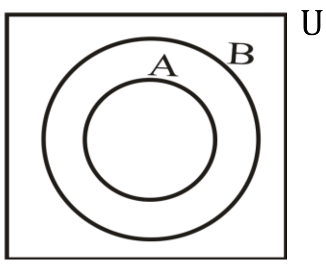
(a) The subsets of $\{a, b\}$ are $\emptyset, \{a\}, \{b\}$ and $\{a, b\}$.
   (i) List all subsets of $\{a\}$.
   (ii) List all subsets of $\{a, b, c\}$.
   (iii) Predict the number of subsets of $\{a, b, c, d\}$ without listing them.

(b) List all proper subset of $A = \{2, 4, 7, 9\}$.

VENN DIAGRAM

A Venn diagram is a way of representing sets visually. These provide useful method of displacing relations between subsets of a given set. The fixed set is called the universal set for the subsets involved. The universal set is composed of all possible elements in the field of discussion. The universal set is represented by interior of a rectangle and subsets of the universal sets are represented by regions. The English mathematician John Venn (1834 – 1923) developed it and was named after him as Venn Diagrams.

In particular, given $A \subset B \subset U$, this can be represented in a Venn Diagram as



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Operation on Sets

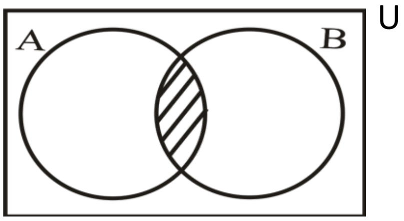
An operation on sets is a rule by which, with one, two or more given sets, we associate a unique set. The three operations on sets are union (∪), intersection (∩) and complement (′)

1. **Intersection of sets**

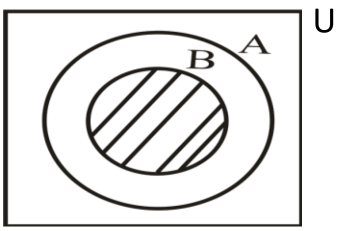
   The intersection of two sets A and B is defined to be the set of all objects that are common to both A and B. It is denoted by A ∩ B which is read as "A intersection B".

   In particular, if
   A $= \{a, b, c, d\}$ and B $= \{a, c, d, f, g\}$
   then
   A ∩ B $= \{a, c, d\}$

The shaded regions show the intersection between sets A and B.



A ∩ B



A ∩ B

Extension to three or more sets

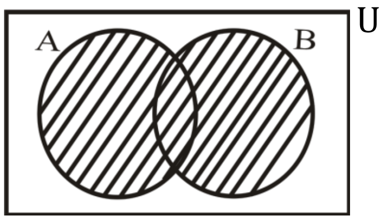
A ∩ B ∩ C is defined to be the set of objects that are common to A, B and C. The order in which the sets appear does not matter.
Also (A ∩ B) ∩ C = A ∩ (B ∩ C). This equation is called the associative property of intersection.

2. **Union of Sets**

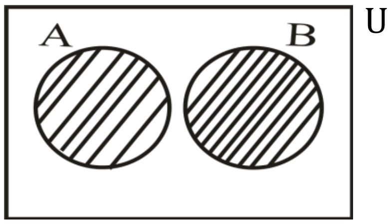
   The union of two sets A, B is defined to be the set consisting of all objects that are members of A or of B or of both. The set is denoted by A ∪ B which is read as "A union B".

   In particular, if
   A = {1, 3, 5, 7, 9} and B = {2, 4, 6, 8, 10}
   then
   A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

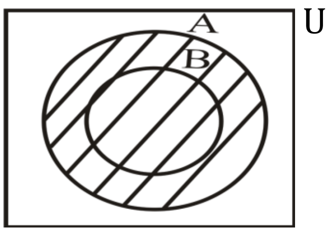
   The shaded regions show the union between the set A and B.



A ∪ B



A ∪ B



A ∪ B

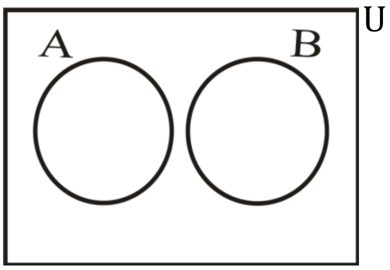
Extension to three or more sets

$A \cup B \cup C$ is defined to be the set consisting of all the objects that are members of at least one of the sets A, B, C. The order in which the sets appear does not matter.

e.g. A ∪ B ∪ C = B ∪ A ∪ C etc.
Also (A ∪ B) ∪ C = A ∪ (B ∪ C). This equation is called the **associative property** of union.

3. Disjoint Sets

When $A \cap B = \emptyset$, i.e. when A, B have no elements common, we say that A, B are disjoint or non-intersecting.
In particular, if $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$. Clearly $A \cap B = \emptyset$ and we say set A and set B are disjoint sets.



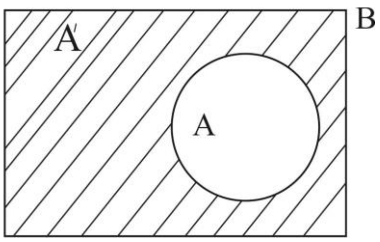
From the Venn diagram above, A and B does not intersect so we say A and B are disjoint sets.

4. Complement of Sets

If $A \subset B$, We define the complement of A with respect to B (or in B) to be the set of all elements of B that are not members of A. It is denoted by A' which is read as "A dashed" or "A prime".

In particular, if $B = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2,\}$ then $A' = \{3, 4, 5\}$.

In the Venn Diagram in Fig. 1 the complement of set A is represented by the shaded region.



Here set B = U

> The complement of a set A is the set of all the elements in the universal set that are not in set A.

Example 2
If $U = \{1, 2, 3, \ldots, 12\}$
$P = \{1, 5, 7, 10\}$, $Q = \{1, 2, 7, 8, 10\}$.
Find the following sets.

(a) P' (e) $P' \cup P$
(b) Q' (f) $Q \cap Q'$
(c) $(P \cup Q)'$ (g) $P \cup Q'$
(d) $(P \cap Q)'$ (h) $P' \cap Q$

Solution…
(a) $P' = \{2, 3, 4, 6, 8, 9, 10, 11, 12\}$

(b) $Q' = \{3, 4, 5, 6, 9, 11, 12\}$

(c) $P \cup Q = \{1, 2, 5, 7, 8, 10\}$
$\therefore (P \cup Q)' = \{3, 4, 6, 9, 11, 12\}$

(d) $P \cap Q = \{1, 7, 10\}$
$\therefore (P \cap Q)' = \{2, 3, 4, 5, 6, 8, 9, 11, 12\}$

(e) $P' \cup P = \{1, 2, 3, 4, 5, 6, 7, 8, 9,10, 11, 12\}$

(f) $Q \cap Q' = \emptyset$

(g) $P \cup Q' = \{1, 3, 4, 5, 6, 7, 9, 10, 11, 12\}$

(h) $P' \cap Q = \{2, 8, 10\}$

POWER SETS

The power set is the set of all possible subsets of an original set. The power set is denoted by $\mathcal{P}(\boldsymbol{A})$. The number of elements in a power set is 2^n, where n is the number of elements in the set.

Example 3
Let $A = \{a, b, c\}$. Find the power set of A.

Solution…
$\mathcal{P}(\boldsymbol{A}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

Exercise 13 **Date:……………….**
If $U = \{1, 2, 3, \ldots, 12\}$, $A = \{2, 3, 4, 7, 9, 11\}$ and $B = \{4, 11\}$.
Find
(i) A' (iv) $A' \cup B'$
(ii) B' (v) $n(A')$
(iii) $A' \cap B'$ (vi) $n(B')$

Exercise 14 **Date:...................**

$A = \{1, 2, 5, 7\}$ and $B = \{1, 3, 6, 7\}$ are subsets of the universal set $U = \{1, 2, 3, \dots, 10\}$.
Find
(i) A' (ii) $(A \cap B)'$ (iii) $(A \cup B)'$

Exercise 15 **Date:...................**

1. Suppose $U = \{2, 3, 4, 5, 6, 7, 8\}$ $A = \{2, 3, 4, 7\}$ and $B = \{2, 5\}$. Find
   (i) $n(U)$
   (ii) $n(A)$
   (iii) $n(A')$
   (iv) $n(B)$
   (v) $n(B')$

2. If $P = \{x: 1 \le x \le 6\}$ and $Q = \{x: 2 < x < 9\}$ where $x \in \mathbb{R}$.
   Find (i) $P \cap Q$ (ii) $P \cup Q$

Exercise 16 **Date:...................**

1. The subset A, B and C of a universal set are defined as follows:
   $A = \{m, a, p, e\}$, $B = \{a, e, i, o, u\}$ and $C = \{l, m, n, o, p, q, r, s, t, u\}$. Find
   (a) $A \cap B$
   (b) $A \cup C$
   (c) $A \cup (B \cap C)$
   (d) $n(A \cup (B \cap C))$

2. The universal set U is the set of all integers P, Q and R are subsets of U defined by $P = \{x: x \leq 2\}$, $Q = \{x: -7 < x < 15\}$, $R = \{x: -2 \leq x < 19\}$. Find
   (a) $P \cap Q$
   (b) $P \cap (Q \cup R')$
   (c) $n(P \cap (Q \cup R'))$

Exercise 17

1. X and Y are subsets of U such that
   $U = \{10, 11, 12, \ldots, 20\}$,
   $X = \{x: 10 \leq x \leq 15\}$ and
   $Y = \{\text{even numbers} < 18\}$.
   Find
   (a) $X \cap Y$
   (b) $X' \cap Y$
   (c) $n(X' \cap Y)$
   (d) $Y \cup (X' \cap Y')$

2. Given that A, B, C are subsets of the universal set μ of real numbers such that:
   $A = \{1, 2, \ldots, 16\}, B = \{x: 0 < x < 16\}$, where x is an odd integer.
   $C = \{p: p < 16\}$, where p is a prime.
   (a) Find $B \cap C$
   (b) Find $(A \cap B)'$

Exercise 18 Date:……………….

1. Given $A = \{2, 4, 6, 8\}$, $B = \{2, 3, 7, 9\}$ and $C = \{x: 3 < x < 9\}$ are subsets of the universal set $U = \{2, 3, 4, 5, 6, 7, 8, 9\}$. Find
   (a) $A \cap (B' \cap C')$
   (b) $(A \cup B) \cap (B \cup C)$

2.
   (a) If $P = \{1, 2, 3, 4\}$, write down all the subsets of P which have exactly two elements.
   (b) $A = \{\text{prime numbers less than } 15\}$
   $B = \{\text{even numbers less than } 15\}$
   $C = \{x: 3 \leq x < 12$, where x is an integer$\}$ are subsets of the universal set $U = \{x: 1 \leq x \leq 14\}$
   List the element of
   (i) $A \cap C$ (iii) $(A \cup B)' \cap C$
   (ii) $B \cap C$

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Exercise 19 **Date:………………..**

1. The sets $A = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$
   $B = \{0 \leq x \leq 9\}$ and
   $C = \{x: -4 < x \leq 0\}$ are subsets of $\mathbb{Z}$, the set of integers.
   (a)
   (i) Describe the members of the set A', where A' is the complement of A.
   (ii) Find $A' \cap B$.

   (b) Represent the sets B and C on a Venn diagram.

2. The set A, B and C are defined as
   $\text{A} = \{1, 3, 5, 7, 9, 11, 13, 15\}$,
   $\text{B} = \{3, 6, 9, 12, 15\}$,
   $\text{C} = \{5, 10, 15, 20, 25\}$
   (a) Draw a Venn diagram to illustrate the above information.

   (b) Find
   (i) $\text{B} \cap \text{C}$
   (ii) $(\text{A} \cup \text{B})' \cap \text{C}$
   (iii) the number of elements in $\text{A} \cup \text{B}$.

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Exercise 20 **Date:..................**

1.
   (a) The universal set $U = \{2, 3, 5, 7\}$, $P = \{2, 5\}$ and $Q = \{5, 7\}$. Find
      (i) $(P \cap Q)'$ (ii) $P' \cup Q'$

   (b) What is the relationship between (i) and (ii).

2.
   (a) If $U = \{1, 2, 3, \dots, 19, 20\}$ and A, B and C are subsets of $\cup$ such that
      $A = \{\text{multiples of five}\}$
      $B = \{\text{multiples of four}\}$
      $C = \{\text{multiples of three}\}$
      List the elements of"
      (i) A (ii) B, (iii) C

   (b) Find:
      (i) $A \cap B$ (ii) $A \cap C$ (iii) $B \cup C$

   (c) Using your results in (b), show that $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$.

3. If S is the set of all positive integers ≤ 8 and A, B and C are the subsets $\{1, 5, 6\}$, $\{2, 3, 5, 7, 8\}$ and $\{1, 3, 6, 8\}$ respectively, of S, find the subsets
   (i) $A \cap B' \cap C$ (ii) $(A \cup C) \cap (B \cup C')$

4. If $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
   $A = \{1, 3, 5\}, B = \{2, 4, 6, 8\}, C = \{2, 5, 10\}$
   and dash $(')$ is used to denote the complement of a subset in S.
   By writing out the elements of the appropriate sets verify that
   (i) $(A \cup B)' = A' \cap B'$
   (ii) $(A \cap B)' = A' \cup B'$
   (iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

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Exercise 21 **Date:……………….**

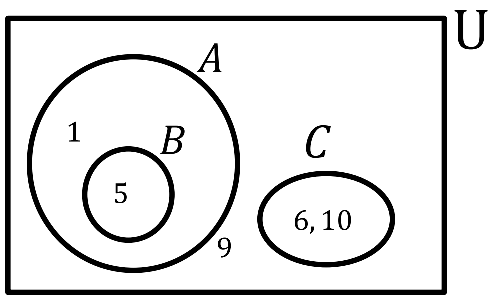
1. If $P = \{1, 2,3\ , 4\}$ and $Q = \{3, 5, 6\}$. Find
   (i) $P \cap Q$
   (ii) $P \cup Q$
   (iii) $(P \cap Q) \cup Q$
   (iv) $(P \cup Q) \cap P$

2. Given that the universal set
   $\text{U} = \{1, 2,3\ 4, \dots, 10\}$,
   $P = \{1, 2, 3, 4, 6, 10\}$ and $Q = \{2, 3, 6, 9\}$,
   show that $(P \cup Q)' = P' \cap Q'$.

3. $A = \{1, 2, 3, 4, 6\}$ and $B = \{1, 2, 3, 6, 9, 18\}$ are subsets of the universal set $\mu = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$. List the elements of
   (i) $A' \cap B$ (ii) $(A' \cap B)'$

4. The sets $P = \{\text{multiples of } 3\}$, $Q = \{\text{factors of } 72\}$ and $R = \{\text{even numbers}\}$ are subsets of $\text{U} = \{10 \leq x \leq 36\}$.
   (a) List the elements of P, Q and R
   (b) Find
      (i) $P \cap Q$
      (ii) $Q \cap R$
      (iii) $P \cap R$
   (c) What is the relationship between $P \cap Q$ and $Q \cap R$?

5. $\text{U} = \{1, 2, 3, \dots, 10\}$, $\text{A} = \{1, 2, 3, 4, 5\}$, $\text{B} = \{2, 3, 5\}$ and $\text{C} = \{6, 8, 10\}$.
   i) Given that the Venn diagram represents the sets above, copy and fill in the elements.



   ii) Find $A \cap C$
   iii) Find $A \cap B'$

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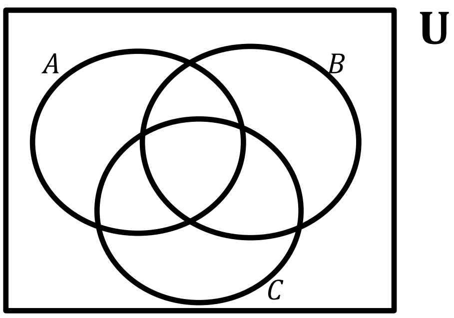
Exercise 22 **Date:……………….**

1. The universal set U is the set of all integers and subsets P, Q, R of U are given by
   $P = \{x: x \leq 0\}$,
   $Q = \{\ldots, 5, -3, -1, 1, 3, 5, \ldots\}$,
   $R = \{x: -2 \leq x \leq 7\}$
   (a) Find $Q \cap R$
   (b) Find R' where R' is the complement of R with respect to U.
   (c) Find $P' \cap R'$
   (d) List the members of $(P \cap Q)'$.

2. The universal set U is the set of all integers. A, B and C are subsets of U defined as follows
   $A = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$,
   $B = \{x: 0 \leq x \leq 9\}$
   $C = \{x: -4 \leq x \leq 10\}$
   (a) Write down set A', where A' is the complement of A with respect to U.
   (b) Find $B \cap C$
   (c) Find the members of the sets $B \cup C$, $A \cap B$ and $B \cap C$ and hence show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. A, B and C are subsets of the universal set U such that $U = \{0, 1, 2, 3, \ldots, 12\}$;
   $A = \{x: 0 \leq x \leq 7\}$
   $B = \{4, 6, 8, 10, 12\}$
   $C = \{1 < y < 8\}$, where y is a prime number.
   (a) Draw a Venn diagram to illustrate the information given above.
   (b) Find
   (i) $(B \cup C)'$ (ii) $A' \cap B \cap C$

4.



The Venn diagram represents three subsets A, B and C of the universal set U. Copy the Venn diagram, shade and indicate the regions respectively.
(i) $A \cap B' \cap C$ (ii) $A' \cap B \cap C'$

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Exercise 23 **Date:………………**

1. If $A = \{x \in \mathbb{R}: 0 < x \le 3\}$ and $B = \{x \in \mathbb{R}: x \le 1 \text{ or } x > 4\}$. Express in simplest form.
   (i) $A \cap B$ (iii) $A \cup B$

2.
   (a) If $A = \{x \in \mathbb{R}: 0 < x < 2\}$ and $B = \{x \in \mathbb{R}: 1 \le x < 4\}$, find, in simplest form,
   (i) $A \cap B$ (ii) $A' \cap B$ (iii) $A' \cup B'$

   (b) If $C = \{x \in \mathbb{R}: 0 \le x \le 2 \text{ or } x \ge 3\}$, find, in simplest form $(A' \cup B') \cap C$.

3. The sets $\mathrm{A} = \{1, 3, 5, 7, 9, 11\}$, $\mathrm{B} = \{2, 3, 5, 7, 11, 15\}$, $\mathrm{C} = \{3, 6, 9, 12, 15\}$ are subsets of $\mathrm{U} = \{1, 2, 3, \dots, 15\}$.
   (a) Draw a Venn diagram to illustrate the given information.
   (b) Use your diagram to find
   i) $C \cap A'$ ii) $A' \cap (B \cup C)$

4.
   (α) $\mathrm{A} = \{1, 2, 5, 7\}$ and $\mathrm{B} = \{1, 3, 6, 7\}$ are subsets of the universal set $\mathrm{U} = \{1, 2, 3, \dots, 10\}$. Find
   i) A'
   ii) $(A \cap B)'$
   iii) $(A \cup B)'$
   iv) the subsets of B each of which has
   (a) two elements
   (b) three elements.

5. List the elements of the sets
   (i) $\{x \in \mathbb{R}: x^3 = x\} \cap \{x \in \mathbb{R}: x^3 + 3x^2 + 2x = 0\}$

   (ii) $\{x \in \mathbb{Q}: x^2 < 1\} \cap \{x \in \mathbb{Q}: 3x \in \mathbb{Z}\}$

   (iii) $\{x \in \mathbb{Z}: x^2 < 50\} \cap \left\{x \in \mathbb{Z}: \frac{1}{2}(x - 1) \in \mathbb{Z}\right\}$

6. The universal set U is defined as follows:
   $\mathrm{U} = \{x: x \in \mathbb{N}, 2 < x < 12\}$. The set M and R are subsets of U such that
   M = {odd numbers}
   R = {square numbers}
   (i) Find the number of the subset of M
   (ii) List the members of the subset of R
   (iii) Draw a Venn diagram that represents the relationship among the defined subsets of U

7. From the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, list the following subsets:
   (i) The subset of even numbers
   (ii) The subset of prime numbers
   (iii) The complement of the subset of perfect squares
   (iv) The subset of members of the form $4n - 1, n \in \mathbb{N}$
   (v) The subset consisting of numbers that are square roots of members of the set.

8.
   (a) Determine which of the following sets are equal to the empty set:
   (i) $\{x \in \mathbb{R}: x^2 = 9 \wedge 2x = 4\}$
   (ii) $\{x \in \mathbb{R}: x \ne x\}$
   (iii) $\{x \in \mathbb{R}: x + 2 = 2\}$
   (iv) $\{x \in \mathbb{R}: 1 < x < 2\}$
   (v) $\{x \in \mathbb{Z}: 1 < x < 2\}$

   (b) Given that $\mathrm{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 20, 30, 40, 60\}$ and $A = \{\text{factors of } 20\}$, $B = \{\text{factors of } 40\}$ and $C = \{\text{factors of } 100\}$ are subsets of U.
   (α) Draw a Venn diagram to illustrate the above information.

   (β) Use your Venn diagram to find the members of the following sets:
   (i) $A \cap B \cap C$
   (ii) $A \cap B'$
   (iii) $A' \cap B \cap C$
   (iv) $(A' \cap C) \cap (A \cup B)$

   (γ) State $n(A \cup B)'$.

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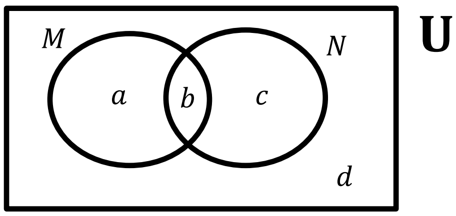
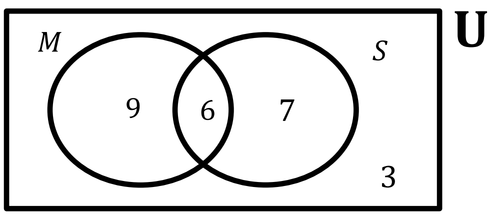
PROBLEM – SOLVING USING VENN DIAGRAMS
TWO SET PROBLEMS



$\text{n(M)} = a + b$
$\text{n(M only)} = a$
$\text{n(N)} = b + c$
$\text{n(N only)} = c$
$\text{n}(\text{M} \cap \text{N}) = b$
$\text{n}(\text{M} \cup \text{N})' = d$
$\text{n(U)} = a + b + c + d$
For any two intersecting sets
$\text{n(U)} = \text{n(M)} + \text{n(N)} - \text{n}(\text{M} \cap \text{N}) + \text{n}(\text{M} \cup \text{N})'$.

Example 4
The Venn diagram show the results of an interview of students of Stevkon's Junior High School.
M = {students who like Mathematics} and
N = {students who like Science}.



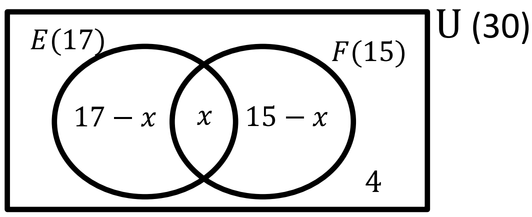
(a) How many students were interviewed?
(b) How many students like Mathematics?
(c) How many students like Science?
(d) How many students like both Mathematics and Science?
(e) How many students like only one subject?
(f) How many students like Mathematics only?
(g) How many students like at least one subject?
(h) How many students like none of the two subjects?

Solution…
(a) The number of students interviewed $= 9 + 6 + 7 + 3 = 25$
(b) The number of students that like Mathematics $= 9 + 6 = 15$
(c) The number of students that like Science $= 6 + 7 = 13$
(d) The number of students that like both Mathematics and Science $= 6$
(e) The number of students that like only one subject $= 9 + 7 = 16$
(f) The number of students who like Mathematics only $= 9$
(g) The number of students that like at least one subject $= 9 + 6 + 7 = 22$
(h) The number of students who like none of the two subjects $= 3$

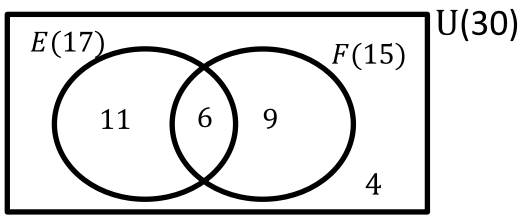
Example 5
Out of 30 students applying for the position of a school prefect, 17 offer English, 15 offer French and 4 offer neither English nor French. How many of them offer
(i) Only English?
(ii) Both English and French?

Solution…
Let U = {students}
E = {students who offer English}
F = {students who offer French}

$\text{n(U)} = 30$ $\quad$ $\text{n}(\text{E} \cup \text{F})' = 4$
$\text{n(E)} = 17$ $\quad$ $\text{n}(\text{E} \cap \text{F}) = x$
$\text{n(F)} = 15$

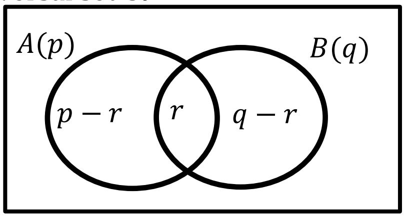


$$\text{n(U)} = 30$$
$$\Rightarrow 17 - x + x + 15 - x + 4 = 30$$
$$36 - x = 30$$
$$-x = 30 - 36$$
$$-x = -6$$
$$\therefore x = 6$$



(i) 11 students offer only English
(ii) 6 students offer both English and French.

Now consider two finite sets A and B, in a universal set U.



Let $\mathrm{n}(A)=p$
$\mathrm{n}(B)=q$
$\mathrm{n}(A\cap B)=r$

From the Venn diagram, we have

$$\begin{aligned}\mathrm{n}(A\cup B)&=(p-r)+r+(q-r)\\&=p+q-r\\&=\mathrm{n}(\mathrm{A})+\mathrm{n}(\mathrm{B})-\mathrm{n}(\mathrm{A}\cap\mathrm{B})\end{aligned}$$

Hence

$$\boxed{\mathrm{n}(\mathrm{A}\cup\mathrm{B})=\mathrm{n}(\mathrm{A})+\mathrm{n}(\mathrm{B})-\mathrm{n}(\mathrm{A}\cap\mathrm{B})}$$

Example 6

A and B are subsets of a universal set U and $\mathrm{n}(\mathrm{A})=23$, $n(B)=14$, $\mathrm{n}(\mathrm{A}\cup\mathrm{B})=29$. Find $\mathrm{n}(\mathrm{A}\cap\mathrm{B})$.

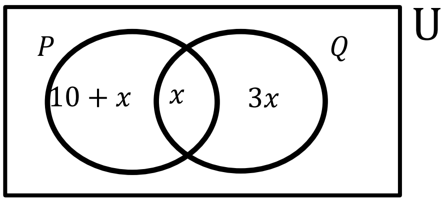
Solution…

$$\begin{aligned}\mathrm{n}(\mathrm{A}\cup\mathrm{B})&=\mathrm{n}(\mathrm{A})+\mathrm{n}(\mathrm{B})-\mathrm{n}(\mathrm{A}\cap\mathrm{B})\\32&=23+14-\mathrm{n}(\mathrm{A}\cap\mathrm{B})\\\mathrm{n}(\mathrm{A}\cap\mathrm{B})&=23+14-32\\&=5\end{aligned}$$

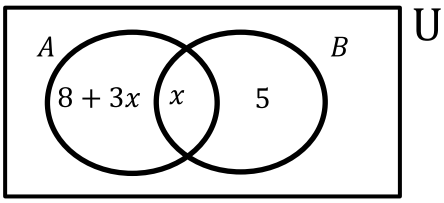
Exercise 24 **Date:………………..**

1. M and N are two intersecting sets. If $\mathrm{n}(\mathrm{M})=20$, $\mathrm{n}(\mathrm{N})=30$ and $\mathrm{n}(\mathrm{M}\cup\mathrm{N})=40$. Find $\mathrm{n}(\mathrm{M}\cap\mathrm{N})$.

2. In the Venn diagram, $\mathrm{n}(\mathrm{P})=\mathrm{n}(\mathrm{Q})$. Find the value of x.

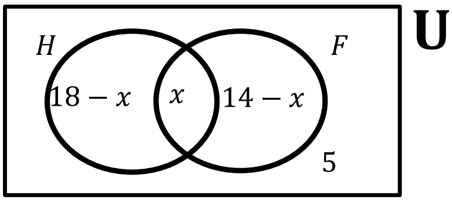


3. In the Venn diagram below, $\mathrm{n}(\mathrm{A})=2\mathrm{n}(\mathrm{B})$.



Find
i) x ii) $\mathrm{n}(B')$

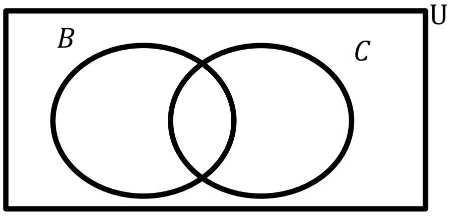
4. The Venn diagram below shows the number of students who study History and French in a class of 30 students.
U = {students in the class}
H = {students who study History}
F = {students who study French}



(i) Write an express, in x in its simplest form, for the total number of students in class.
(ii) State whether the following relations are True or False.
- $H\cup F=\mathrm{U}$
- $H\cap F'=\emptyset$

(iii) Determine the number of students who study both History and French.

5. In a survey of 39 students, it was found that
18 can ride a bicycle
15 can drive a car
x can ride a bicycle and drive a car
$3x$ can do neither
B is set of students in the survey who can ride a bicycle, and C the set of students who can drive a car.
(i) Copy and complete the Venn diagram to represent the information.



(ii) Write an expression in x for the number of students in the survey.
(iii) Calculate the value of x.
(iv) Find $n(B')$.

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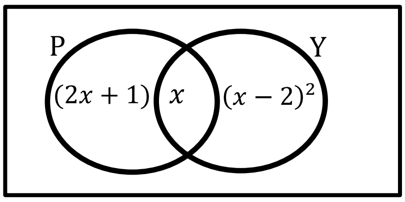
Exercise 25 **Date:……………….**

1. In a class of 50 students, 30 offer Economics, 17 offer Government and 7 offer neither Economics nor Government. How many students offer both subjects?

2. In a class of 31 students, 16 play football, 12 play table – tennis and 5 play both games. Find the number of students who play
   (a) only one of the games
   (b) at least one of the games
   (c) none of the games

3. In a class, the number of students studying French or History is 40. Twenty study both subjects and the number who study French is 10 more than the number who study History.
   (a) How many study French?
   (b) How many study History?

4. In a hotel breakfast menu is a choice between yam (Y) or plantain (P) or both.



   The Venn diagram shows the choices made by 25 guests of the Hotel.
   (a) Find the value of x
   (b) What is the probability that a guest chosen at random chose only one of the two?

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Exercise 26 **Date:……………….**

1. In a class of 45 students, 32 offered Physics (P), 28 offered Government (G) and 12 did not offer any two subjects.
   (a) Draw a Venn diagram to represent the information.
   (b) How many students offered both Physics and Government?
   (c) What is n($P \cup G$)?

2. Out of 30 candidates applying for a post, 17 have degrees, 15 have diplomas and 4 neither degree nor diploma. How many of them have both?

3. A number of tourists were interviewed on their choice of means of travel. Two – thirds said that they travel by road, $\frac{13}{30}$ by air and $\frac{4}{15}$ by both road and air. If 20 tourists did not travel by either air or road,
   (i) Represent the information on a Venn diagram;
   (ii) How many tourists
      (α) were interviewed?
      (β) travelled by air only?

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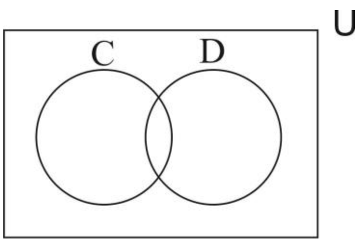
Exercise 27 **Date:……………….**

1. Sets A and B are such that $n(A) = 11$, $n(B) = 13$ and $n(A \cup B) = 18$. Find $n(A \cap B)$

2. In a class of 30 students, 17 are studying politics, 14 are studying Economics and 10 are studying both of these subjects.
   Illustrate this information using a Venn diagram.
   Find the number of student studying
   (a) neither of these subjects
   (b) exactly one of these subjects.

3. A group of 40 students were asked whether they like Mathematics (M) or French (F). The number liking both Mathematics and French was three times the number liking only Mathematics. Adding 3 to the number liking only Mathematics and doubling the answer equals the number of students liking only French. 4 said they did not like any of the subjects.
   (a) Draw a Venn diagram to represent this information.
   (b) Calculate $n(M \cap F)$
   (c) Calculate $n(M \cap F')$
   (d) Calculate $n(M' \cap F)$

4. A survey of the reading habits of 130 students showed that 30 read both Comics and Novels, 10 read neither Comics nor Novels and twice as many students read Comics as read Novels.
   (a) How many students read Novels?
   (b) How many read Comics?
   (c) How many read only Comics?

5. 

   The Venn diagram above represents the sets

   U = {homes in a certain town}
   C = { homes with computer}
   D = {homes with a dish washer}
   It is given that
   $n(C \cap D) = k$
   $n(C) = 7 \times n(C \cap D)$
   $n(D) = 4 \times n(C \cap D)$ and
   $n(U) = 6 \times n(C' \cap D')$

   i) Copy the Venn diagram above and insert, in each of its four regions, the number, in terms of k, of homes represented by that region.

   ii) Given that there are 16500 homes which do not have both a computer and a dish washer, calculate the number of homes in the town.

6. The Universal set U and sets P and Q are such that $n(U) = 15$, $n(P) = 13$ and $n(P \cap Q) = 4$, find
   (i) $n(Q)$
   (ii) $n((P \cup Q)')$
   (iii) $n(P \cap Q')$

7. Out of 120 customers in a shop, 45 bought both bags and shoes. If 11 customers bought shoes than bags,
   (a) Illustrate this information on a Venn diagram.
   (b) Find the number of customers who bought shoes.
   (c) Find the probability that a customer bought bags.

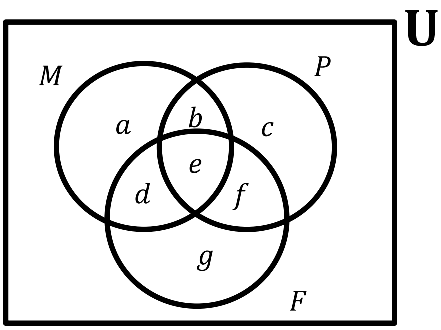
8. Given U = {students in class}
   F = {students who like football}
   B = {students who like basketball}
   $n(U) = 24$
   $n(F \cap B) = 12$
   $n(F \text{ only}) = 7$
   $n(B \text{ only}) = 2$
   (i) Draw a Venn diagram to represents this information.
   (ii) How many students do not like either sport?
   (iii) Find the value of $n(F \cup B)$
   (iv) Find the value of $n(F' \cap B)$
   (v) A students from the class is selected at random. What is the probability that this student likes basketball?
   (vi) A student who likes football is selected at random. What is the probability that this student likes basketball?

9. A team of 24 swimmers took part in a competition. 15 competed in free style, 11 competed in back stroke, and 6 competed in both of these strokes. Display this information on a Venn diagram, and hence determine the number of swimmers who competed in:
   (a) back stroke but not free style
   (b) at least one of these strokes
   (c) freestyle but not backstroke
   (d) neither stroke
   (e) exactly one of these strokes.

THREE – SET PROBLEMS

Example 7

The Venn diagram below shows the result of interviewing some students in a certain school to as which subject they like.
$M = $ {students who like Mathematics}
$P = $ {students who like Physics}
$F = $ {students who like French}



(a) How many students were interviewed?
(b) How many students like French?
(c) How many students like Mathematics only?
(d) How many students like Mathematics and French?
(e) How many students like Mathematics and French only?
(f) How many students like only one subject?
(g) How many students like only two subjects?
(h) How many students like all three subjects?
(i) How many students like at least two subjects?
(j) How many students like Mathematics or French but not Physics?
(k) How many students like Mathematics or Physics?

Solution…

(a) The number of students interviewed $= a + b + c + d + e + f + g$

(b) The number of students who like French $= d + e + f + g$

(c) The number of students who like Mathematics only $= a$

(d) The number of students who like Mathematics and French $= a + b + d + e + f + g$

(e) The number of students who like Mathematics and French only $= d$

(f) The number of students who like only one subject $= a + c + g$

(g) The number of students who like only two subjects $= d + b + f$

(h) The number of students who like all three subjects $= e$

(i) The number of students who like at least two subjects $= b + d + f + e$

(j) The number of students who like Mathematics or French but not Physics $= a + d + g$

(k) The number of students who like Mathematics or Physics $= a + b + e + d + c + f$

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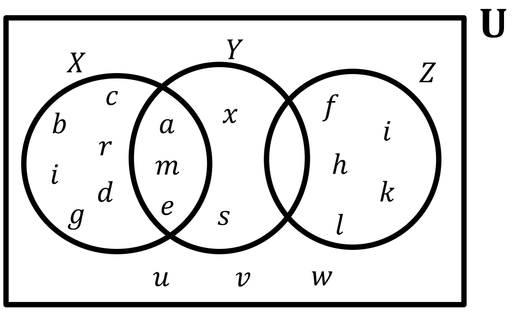
Exercise 28 **Date:………………..**

1.



(a) Use set notation to complete the statements for the Venn diagram above

(i) c … … … … … . X

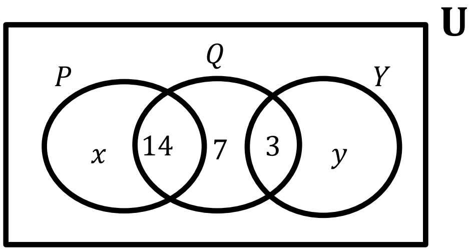
(ii) … … … = $\{a, m, e\}$

(iii) Y ∩ Z = …………………..

(b) List the elements of $(X \cup Y \cup Z)'$

(c) Find $n(X' \cap Z)$

2. P, Q and R are sets such that $U = P \cup Q \cup R$

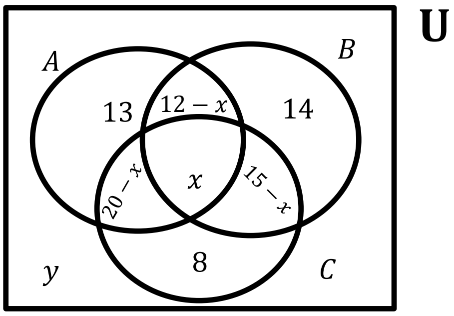


(i) Find x, given that $n(P) = n(Q)$

(ii) Find y, given that $n((P \cup Q)') = n(P \cap Q)$

(iii) Find $n(Q)$

3.



The Venn diagram shows the number of elements in sets A, B and C.

(a) $n(A \cup B \cup C) = 74$
Find x.

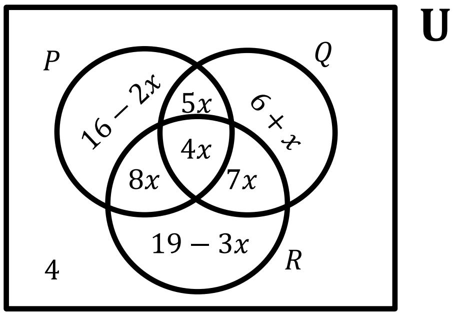
(b) $n(U) = 100$
Find y.

(c) Find the value of $n((A \cup B)' \cap C)$.

4. In the Venn diagram, P, Q and R are subsets of the universal set U. If $n(U) = 125$, find;

(i) the value of x

(ii) $n(P \cup Q \cap R')$



5. If P, Q and R are sets such that $n(P) = 20$, $n(Q) = 16$, $n(R) = 21$, $n(P \cap Q) = 7$, $n(P \cap R) = 8$, $n(Q \cap R) = 5$ and $n(P \cap Q \cap R) = 3$.

(a) Represent this information on a Venn diagram;

(b) Find;

(i) $n(P \cup Q \cup R)$

(ii) The probability of $((P \cup Q)' \cap R)$

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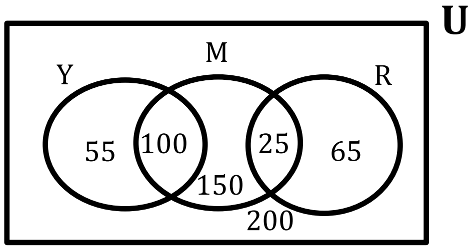
Exercise 29 **Date:………………..**

1. There are 30 students in a class. 20 of them play football, 16 play hockey and 16 play volley, 9 play all three games, 15 play football and volley, 11 play football and hockey, while 10 play hockey and volley.
   (a) Illustrate the information on a Venn diagram.
   (b) Using your Venn diagram, find the number of students who play **at least** two games.
   (c) What is the probability that a student chosen at random from the class does **not** play any of the three games?

2. In a class of 32 students, 18 offer Chemistry, 16 offer Physics and 22 offer Mathematics, 6 offer all three subjects, 3 offer Chemistry and Physics only and 5 offer Physics only. Each student offers at least one subject. Find the number of students who offer
   (a) Chemistry only;
   (b) only one subject;
   (c) only two subjects.
   (d) at least two subjects.

3. The number of farmers growing rice, maize and yams is illustrated in the Venn diagram below.



U = {farmers}
R = {rice farmers}
M = {maize farmers}
Y = {yam farmers}

   (a) How many farmers grow only one crop?
   (b) How many grow exactly two crops?
   (c) How many grow all three crops?
   (d) How many grow only yam or only maize?
   (e) How many grow maize but not rice?
   (f) How many grow yam and rice?
   (g) How many grow rice and maize not yam?
   (h) How many grow at least one of the three crops?

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Exercise 30 **Date:………………..**

1. Mathematics, English and Life Skills books were distributed to 50 students in a class. 22 had Mathematics books, 21 English books and 25 Life Skills books, 7 had Mathematics and English books, 6 Mathematics and Life Skills books and 9 English and Life Skills books. Find the number of students who had
   (a) all three books;
   (b) exactly two of the books
   (c) only Life Skills books

2. 100 members of a community were asked to state the activities they undertake during the day.
   38 go to School.
   54 go for Fishing.
   50 engage in Trading.
   10 go to School and also Fish.
   18 go to School and also Trade.
   22 go for Fishing and also Trade.
   Each of these members undertakes at least one of the activities. The number of people who go to School only is the same as the number who engage in Trading only.
   Use the information to find the number of people who
   (a) undertake all three activities;
   (b) go to School only.
   (c) exactly two of the activities.
   (d) at least two of the activities.

3. In an examination each of 1000 students sat for Biology, Chemistry and Physics. All passed at least one of the three subjects. 600 pass Biology, 500 passed Chemistry and 290 passed Physics. 175 passed both Biology and Chemistry, 150 passed both Biology and Physics, and 120 passed both Chemistry and Physics.
   (a) How many students passed exactly one subject?
   (b) How many passed exactly two subjects?
   (c) How many passed all three subjects?

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Exercise 31 **Date:………………..**

1. In an examination, 60 candidates passed Integrated Science or Mathematics. If 15 passed both subjects and 9 more passed Mathematics than Integrated Science, find the:
   (a) number of candidates who passed in each subject;
   (b) probability that a candidate passed exactly one subject.

2. In a class of 50 students, 24 like football, 21 basketball and 18 cricket. Six like football and basketball only, 3 like basketball only, 5 like all the three games and 14 did **not** like any of the three games.
   (i) Illustrate this information on a Venn diagram.
   (ii) Find the number of students who like:
      (a) football and cricket **only**;
      (b) exactly **one** of the games;
      (c) at least two of the games;
      (d) exactly two of the games.

3. Some students were interviewed to find out which of the following three sports they liked; football, boxing and volleyball. 70% of the students liked football, 60% boxing and 45% volleyball. 45% liked football and boxing, 15% boxing and volleyball, 25% football and volleyball and 5% liked all three sports.
   (a) Draw a Venn diagram to illustrate this information.
   (b) Use your diagram to find the percentage of students who liked
      (i) Football and boxing but not volleyball
      (ii) Exactly two sports
      (iii) None of the three sports

4. In a survey, 74 out of 88 tourist interviewed said they had visited at least one of Africa (A), Europe (E) and South America (S). Of these, 19 had visited Europe and Africa, 30 Europe and South America, 26 South America and Africa. No one had visited only Africa, 10 only Europe, 7 only South America and x had visited all the three continents.
   (a) Draw a Venn diagram to represent this information.

(b) Write down a suitable equation in x and find the value of x.

(c) Find

(i) $n[(E \cap A) \cup S]$

(ii) $n[(E \cup A)' \cap A]$

(iii) $n(A)$

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Exercise 32 **Date:………………..**

1. A group of 50 girls were asked which of the three colours red, yellow and green they liked. 5 of them said they liked all three colours, 25 liked red and 22 liked green. 15 liked red and yellow, 12 liked red and green. 4 liked only yellow and 2 liked only green.
   (a) Illustrate the information on a Venn diagram.
   (b) How many girls did **not** like any of the three colours?

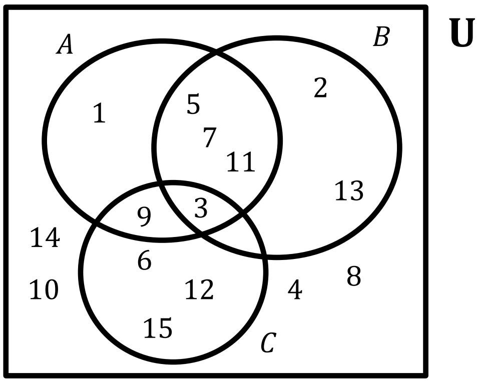
2. A survey of 150 traders in a market shows that 90 of them sell cassava, 70 sell maize and 80 sell yam. Also, 26 sell cassava and maize, 30 sell cassava and yam and 40 sell yam and maize. Each of the traders sells at least one of these crops.
   (a) Represent the information on a Venn diagram.
   (b) Find the number of traders who sell all the three food crops.
   (c) How many of the traders sell one food crop only?

3. Out of 95 travelers interviewed, 7 traveled by bus and train only, 3 by train and car only and 8 traveled by all three mans of transport. The number, x, of travelers who traveled by bus only, was equal to the number who traveled by bus and car only. If 47 people traveled by bus and 30 by train:
   (i) Represent this information in a Venn diagram;
   (ii) Calculate the
      (α) value of x
      (β) number who traveled by at least two means.

4. In a certain class, 22 pupils take one or more of Chemistry, Economics and Government. 12 take Economics (E), 8 take Government (G) and 7 take Chemistry (C). Nobody takes Economics and Chemistry and 4 pupils take Economics and Government.
   (a)
      (i) Using set notation and the letters indicated above, write down the two statements in the last sentence.
      (ii) Draw a Venn diagram to illustrate the information.
   (b) How many pupils take
      (i) Both Chemistry and Government?
      (ii) Government only?

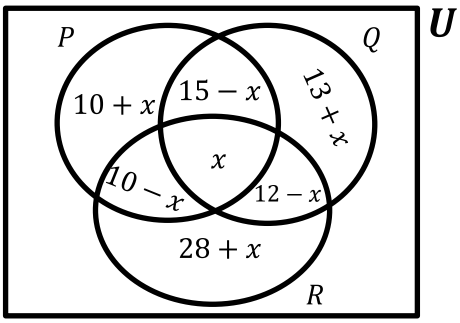
5. Out of 40 customers in a shop, 25 bought plantain, 16 bought yam and 21 bought corn. Each of the customers bought **at least** one of the three items. Eight bought **both** plantain and yam, 11 bought plantain and corn and 6 bought yam and corn.
   (a)
      (i) Represent the information on a Venn diagram.
      (ii) How many customers bought all the **three** items?
   (b) What is the probability that a customer selected at random bough
      (i) **either** plantain **or** corn only?
      (ii) **at least** two items?

6.



   (i) List the elements of $(A \cap B) \cup C$
   (ii) What is $n[(A \cup B)' \cap C]$

7.



If $n(P \cup Q \cup R) = 93$, calculate:
(a) x
(b) $n(P)$
(f) $n(Q \cap R)$
(g) $n(P \cap R)$

(c) $n(Q)$
(d) $n(R)$
(e) $n(P \cap Q)$
(h) $n(R \cup Q)$
(i) $n(P \cap Q)'$

Exercise 33 **Date:………………..**

1. If $P = \{n^2 + 1 : n = 0, 2, 3\}$ and $Q = \{n + 1 : n = 2, 3, 5\}$, find
   (i) $n(P)$
   (ii) $n(Q)$
   (iii) $P \cup Q$
   (iv) $n(P \cap Q)$

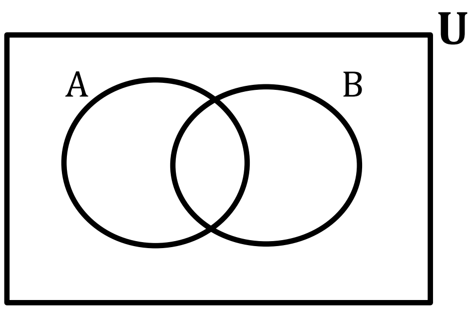
2. From a Universal set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, three subsets defined:
   $A = \{x : x \text{ is divisible by } 3\}$
   $B = \{x : x \text{ is a factor of } 18\}$
   $C = \{x : x \text{ is even and not divisible by } 5\}$
   Draw a Venn diagram to show the Universal set and the three subsets.

3. If $A = \{x : -10 < 2(x - 2) \leq 10\}$ and $B = \{x : -14 \leq 3x - 2 < 4\}$ are subset of $U = \{-5, -4, -3, -2, \ldots, 10\}$, find $n(A' \cap B)$.

4. A, B and C are subsets of the same universal set.
   (i) Write each of the following statements in words.
   (a) $A \not\subset B$
   (b) $A \cap C = \emptyset$

   (ii) Write each of the following statements in set notation.
   (a) There are three elements in the set A or B or both.
   (b) x is an element of A but it is not an element of C.

5.
   (a) On the Venn diagram below, shade the region that represents $A \cap B'$.



   (b) The universal set U and sets P, Q and R are such that
   $(P \cup Q \cup R) = \emptyset$ $P' \cap (Q \cap R) = \emptyset$
   $n(Q \cap R) = 8$ $n(P \cap R) = 8$
   $n(P) = 21$ $n(Q) = 15$
   $n(P \cap Q) = 10$
   $n(U) = 30$

   Compute the Venn diagram to show this information and state the value of $n(R)$.

6.
   (a) Illustrate the statements $A \subset B$ and $B \subset C$ using a Venn diagram.

   (b) It is given that the elements of set U are the letters of the alphabet, the elements of set P are the letters in the word **MATHS**, the elements of set Q are the letters in the word **EXAM**.
   (i) Write the following using set notation.
   The letter h is in the word **MATHS**.
   (ii) Write the following using notation.
   The number of letters occurring in both of the words **MATHS** and **EXAM** is two.
   (iii) List the elements of the set $P \cap Q'$.

7.
   (a) Let D(n) denotes the set of all factors of the natural numbers n. For example $D(8) = \{1, 2, 4, 8\}$.
   (i) List the elements of D(12), D(15), $D(12) \cup D(15)$.
   (ii) State the least value of r such that $D(12) \cup D(15) = D(r)$.

   (b) If F(n) denotes the set of factors of the natural number, including n but excluding 1, find a number p such that $F(12) \cap F(18) = F(p)$.

8. $U = \{p, q, r, s, t, u, v, w\}$
   $A = \{r, t, v, w\}$
   $B = \{q, r, s, u, v\}$
   $C = \{q, s, u\}$
   Find
   (i) $n(A \cap B)$
   (iv) $A \cap B'$

(ii) $n(B \cup C)$ (v) $A \cap (B \cap C')$
(iii) $(A \cup B)'$

9. $U = \{x: 1 \leq x \leq 30\}$
$A = \{x: x \text{ is a multiple of } 4\}$
$B = \{x: x \text{ is a multiple of } 3\}$
$C = \{x: x \text{ is a multiple of } 12\}$
(a) List the elements of the set A
(b) Find $n(A \cap B')$
(c) Write down in set notation an equation involving the three sets A, B and C.

10.
(a) If R = {rhombuses} and P = {parallelograms}, simplify $R \cap P$.
(b) The sets C and D are such that $n(C \cup D) = 44$, $n(C \cap D) = 11$ and $n(C) = 31$. Find the value of $n(D)$.

11. If $U = \{x: \text{is an integer and } 1 \leq x \leq 10\}$, A = {prime numbers}, B = {even numbers} and C = {multiples of 3}, list members of the sets
(i) $A \cap B$
(ii) $A \cup B$
(iii) $(A \cap C') \cup B'$

12.
(α) $U = \{x: x \text{ is an integer}, 1 \leq x \leq 100\}$
$A = \{x: x \text{ is divisible by } 7\}$
$B = \{x: x \text{ is divisible by } 14\}$
$C = \{x: x \text{ is divisible by } 21\}$
(i) Find
(a) $n(A)$ (b) $n(B \cap C)$

(ii) Represent the three sets on a clearly labelled Venn diagram.

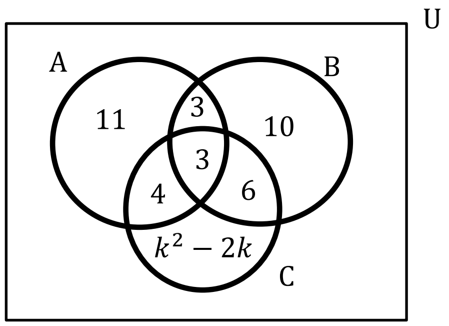
(β) $U = \{1, 2, 3, 4, 5, 6\}$
The sets A, B and C each contains two elements and $A \cup B \cup C = U$. Given that $(A \cup B)' = \{1, 2\}$, write down;
(i) the set C;
(ii) a possible set A and the corresponding set B.

13. $U = \{x: x \text{ is an integer. } 10 \leq x \leq 100\}$
$A = \{x: x + 7 < 57 < x + 41\}$
$B = \{x: \sqrt{x} \text{ is a positive integer}\}$
$C = \{x: x \text{ is a multiple of } 12\}$
(a) Find
(i) $n(A)$ (iii) $n(A \cup C)$
(ii) $n(B')$

(b) List the elements of
(i) $A \cap B$ (iii) $B \cap (A \cup C)'$
(ii) $A' \cap C$

(c) List the elements of x such that $x \in B \cup C$ and $x \notin A$.

14. A, B and C are the three sets and the number of elements are as shown in the Venn diagram below.



The universal set $U = A \cup B \cup C$
(a) State the value of $n(B \cup C)'$
(b) If $x \in (A \cup B) \cap C$, find the probability that $x \in A$
(c) If $n(C) = n(A)$, find the two possible values of k.

15.
(a) $U = \{x: x \text{ is an integer and } 30 \leq x \leq 100\}$
$A = \{x: x \text{ is divisible by } 3\}$
$B = \{x: x \text{ is a perfect square}\}$
$C = \{x: \text{units digit of } x \text{ is } 7\}$
Find
(i) $A \cap B$ (iii) $n(B \cap C)$
(ii) $n(A \cap C)$

(b) Given that
$U = \{p, q, r, s, t, u, v\}$
$A = \{p, q, r, s\}$
$B = \{r, t, u, v\}$
$C = \{r, s, u, v\}$
(α) Find $n(A \cap C)$
(β) List the elements of
(i) $(B \cup C)'$
(ii) $(A \cup C) \cap B$

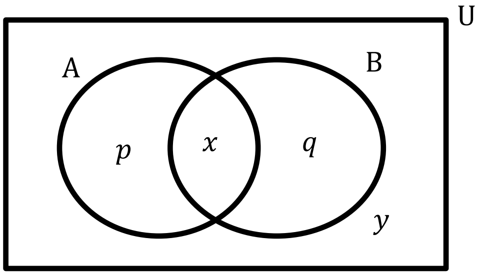
16. Construct a Venn diagram, illustrating the following sets:
(a) $U = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}$

A = $\{a, c, d, e, g, f, h\}$
B = $\{b, c, e, f, i, j, m, n\}$
C = $\{c, g, h, i, j\}$

(b) U = $\{x: x$ is a natiral number, $2 < x \leq 12\}$
P = $\{x: x$ is an even number$\}$
Q = $\{x: x$ is a multiple of $3\}$
R = $\{x: x$ is a prime number$\}$

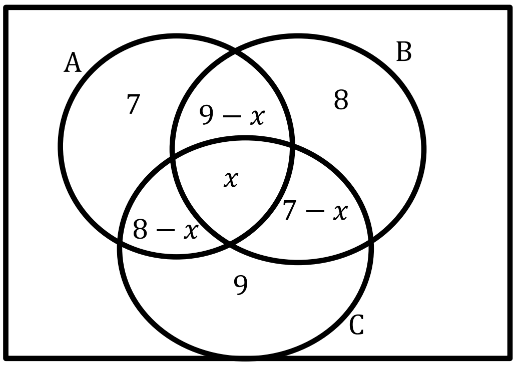
17.
(α) Given U = $\{a, b, c, d, e, f\}$
A = $\{a, c, d, f\}$
B = $\{b, c, e\}$
C = $\{a, d, g\}$
Find
(a) $(A \cup B) \cap C$
(b) $A \cup (B \cap C)$
(c) $(A \cup B) \cap (A \cup C)$

(β) In the Venn diagram, U is the set of all children in a certain chosen group,
A = {children in Youth Club A}
B = {children in Youth Club B}



The letters p, q, x and y in the diagram represent the number of children in each subset. Given that $n(U) = 200, n(A) = 75$ and $n(B) = 35$,
(a) Express p in terms of x
(b) Find the smallest possible value of y
(c) Find the largest possible value of x
(d) Find the value of q if $p = 45$

18. A, B and C are three sets and the numbers of elements are shown in the Venn diagram.



Given that $U = A \cup B \cup C$ and that $n(U) = 34$, find
(a) the value of x
(b) $n(A \cap B \cap C')$

19. If X, Y and Z are subsets of U and $n(U) = 80, n(X) = 32, n(Y) = 27, n(Z) = 29, n(X \cap Y) = 12, n(X \cap Z) = 13, n(Y \cap Z) = 10, n(X \cap Y \cap Z) = 3$, find $n(X' \cap Y' \cap Z')$.

20. The 89 members of the Fifth Form belong to one or more of the Chess Club, the Debating Society and the Jazz Club. Denoting these sets by C, D and J respectively, it is known that 20 pupils belong to C only, 15 to J only and 12 to D only. Given that $n(C \cap J) = 18$, $n(C \cap D) = 20$ and $n(D \cap J) = 16$, calculate
(a) $n(C \cap J \cap D)$, (b) $n(D')$

21. In a class of 25 students, 6 study French (F), 14 study Physics (P) and 3 study both French and Physics. Find $n(F' \cap P')$.

22. In a survey carried out at a sports centre, men were asked about their sporting activities. Of the men questioned, 12 played rugby, 16 played squash, 13 played tennis. 8 played none of these games, 3 played both rugby and squash, 5 played both rugby and tennis. 2 men played tennis only.
Let R, S and T be the sets of rugby, squash and tennis players respectively. Let the number of men playing all three games be x. Draw a Venn diagram and show, in terms of x, the number in each region of the diagram in set R. Also show the number in each of the other four regions. Find the total number of

men questioned and state the possible values of x.

23. Each student in a class of 17 studies at least one subject. Of these, 7 study Art (A), 5 study Biology (B), 12 study Home Management (H), 3 study Art only, 5 study Home Management only but no one studies both Art and Biology. Find $n(A' \cap H' \cap B)$.

24. A number of students prepared for an examination in Physics, Chemistry and Mathematics. Out of this number, 15 took Physics, 20 took Chemistry and 23 took Mathematics. 9 students took both Chemistry and Mathematics, 6 took both Physics and Mathematics all those who took Physics also took Chemistry. One student fell ill and failed to write any of the papers.

    (i) How many students took exactly one of the examination papers?

    (ii) How many students wrote exactly two of the examination papers?

    (iii) How many students prepared for the examination?

25. In a class of 40 students, 18 passed Mathematics, 19 passed Accounts, 16 passed Economics, 5 Mathematics and Account only, 6 Mathematics only, 9 Accounts only, 2 Accounts and Economics only. If each student offers at least one of the subjects,
    (a) how many of the students failed in all the subjects?
    (b) Find the percentage number who failed in at least one of Economics and Mathematics
    (c) Calculate the probability that a students selected at random failed in Account.

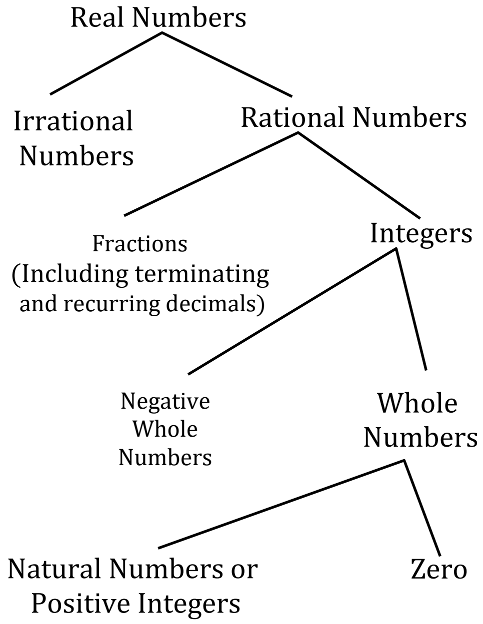
26. In a survey of 100 out – patients who reported at a hospital one day, it was found that 70 complained of fever, 50 had stomach ache and 30 were injured. All the 100 patients had at least one of the complaints and 44 had exactly two of the complaints. How many patients had all three complaints?

27. A universal set U includes subsets A, B and C. There are 5 members of U which are not in any of these subsets. Every member of B is a member of A but C contains members which do not belong either to A or B. Draw a Venn diagram to incorporate these features.
    Given further that,
    (a) there are 20 members of C not in A or B,
    (b) there are 3 members common to A, B and C,
    (c) there are 13 members of B,
    (d) there are 27 members of A not in C,
    (e) there are 10 members common to A and C,
    insert appropriate numbers in the regions of your diagram. Hence calculate,
    (i) $n(\text{A})$ (iii) $n(\text{U})$
    (ii) $n(\text{A} \cap \text{C} \cap \text{B}')$

28. A number of girls bought red, green and black ball pen. Three bought one of each colour. Of the girls who bought two colours, 3 did not buy red, 5 not green and 2 not black. The same number of girls bought red only as bought red with other colours. The same number bought black only as bought green only. More girls bought red and black but not green than bought black only, but more girls bought green only than bought green and black but not red.
    (i) How many girls are there?
    (ii) How many girls bought only one colour?
    (iii) How many girls bought at least two colours?
    (iv) How many girls bought exactly two of the colours?
    (v) The probability that a girl bought black and red but not green?

REAL NUMBER SYSTEM

The real numbers include all the numbers you encounter in arithmetic. The set of natural numbers, whole numbers, integers, rational and irrational numbers are all subsets of real numbers. The set of real numbers is denoted by $\mathbb{R}$.
We can draw a "tree diagram" which brings together all these sets of numbers.



PROPERTIES OF NUMBERS

(i) **NATURAL NUMBERS ($\mathbb{N}$)**
A natural number is any whole number from 1 to infinity. Zero (0) is not part of natural numbers. Natural numbers are used to count items and to make lists.
Natural numbers are denoted by $\mathbb{N}$ and is given by $\{1, 2, 3, \ldots\}$. Natural numbers are also called "counting numbers" because they are used for counting.

(ii) **WHOLE NUMBERS**
Whole numbers are the natural numbers and zero (0). It is denoted by $\mathbb{W}$. Whole numbers exclude decimals and fractions.
$\mathbb{W} = \{0, 1, 2, 3, \ldots\}$

(iii) **INTEGERS**
Integers are the set of positive and negative whole numbers including zero (0). It is denoted by $\mathbb{Z}$.
$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$

Rational Numbers
A rational number is any number that can be expressed in the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$. All terminating and recurring decimals are rational numbers as they can be expressed in the form $\frac{a}{b}$.
e.g. $2 = \frac{2}{1}, 0.3 = \frac{3}{10}, 0.\dot{2} = \frac{2}{9}$.
Rational numbers are denoted by $\mathbb{Q}$.

Note:
In Rational Numbers, decimals ends somewhere or it has a repeating pattern to it.

Irrational Numbers
Irrational numbers are numbers that cannot be expressed in the form $\frac{a}{b}$. E.g. $\sqrt{p}$, where p is not a perfect square.

Examples of irrational numbers:
$\sqrt{2}, \sqrt{3}, \pi$.

Exercise 1 **Date:………………….**

1. Which of the following numbers are irrational?
   $\frac{2}{3},\ \sqrt{36},\ \sqrt{3} + \sqrt{6},\ \pi,\ 0.75,\ 48\%,\ 8^{\frac{1}{3}}$.

2. Put a ring around the irrational number in the list below.
   $\frac{2}{3},\ \sqrt{5},\ -\frac{5}{7},\ \sqrt{49},\ 1\frac{4}{5}$

3. $\pi,\ 3^{-2},\ 3\frac{4}{7},\ 33.3\%,\ \sqrt{3},\ 0.3,\ 3^{999}$.
   From this list, write down the numbers that are irrational.

4. State whether the following numbers are rational or irrational.

(i)	1.6	(iv)	$0.\dot{5}\dot{3}$
(ii)	$\sqrt{11}$	(v)	$\sqrt{121}$
(iii)	$0.\dot{4}$	(vi)	π

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 2 **Date:…………………..**

State whether each of the following is true or false.

1. $Q \subset \mathbb{R}$
2. $\mathbb{R} \subset \mathbb{Z}$
3. $\mathbb{Z} \subset \mathbb{N}$
4. $\mathbb{N} \subset \mathbb{Q}$
5. $\mathbb{Q} \subset \mathbb{W}$
6. $\mathbb{Z} \subset \mathbb{Q}$
7. If $x \in \mathbb{Q}$ then x is an integer.
8. If x is a fractional number then $x \in \mathbb{Z}$.
9. If x is an irrational number then $x \notin \mathbb{Q}$.
10. If $x \in \mathbb{R}$, then x is an irrational number.
11. If $x \in \mathbb{Z}$, then $x \in \mathbb{Q}$.
12. If $x \in \mathbb{Q}$, then x is a natural number.

Factors

Factors are numbers that can divide other numbers exactly without leaving a remainder. 2 is a factor of 12, 5 is a factor of 15.

Multiples

Multiples are the continuous addition of a given number.

E.g. Multiples of 2 $= \{2, 4, 6, 8, \ldots\}$

Multiples of 3 $= \{3, 6, 9, 12, \ldots\}$

Multiples of 4 $= \{4, 8, 12, 16, \ldots\}$

Prime Numbers

A prime number is any positive number that is exactly divisible by itself and one (1). In other words, prime numbers are numbers that have two factors. i.e. 1 and the number itself.

(Note that 1 is not a prime number).

For example, 2, 3, 5, 7, 11, 13, … are prime numbers.

Composite Numbers
A whole number that can be divided evenly by numbers other than 1 and itself is a composite number. In other words, a composite number has more than two factors.
Example: 4, 6, 8, 9, 10, 12, …

Note: 0 and 1 are neither prime nor composite numbers.

Even Numbers
An even number is a number that divides evenly (exactly) by two (2).
Example: 2, 4, 6, 8, …

Odd Numbers
An odd number is one that does not divide evenly (exactly) by two (2).
Example: 1, 3, 5, 7, …

Square Numbers
When an integer (whole number) is multiplied by itself, the result is a square number.
i.e. $1 \times 1 = 1^2 = 1$
$2 \times 2 = 2^2 = 4$
$3 \times 3 = 3^2 = 9$
$4 \times 4 = 4^2 = 16$
i.e. the examples, 1, 4, 9 and 16 are all square numbers.

Cube Numbers
When an integer is multiplied by itself and then by itself again, the result is a cube number.
i.e. $1 \times 1 \times 1 = 1^3 = 1$
$2 \times 2 \times 2 = 2^3 = 8$
$3 \times 3 \times 3 = 3^3 = 27$
$4 \times 4 \times 4 = 4^3 = 64$
In the examples above, 1, 8, 27, 64 are all examples of cube numbers.

Exercise 3 **Date:………………….**

1. $\sqrt{4}$, $\sqrt{14}$, $\sqrt{36}$, $\sqrt{64}$, $\sqrt{81}$, $\sqrt{100}$.
   From the list above, write down
   (i) a prime number
   (ii) a factor of 27
   (iii) a multiple of 4
   (iv) an irrational number.

2. 4, $\sqrt{18}$, $\sqrt{25}$, $\frac{5}{2}$, 0.3333.
   From the list above, write down
   (i) a prime number
   (ii) an irrational number

Exercise 4 **Date:………………….**

1. $\sqrt{5}, -7,\ 343, -11,\ 0.4,\ 2.5,\ \frac{1}{3}$
   From this list of numbers, write down
   (a) a cube number
   (b) the smallest number
   (c) a natural number

2. Write down a number between 20 and 30 that is
   (i) a multiple of 6
   (ii) a square number
   (iii) a cube number
   (iv) a prime number

Exercise 5 **Date:……………………**

1. n is an integer and $120 < n < 140$. Find the value of n when it is
   (i) a multiple of 45
   (ii) a square number
   (iii) a factor of 402
   (iv) a cube number.
2. Find
   (i) the smallest multiple of 7 that is greater than 100.
   (ii) the largest cube number that less than 100
   (iii) the six factors of 45
   (iv) an irrational number between 6 and 7

3. $\mathrm{U} = \left\{-2\frac{1}{2}, -1, \sqrt{2}, 3.5, \sqrt{30}, \sqrt{36}\right\}$
   $\mathrm{X} = \{\text{integers}\}$
   $\mathrm{Y} = \{\text{irrational numbers}\}$.
   List the members of;
   (a) X, (b) Y

Exercise 6 **Date:.....................**

List five members in each of the following sets.

1. {rational numbers less than -20.13}
2. {rational numbers between -15 and 9.9}
3. $\{x: x \in \mathbb{Q}, \text{and } -4.8 < x \leq 12.6\}$
4. $\left\{x: x \in \mathbb{Q}, \text{and } x > -27\frac{1}{3}\right\}$
5. $\left\{x: x \in \mathbb{Q}, \text{and } x \leq -64\frac{2}{5}\right\}$

Exercise 7 **Date:.....................**

Represent the following rational numbers on the same number line.

1. $-6, -5\frac{1}{2}, -3\frac{1}{4}, -2.8, -2$
2. $-3.1, -2.2, -1.3, 0.4, 1.5$
3. $-14, -7.5, \frac{1}{2}, 7\frac{1}{4}, 13\frac{2}{5}$
4. $-80.5, -60\frac{1}{2}, -40.5, -10\frac{1}{2}, 20.5$
5. $-16\frac{1}{3}, -6\frac{1}{3}, 3\frac{2}{3}, 13\frac{2}{3}, 23\frac{2}{3}$
6. $-34.5, -24.5, -19.5, -9.5, -4.5$

Properties Of Real Numbers

Addition

It is assumed that there is a mode of combining two real numbers $\boldsymbol{a}$ and $\boldsymbol{b}$ so as to produce a definite real number called their sum. This mode of combination is called addition. The sum of $\boldsymbol{a}$ and $\boldsymbol{b}$ is denoted by $\boldsymbol{a+b}$. In this sum $\boldsymbol{a}$ and $\boldsymbol{b}$ are called terms.

Multiplication

It is assumed that there is a mode of combining any two real numbers $\boldsymbol{a}$ and $\boldsymbol{b}$ to produce a definite real number called their product. This mode of combination is called multiplication. The product of $\boldsymbol{a}$ and $\boldsymbol{b}$ is denoted by $\boldsymbol{a \cdot b}$, $\boldsymbol{ab}$ or $\boldsymbol{a \times b}$. The individual numbers $\boldsymbol{a}$ and $\boldsymbol{b}$ are called factors of the product.

Commutative Law for Addition

If $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}$ then, $\boldsymbol{a+b=b+a}$

Thus, the sum of two numbers is the same regardless of the order in which they are added. i.e. $2+3=3+2$

Commutative Law for Multiplication

If $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}$, then $\boldsymbol{ab=ba}$

That is, the product of two numbers is the same regardless of the order in which they are multiplied. i.e. $2 \times 3 = 3 \times 2$.

Associative Law for Addition

If $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}$ then,

$$(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c}=\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$$

That is, we obtain the same result whether we add the sum of $\boldsymbol{a}$ and $\boldsymbol{b}$ to $\boldsymbol{c}$, or we add $\boldsymbol{a}$ to the sum of $\boldsymbol{b}$ and $\boldsymbol{c}$. Since the way in which we associate or group these numbers is immaterial, we may write this common value as $\boldsymbol{a+b+c}$ without fear of ambiguity.

i.e. $2+3+4=(2+3)+4=2+(3+4)$

Exercise 8 **Date:…………………..**

Use the associative rule to add this as quickly as you can.

1. $2+9+4$
2. $17+13+29$
3. $103+172+98$
4. $1245+225+163$
5. $819+147+653$
6. $1297+1363+4703$

Associative Law for Multiplication
If $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}$ then, $(\boldsymbol{ab})\boldsymbol{c} = \boldsymbol{a}(\boldsymbol{bc})$
That is, we obtain the same result whether we multiply the product of $\boldsymbol{a}$ and $\boldsymbol{b}$ by $\boldsymbol{c}$, or we multiply $\boldsymbol{a}$ by the product of $\boldsymbol{b}$ and $\boldsymbol{c}$. Since the way in which we associate or group these numbers is immaterial, we may write the result as $\boldsymbol{abc}$ without fear of ambiguity. That is,
$2 \times 3 \times 4 = (2 \times 3) \times 4 = 2 \times (3 \times 4)$

Distributive Law
If $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}$ then, $\boldsymbol{a}(\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{ab} + \boldsymbol{ac}$
i.e. $2(3 + 4) = 2 \times 3 + 2 \times 4$
This law can be extended to the case where the sum consists of three or more items, as in the following example.
$2a(x + 3y - 2z) = 2ax + 6y - 4z$

Zero
It is assumed that there is a special number called zero and denoted by 0, such that, for every real number a, $a + 0 = a$.

Negative of a Number
It is assumed that for every real number a there exist a corresponding number, called the negative of a a and designated by $-a$, such that
$a + (-a) = 0$.

For example,
$1 + (-1) = 0, (-2) + 2 = 0$

The Unit
It is assumed that there is a special number called the number unit and denoted by 1, such that for every real number $\boldsymbol{a}$,
$\boldsymbol{a} \cdot \boldsymbol{1} = \boldsymbol{a}$.
i.e. $2 \times 1 = 2$, $5 \times 1 = 5$.

Reciprocal of a Number
It is assumed that for every number $\boldsymbol{a}$ which is not 0, there is an associated number $\frac{1}{\boldsymbol{a}}$, called the reciprocal of a, such that;
$$\boldsymbol{a} \cdot \frac{1}{\boldsymbol{a}} = 1.$$

Subtraction
The difference $\boldsymbol{a} - \boldsymbol{b}$, of any real numbers $\boldsymbol{a}$ and $\boldsymbol{b}$, is defined by:
$$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b}).$$
The operation indicated by the sign minus which produces for any two real number a and $\boldsymbol{b}$ the number $\boldsymbol{a} - \boldsymbol{b}$ is called subtraction.

Division
The quotient $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ or $\boldsymbol{a} \div \boldsymbol{b}$ of any real numbers $\boldsymbol{a}$ and $\boldsymbol{b}$, where $\boldsymbol{b} \neq 0$ is defined by:
$$\frac{\boldsymbol{a}}{\boldsymbol{b}} = \boldsymbol{a} \times \left(\frac{\boldsymbol{1}}{\boldsymbol{b}}\right)$$

Operations On Directed Numbers
Positive and negative numbers are collectively known as **directed numbers**. In each of the following relationships, a and b are any two real numbers, except that the denominator of a fraction may not be zero.
(i) $-(-a) = a$
(ii) $-(a + b) = -a - b$
(iii) $-(a - b) = -a + b$
(iv) $(-a)b = -(ab)$
(v) $(-a)(-b) = ab$
(vi) $\frac{1}{-b} = -\frac{1}{b}$
(vii) $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$
(viii) $\frac{-a}{-b} = \frac{a}{b}$

Exercise 9 **Date:………………….**

1. Identify the properties of real number that justify each of the following equations.
   (i) $x + y = y + x$
   (ii) $rs = sr$
   (iii) $2(3 \cdot 5) = (2 \cdot 3)5$
   (iv) $5(a + b) = 5a + 5b$
   (v) $(a + 2)(b - 3) = (b - 3)(a + 2)$
   (vi) $(a + b)c = c(a + b) = ca + cb$

2. Find the value of each of the following without the use of calculator.
   (i) $(-3) + (+5)$ (vi) $0 - (-2)$
   (ii) $(-5) + (-3)$ (vii) $(-5) - 0$
   (iii) $(-1) - (-2)$ (viii) $15 + (-3)$
   (iv) $(+7) - (+2)$ (ix) $(-7) - (-5)$
   (v) $(-8) - (-9)$ (x) $(+32) - (-23) + (-45)$

Exercise 10 **Date:…………………..**

1. Evaluate each of the following without the use of a calculator.
   (i) $(+2)(-3)$
   (ii) $(-3)(-5)$
   (iii) $(-7)(+5)$
   (iv) $2(-5)$
   (v) $(-5)(-9)$
   (vi) $(-7)0$
   (vii) $(-1)(-2) + (-3)(0)$
   (viii) $(+2)(-3) - (+7)(-5)$
   (ix) $(-4)(-5) - (-2)(-1)$

2. Evaluate the following without the use of a calculator.
   (i) $(-13) - (-13)$
   (ii) $-18 - (-10)$
   (iii) $(7 + (9 - (8 - 3)))$
   (iv) $(9 - (7 - (6 - 5)))$
   (v) $(5 - 3)(7 - 4)$
   (vi) $(7 \times 6) - (4 \times 2) + (16 \div 2)$

Exercise 11 **Date:…………………..**

1. Determine the negative of each of the following
   (i) 5
   (ii) -3
   (iii) 0
   (iv) $2x$
   (v) $\frac{2}{3}$
   (vi) $2-3$
   (vii) $2a-3b$
   (viii) $-(x-y)$
   (ix) $-[(a)(-b)]$
   (x) $3x+2$

2. Find the reciprocal of each of the following:
   (i) 1
   (ii) $\frac{2}{3}$
   (iii) $3+\frac{2}{5}$
   (iv) $2+\frac{1}{2}$
   (v) 1.02
   (vi) $a+b$
   (vii) $-\frac{0.4}{3}x$
   (viii) $-\frac{1}{a}$
   (ix) $\frac{1}{x+y}$
   (x) $\frac{5}{2-0.3x}$

Exercise 12 **Date:……………………**

1. p is the largest prime number between 50 and 100. q is the smallest prime number between 50 and 100. Calculate the value of $p - q$.

2. The sum of the prime numbers less than 8 is equal to 17.
   (a) Find the sum of the prime numbers less than 21.
   (b) The sum of the prime numbers less than x is 58. Find x.

3.
   (a) Find the sum of the first ten odd numbers
   (b) Find the sum of the first six prime numbers
   (c) Find the sum of the first eight composite numbers.

Exercise 13 **Date:……………………**

1. Fill in the missing number in each calculation.
   (i) $6 + 2 \times$ ______ $= 24$
   (ii) $(10 -$ ________$) \div 3 = 2$

2. Put in brackets to make the sentence correct.
   (i) $2 \times 3 + 4 = 14$
   (ii) $6 + 4 \times 3 + 2 = 20$
   (iii) $36 = 4 \times 3 + 6 \times 4$
   (iv) $7 + 2 \times 9 = 81$
   (v) $5 \times 3 + 6 \times 2 = 90$
   (vi) $6 + 12 \div 3 \times 2 = 8$
   (vii) $8 \times 8 - 5 - 4 = 15$
   (viii) $7 + 14 \div 4 - 1 \times 2 = 14$

Exercise 14 **Date:.......................**

1. State whether each of the following is a prime number (P) or composite (C) number.

(i) 16	(iv) 37
(ii) 17	(v) 51
(iii) 50	(vi) 79

2. Use the Sieve of Eratosthenes to identify prime numbers up to 100.

Prime Factorization

It is the determination of the set of prime numbers which multiply together to give original integer. It is also known as prime decomposition. All composite numbers can be broken down (decomposed) into a unique set of prime factors. A factor tree can be used to decompose the composite number. Beginning with the given number, 'branches' come down in pairs, representing a pair of factors that multiply to give the number above. This process continues until prime factors are reached.

Example 1

Write 48 as a product of prime numbers.

METHOD 1

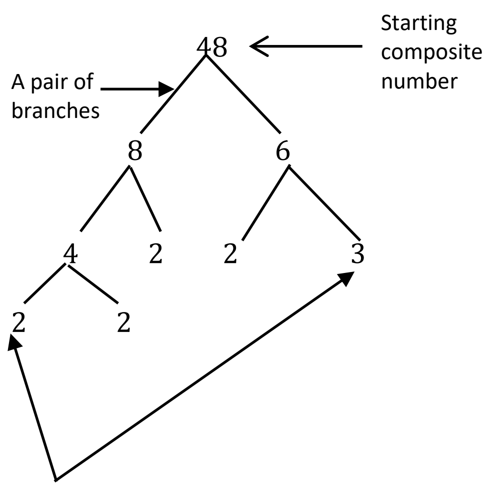
$48 = 8 \times 6$

$= 2 \times 4 \times 2 \times 3$

$= 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

METHOD 2

Write 48 as a product of prime numbers.



'branches' terminate on prime factors.

i.e. $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

Exercise 15 **Date:.....................**

Write each of number as a product of prime factors.

(i) 10	(iii) 168
(ii) 42	(iv) 2700

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Exercise 16 **Date:…………………**

Express each number as a product of prime factors.

(i) 72 (iii) 98

(ii) 350 (iv) 1352

Highest Common Factors (HCF)
It refers to the highest (i.e. largest) factor that is common to the numbers given in the question.

Example 2
Find the HCF of 12 and 15.

Solution
Factors of 12 $= \{1, 2, 3, 4, 6, 12\}$
Factors of 15 $= \{1, 3, 5, 15\}$
HCF $= \{3\}$.

Example 3
Find the HCF of $3^3 \times 5^2$ and $3^2 \times 5^4$.

Solution
Let $P = 3^3 \times 5^2$
$= 3^2 \times 3 \times 5^2$
$Q = 3^2 \times 5^4$
$= 3^2 \times 5^2 \times 5^2$
Now, common factors of P and Q: $3^2 \times 5^2$.

Exercise 17 **Date:…………………**
Find the HCF of the following pairs of numbers.
(i) 15 and 21.
(ii) 35 and 70.
(iii) 15 and 60.
(iv) 28 and 42.
(v) 84 and 90.

Exercise 18 **Date:………………**
Find the HCF of the following groups of numbers.
(i) 6, 15 and 42.
(ii) 18, 13 and 21
(iii) 20, 12 and 28.
(iv) 48, 30 and 18.

Least Common Multiples (LCM) Up To 2 – Digit Numbers

It refers to the lowest (i.e. smallest) multiple that is common to the numbers provided in the question.

Example 4

What is the lowest common multiple of 2 and 3?

Solution

Multiples of $2 = \{2, 4, 6, 8, 10, 12, \dots\}$

Multiples of $3 = \{3, 6, 9, 12, 15, \dots\}$

LCM $= \{6\}$

Exercise 19 **Date:……………..**

Find the least common multiple (LCM) of:

(i) 16, 30 and 36.

(ii) 9, 18 and 16

(iii) $2^2 \times 3 \times 5^2$ and $2^3 \times 3^2 \times 5$.

(iv) $2^3 \times 3^2 \times 5$ and $2^4 \times 3^3 \times 5^2$.

(v) $2 \times 3^4 \times 5, 2^3 \times 3^3 \times 5$ and $2^2 \times 3^2 \times 5^2$

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Exercise 20 **Date:…… ………….**

1. The operation $(\cdot)$ is defined on the set $\{2, 4, 6\}$ by $m \cdot n =$ the unit digit in the product mn.

   (i) Copy and complete the table

$\cdot$	2	4	6
2	4	8	2
4		6	
6			

   (ii) Use the table to solve the following equations

   (a) $x \cdot 4 = 8$

   (b) $e \cdot e = e$

   (c) $(4 \cdot f) \cdot 4 = f$

2. What is the least value of p if $2^3 \times 3^3 \times 7 \times p$ is a perfect square?

3. If $C = 216k$, find the least integral value of k that will make C a perfect square.

Exercise 21 **Date:…………………..**

1.

(a) Written as a product of its prime factors, $T = 2^2 \times 3 \times 5^2$.

(i) Work out the value of T

(ii) Write 80 as a product of its prime factors.

(iii) Find the highest common factor of T and 80.

(b) Write down the next two prime numbers after 47.

(c)

(i) Write 180 as a product of its prime factors.

(ii) Find the lowest common multiple (LCM) of 180 and 54.

2. An integer, X, written as a product of its prime factors is $a^2 \times 7^{b+2}$
An integer, Y, written as a product of its prime factors is $a^3 \times 7^2$.
The highest common factor (HCF) of X and Y is 1225. The lowest common multiple (LCM) of X and Y is 42,875.
Find the value of X and Y.

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FRACTIONS

The word 'fraction' comes from the Latin word 'frangere' which means to break into pieces.

A fraction is made up of a numerator (up) and a denominator (down). For example

$$\frac{a}{b} \begin{array}{l} \leftarrow \text{numerator} \\ \leftarrow \text{denominator} \end{array}$$

where $b \neq 0$.

- The denominator tells you how many parts the whole is divided up into.
- The numerator tells you how many of the divided parts you have selected
- The horizontal line separating the numerator and denominator is called **vinculum**

TYPES OF FRACTION

PROPER FRACTION

It is a fraction where the numerator is less than the denominator.

For example $\frac{1}{3}, \frac{2}{3}, \frac{4}{7}$ etc

IMPROPER FRACTION

It is a fraction where the numerator is greater than the denominator, as in, $\frac{3}{2}, \frac{5}{4}, \frac{7}{6}$ etc.

MIXED FRACTION

It is a mixture of a whole number and a fraction. For example $2\frac{1}{3}, 4\frac{3}{5}, 1\frac{3}{5}$ etc.

LIKE FRACTION

They are fraction with common denominator. For example $\frac{3}{4}, \frac{5}{4}, \frac{1}{4}$

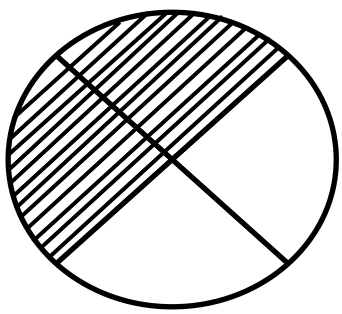
UNLIKE FRACTION

They are fraction with different denominator, for example $\frac{1}{3}, \frac{4}{5}, \frac{2}{7}$ etc.

NOTE:

We can represent fractions using area. If a shape a divided into regions of equal areas, then shading a certain number of these regions will create a fraction of the whole shape.

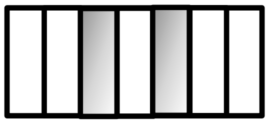
For example



Fraction shaded $= \frac{3}{6}$

Exercise 22 **Date:……………………..**

The diagram below is a picture of a bread



a. Into how many pieces has the whole bread been divided?
b. How many pieces have been selected (shaded)?
c. In representing the shaded fraction of the bread.
   (i) What must the denominator equal?
   (ii) What must the numerator equal?
   (iii) Write the amount of bread selected (shaded) as a fraction.

Ordering Fractions

- To order (or arrange) fractions we must know how to compare different fractions. This is often done by considering equivalent fractions.
- If the numerators are the same, the smallest fraction is the one with the biggest denominator, as it has been up divided into the most pieces. For example $\frac{1}{5} < \frac{1}{2}$.
- If the denominator are the same, the smallest fraction is the one with the smallest numerator. For example : $\frac{3}{11} < \frac{5}{11}$
- To order two fractions with different numerator and denominator we can use our knowledge of equivalent fractions to produce fractions and then compare the numerators.
- The lowest common denominator (LCD) is the lowest common multiple of the different denominators.
- Ascending order is when numbers are ordered going up, from smaller to largest.
- Descending order is the numbers ordered going down, from largest to smallest.

Example 5

Arrange the fractions $\frac{3}{4}, \frac{2}{3}, \frac{4}{5}$ in ascending order of magnitude.

Solution

$\frac{3}{4}, \frac{2}{3}, \frac{4}{5} = \frac{45,40,48}{60}$

LCD of 3,4 and 5 is 60

Produce equivalent fractions with denominator of 60. Order fractions in ascending order

$= \frac{40}{60}, \frac{45}{60}, \frac{48}{60}$

$= \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$

Exercise 23 **Date:......................**

Arrange the following fractions in ascending order.

(i) $\frac{5}{6}, \frac{3}{5}, \frac{2}{3}$

(ii) $4\frac{2}{5}$, 4.03, $\frac{28}{5}$, 5.75

(iii) $\frac{7}{75}, \frac{2}{5}$ and $\frac{1}{3}$

Fundamental Operations On Fractions

$a, b, c, d \in \mathbb{R}$, except that no factor in the denominator of a fraction may be zero.

(i) $\frac{ac}{bc} = \frac{a}{b}$

(ii) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

(iii) $\frac{a}{c} + \frac{b}{d} = \frac{ad+bc}{cd}$

(iv) $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

(v) $\frac{a}{c} - \frac{b}{d} = \frac{ad-bc}{cd}$

(vi) $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

(vii) $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Order Of Operation

When a problem (be it an integer or a fraction) involves more than one operation, the following order must be observed;

BODMAS

( ), orders , $\div$, $\times$, $+$, $-$

Thus,
Bracket first if there is .... ( )
'Order of' next (if there is)................. ($\times$)
Division next (if there is).................. $\div$
Multiplication (if there is)................. $\times$
Addition (if there is)......................... $+$
Subtraction last (if there is)............ $-$

Exercise 24 **Date:......................**

Evaluate the following without the use of calculator.

1. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
2. $\left(\frac{1}{2} - \frac{1}{3}\right) \times \left(\frac{1}{4} - \frac{1}{5}\right)$
3. $\left(1\frac{2}{7} - \frac{1}{3}\right) \times 1\frac{3}{4}$
4. $6\frac{2}{3} \div \left(3\frac{4}{15} - 1\frac{3}{5}\right)$

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Exercise 25 **Date:…………………..**

Simplify the following without the use of calculator.

1. $1\frac{1}{2} - \frac{3}{5} \div \frac{2}{3}$
2. $\left(\frac{3}{4} - \frac{1}{3}\right) \times 4\frac{1}{3} \div 3\frac{1}{4}$
3. $\frac{\frac{3}{4}\left(3\frac{3}{8}+1\frac{5}{6}\right)}{2\frac{1}{8}-1\frac{1}{2}}$
4. $37\frac{1}{2} \div \frac{5}{9}$ of $\left(\frac{4}{7} + \frac{1}{5}\right) - 80\frac{1}{3}$

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Exercise 26 **Date:…………………..**

1. By how much is the sum of $3\frac{2}{3}$ and $2\frac{1}{5}$ less than 7?

2. Simplify, without using mathematical tables or calculators.
   (i) $\frac{\frac{3}{4}-\frac{7}{8}+\frac{1}{2}}{\frac{3}{4}\text{ of }\left(\frac{7}{8}-\frac{1}{2}\right)}$
   (ii) $3\frac{4}{9} \div \left(5\frac{1}{3} - 2\frac{3}{4}\right) + 5\frac{9}{10}$
   (iii) $\left(\frac{2}{3}\text{ of }2\frac{1}{4}\right) \div \left(3\frac{1}{2} - 2\frac{1}{4}\right)$

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Exercise 27 **Date:…………………..**

1. Simplify $\sqrt{5\frac{44}{49}} \times \left(11\frac{1}{3}\right)^{-1}$, leaving your answer in the form $\frac{p}{q}$ where p and q are positive integers.

2. Simplify, without using a calculator
   (i) $\frac{3\frac{4}{7}-1\frac{1}{3}\div 2\frac{2}{5}}{\frac{13}{3}-1\frac{6}{7}}$
   (ii) $\left(4\frac{3}{4} - 1\frac{5}{6}\right) \div 1\frac{1}{24} \times \left(1\frac{2}{3} + 2\frac{1}{2}\right)$
   (iii) $\frac{4\frac{1}{4}-3\frac{1}{2}+3\frac{1}{8}}{3\frac{2}{5}\text{ of } 1\frac{1}{4}\div 2\frac{5}{6}}$

3. When a fraction is reduced to its lowest term, it is equal to $\frac{3}{4}$. The numerator of the fraction when doubled would be 34 greater than the denominator. Find the fraction.

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Applications of Fractions

Example 6

A man saves $\frac{3}{10}$ of his income and pay $\frac{1}{6}$ of the remainder as rent. Find what fraction of his income is left for other purposes.

Solution…

Fraction on saving $= \frac{3}{10}$

Remainder $= 1 - \frac{3}{10} = \frac{7}{10}$

Fraction on Rent $= \frac{1}{6}$ of the Remainder

$= \frac{1}{6} \times \frac{7}{10} = \frac{7}{60}$

Total fraction used $= \frac{3}{10} + \frac{7}{60} = \frac{5}{12}$

Fraction left for other purposes

$= 1 - \frac{5}{12} = \frac{7}{12}$

Example 7

A girl spent $\frac{3}{5}$ of her pocket money and was left with GH¢1,800.00. How much was her pocket money?

Solution…

Fraction spent $= \frac{3}{5}$

Fraction left $= 1 - \frac{3}{5} = \frac{2}{5}$

Let her pocket money be GH¢ x

If amount left = GH¢1,800.00

$\Rightarrow \frac{2}{5}x = 1800$

$\therefore x = \frac{5}{2} \times 1800 = 4500$

$\therefore$ Her pocket money was GH¢4,500.00

Alternative Method 1

Let her pocket money be GH¢ x

Amount spent $= \frac{3}{5}x$

Amount left = GH¢1,800.00

$$\Rightarrow \frac{3}{5}x + 1800 = x$$

$$\Rightarrow 3x + 5 \times 1800 = 5x$$
$$3x - 5x = -9000$$
$$-2x = -9000$$
$$x = 4500$$

∴ Her pocket money was GH¢4,500.00

Alternative Method 2
Suppose her pocket money be x
One (1) corresponds to total amount.

$\Rightarrow 1 \longrightarrow x$

Fraction spent $= \frac{3}{5}$
Fraction left $= 1 - \frac{3}{5} = \frac{2}{5}$

∴ Given amount left $=$ GH¢1,800.00

$\Rightarrow \frac{2}{5} \longrightarrow 1{,}800.00$

If more, less divide
$\therefore x = \frac{1}{\frac{2}{5}} \times 1{,}800$
$= \frac{5}{2} \times 1{,}800$
$= 4{,}500$
∴ Her pocket money was GH¢4,500.00

Exercise 28 **Date:………………….**

1. Kofi spent $\frac{2}{5}$ of his pocket money on snacks and $\frac{1}{3}$ of the remaining on transport. What fraction of his money is left?

2. A farmer uses $\frac{2}{5}$ of his land to grow cassava, $\frac{1}{3}$ of the remainder for yams and the rest for maize. Find the part of the land used for maize.

3. A boy spent $\frac{1}{2}$ of his money on food and $\frac{1}{3}$ of the rest on clothes. He had GH¢150.00 left in his pocket. How much money had he originally?

4. A clerk spends $\frac{1}{5}$, $\frac{1}{3}$ and $\frac{1}{8}$ of his annual salary on rent, transport, and entertainment respectively. If after all these expenses he had GH¢4,100.00 left, find how much he earns per annum.

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Exercise 29 **Date:…………………..**

1. A man spent $\frac{1}{4}$ of his monthly salary on rent, $\frac{2}{5}$ on food and $\frac{1}{6}$ on his children's education. What fraction of his salary is left?

2. A clerk spends $\frac{1}{5}$, $\frac{1}{3}$ and $\frac{1}{8}$ of his annual salary on rent, transport and entertainment respectively. If after all these expenses he had GH¢820.00 left, find how much he earns per annum.

3. A man spends $\frac{1}{9}$ of his monthly salary on rent, $\frac{1}{2}$ on food and $\frac{1}{4}$ on clothes and other items. If he had GH¢19,500.00 left at the end of the month, how much does he earn?

4. A woman spent $\frac{1}{6}$ of her monthly salary on foodstuffs, $\frac{1}{3}$ on drugs, $\frac{1}{4}$ on utility bills and had GH¢285.00 left. Calculate her monthly salary.

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Approximating And Rounding Numbers

Decimal Places

A decimal place(s) (d.p) in a number is that which comes after a decimal point.

Steps

1. Identify the position of the last digit.
2. Then look at the next digit to the right called the **decider**.
3. If the decider is 5 or more, then round – up the last. If the decider is 4 or less, then leave the last digit as it is.

Exercise 30 **Date:.....................**

1. Correct the following to 1 d.p.
   (i) 4.92 (iv) 7.38
   (ii) 3.56 (v) 5.265
   (iii) 2.43

2. Correct the following to 2 d.p.
   (i) 4.377 (iv) 2381.597
   (ii) 7.5237 (v) 1045.2781
   (iii) 2.371 (vi) $(3.09)^2$

Exercise 31 **Date:.....................**

(a) Correct 0.18375 to
   (i) 1 d.p
   (ii) 2 d.p
   (iii) 3 d.p
   (iv) 4 d.p

(b) Round each of the following number to the nearest
   (i) 10 (ii) 100 (iii) 1000

1. 34	8. 497
2. 23	9. 501
3. 63	10. 892
4. 98	11. 1096
5. 105	12. 1956
6. 147	13. 4464
7. 287	14. 9600

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Exercise 32 **Date:………………….**

1. Round each of the following measurements
   (a) to the nearest 10 units
   (b) to the nearest 100 units
   (i) 175mm
   (ii) 168cm
   (iii) 237m
   (iv) 384km
   (v) 351m
   (vi) 542mm
   (vii) 2395km
2. Express the following to the nearest thousands.
   (i) 196650
   (ii) 209802
   (iii) 14835
3. Round 19.4863027
   (a) to the nearest unit
   (b) to 2 decimal places
   (c) to 3 decimal places
   (d) to 5 decimal places

Significant Figure

The first significant figure of a decimal number is the first (left – most) non – zero digit. For example,

(i) The first significant figure of 13456 is 1.
(ii) The first significant figure of 0.00024678 is 2.

Every digit to the right of the first significant figure is regarded as another significant figure.

Steps

Count off the specified number of significant figures then look at the next digit.

(i) If the digit is less than 5, do not change the last significant figure.
(ii) If the digit is 5 or more, then increase the last significant figure by 1.

Note:

Delete all figures following the significant figures, replacing with 0s where necessary.

Exercise 33 **Date:………………….**

Round correct to the number of significant figures shown in brackets.

(i)	33.4	[2]
(ii)	7.327	[3]
(iii)	1045.2781	[2]
(iv)	0.00578	[2]
(v)	1426.3075	[2]
(vi)	1.02616	[3]
(vii)	7.30713	[3]
(viii)	0.0030071	[3]
(ix)	0.024561	[3]
(x)	5045.0049	[3]
(xi)	46.23067	[5]

Exercise 34 **Date:………………….**

Round correct to the number of significant figures shown in brackets.

(i)	0.037696	[3]
(ii)	0.002473	[3]
(iii)	0.0033780	[3]
(iv)	0.005854	[2]
(v)	0.081778	[3]
(vi)	0.000407	[2]
(vii)	0.037696	[3]
(viii)	0.0395387	[3]
(ix)	7.0959	[3]
(x)	0.006586	[3]
(xi)	0.0063075	[3]
(xii)	0.00268	[2]

Exercise 35 **Date:………………….**

1. Round correct to the number of significant figures shown in brackets.

(i)	48976	[3]
(ii)	1975	[2]
(iii)	57774	[3]
(iv)	39763	[3]
(v)	75882	[3]
(vi)	10100	[2]
(vii)	6857	[3]

2.

(i)	$(0.13)^3$	[3]
(ii)	$2^{\sqrt{3}}$	[4]
(iii)	$\sqrt{25.65}$	[4]
(iv)	$\sqrt[3]{2.35^2 - 1.09^2}$	[4]
(v)	$\sqrt[3]{7^{1.5} + 22^{0.9}}$	[4]

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Exercise 36 **Date:…………………..**

1.
   (a)
      (i) Write the number five million, two hundred and seven in figures.
      (ii) Write thirty thousand, one hundred and eleven in figures.

   (b) Write down the following numbers in words.
      (i) 604925
      (ii) 1111111

2.
   (i) Write 84 as a product of its prime factors.
   (ii) Find the highest common factor (HCF) of 84 and 126.
   (iii) Write 4647 correct to the nearest 100.
   (iv) Find the LCM of 18 and 21.

3.
   (a) Find the value of
      (i) the square root of 19044.
      (ii) 2^7
      (iii) 999^0
      (iv) $12 - (9 - (7 - 5))$
      (v) $2(17 + 3) - \big(17 + (10 - 3)\big) + 3.4$
      (vi) $2(11 - 10) + 3(10 - 8) - 4(9 - 7)$
      (vii) $(12 - 7) + (6 - 3) + 18 + 20$
      (viii) $3^2 + 2^2$
      (ix) $2^3 + 3^2$
      (x) $7^2 - 5^2$
      (xi) $(8^2 - 2^2) + 3^3$
      (xii) $5^2 - 4^2 + (5 - 4)^2$
      (xiii) $4 - 5(4^2 - 3^3)^2$

   (b) Evaluate the following.
      (i) $10 - 4 \times 5$
      (ii) $5 - 3 \times 8 - 6 \div 2$
      (iii) $7 + 3 \div 4 + 1$
      (iv) $100 - 30 \times (4 - 3)$
      (v) $(8 + 8) - 6 \times 2$
      (vi) $[(12 + 6) \div 9] \times 4$
      (vii) $[(60 - 40) - (53 - 43)] \times 2$
      (viii) $6 \times [(20 \div 4) - (6 - 3) + 2]$
      (ix) $\{6 + [5 \times (2 + 30)]\} \times 10$
      (x) $100 - [6 \times (4 + 20) - 4 \times (3 + 0)]$

4. The price of a ticket for a football match is GH¢124.00.
   (a) Calculate the amount received when 76500 tickets are sold.
   (b) Write your answer in (a) to the nearest 100,000.

5.
   (α) Here is a set of numbers $\{-4, -1, 0, 3, 4, 6, 9, 15, 16, 19, 20\}$. Which of these numbers are
      (a) natural numbers?
      (b) square numbers?
      (c) negative numbers?
      (d) prime numbers?
      (e) multiples of 2?
      (f) factors of 80?
   (β)
      (a) Use a factor tree to express 400 as a product of prime factors.

      (b) Use division method to express 1080 as a product of prime factors.

      (c) Use your answers to find:
         (i) the LCM of 400 and 1080.
         (ii) the HCF of 400 and 1080.
         (iii) $\sqrt{400}$.
         (iv) whether 1080 is a cube number; how can you tell?

Exercise 37 **Date:…………………..**

1.
   (α) 1, 3, 8, 9, 10.
   From these numbers, write down
      (a) the prime number
      (b) a multiple of 5
      (c) two square numbers
      (d) two factors of 32
      (e) find two numbers m and n from the list such that $m = \sqrt{n}$ and $n = \sqrt{81}$
      (f) If each of the numbers in the list can be used once, find p, q, r, s, t such that $(p + q)r = 2(s + t) = 36$ if $p < q < t < s$.

(β)

(i) Insert one of the symbols $>, =, <$ to make each of the statements correct.

(a) $(0.2)^2$ ________ 4×10^{-2}

(b) $\frac{37}{73}$ ________ 0.507

(ii) $\frac{82}{99}$, 82%, $\sqrt{0.674}$

(a) Write these in order of size, starting with the smallest.

(b) Find the difference between the largest and the smallest, giving your answer correct to two significant figures.

2.

(a) Write 135, 210 and 1120 as the product of their prime factors.

(b) Copy this grid

$a = 1$	$b =$	$c =$
$d =$	$e =$	$f =$
$g =$	$h =$	$i = 8$

The nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are to be placed in your grid in such a way that the following four statements are all true.

$a \times b \times d \times e = 135$
$b \times c \times e \times f = 1080$
$d \times e \times g \times h = 210$
$e \times f \times h \times i = 1120$

The digits 1 and 8 have already been placed for you.

Use your answers to part (a) to answer the following questions.

(i) Which is the only digit, other than 1, that is a factor of 135, 1080, 210 and 1120?

(ii) Which is the only letter to appear in all four statements above?

(iii) 7 is a factor of only two of the numbers 135, 1080, 210 and 1120. Which two?

(c) Now complete the grid.

Recurring Decimals To Fractions

A rational number $\left(\text{i. e.} \frac{p}{q}, p, q \in \mathbb{R}, q \neq 0\right)$ can be expressed as either a terminating or recurring (repeating) decimal.

$\frac{3}{4} = 0.75$ (terminates)

$\frac{7}{4} = 1.75$ (terminates)

$\frac{1}{3} = 0.333 \ldots = 0.\dot{3}$ (does not terminate)

$\frac{2}{8} = 0.181818 \ldots = 0.\dot{1}\dot{8}$ (does not terminate)

Note: $0.\dot{1}2\dot{6} = 0.126126126 \ldots$

Recurring decimals can be converted to a rational number.

Example 8

Show that the following are rational numbers.

(i) $0.\dot{3}$ (iii) $1.\dot{2}\dot{4}$

(ii) $0.1\dot{4}$ (iv) $0.1\dot{3}\dot{7}$

Solution…

(i) $0.\dot{3} = 0.3333 \ldots$

Let $x = 0.3333 \ldots.$ ………………… (1)

Multiply through by 10
(Since only 3 recurs)

$\Rightarrow 10x = 3.3333$…………………… (2)

$(2) - (1) \Rightarrow 9x = 3$

$x = \frac{3}{9}$

$x = \frac{1}{3}$

$\therefore 0.\dot{3} = \frac{1}{3}$

(ii) $0.1\dot{4} = 0.14444 \ldots$

Let $x = 0.14444$

Multiply through by 10
(Since 1 does not recur)

$\Rightarrow 10x = 1.4444$……………….. (1)

Multiply (1) by 10
(Since only 4 recurs)

$\Rightarrow 100x = 14.4444$…………… (2)

$(2) - (1) \Rightarrow 90x = 13$

$x = \frac{13}{90}$

$\therefore 0.1\dot{4} = \frac{13}{90}$

(iii) $1.\dot{2}\dot{4} = 1.242424 \ldots$

Let $x = 1.242424$………………… (1)

Multiply through by 100

$\Rightarrow 100x = 124.242424$……….. (2)

(1) is multiplied by 100 here, so that the decimal fraction to the right of the decimal point, still equal (1).

$(2) - (1) \Rightarrow 99x = 123$

$x = \frac{123}{99} = \frac{41}{33}$

$\therefore 1.\dot{2}\dot{4} = \frac{41}{33}$

(iv) $0.1\dot{3}\dot{7} = 0.1373737 \ldots$

Let $x = 0.1373737 \ldots$

$10x = 1.373737 \ldots$ …………….. (1)

$1000x = 137.373737 \ldots$ ………. (2)

$(2) - (1) \Rightarrow 990x = 136$

$x = \frac{136}{990}$

$x = \frac{34}{225}$

$\therefore 0.\dot{1}\dot{3}\dot{7} = \frac{34}{225}$

Exercise 38 **Date:…………………..**

Write the following recurring decimal as a fraction. Show all your working.

(1) $0.\dot{2}$
(2) $0.\dot{1}\dot{2}$
(3) $1.\dot{2}$
(4) $0.\dot{4}\dot{8}$
(5) $0.1\dot{5}$
(6) $0.\dot{1}\dot{4}$
(7) $0.\dot{2}\dot{5}$
(8) $0.\dot{3}\dot{5}$
(9) $2.3\dot{4}$
(10) $0.\dot{1}23\dot{4}$

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Exercise 39 **Date:……………………**

Write the following recurring decimal as a fraction. Show all your working.

(1) $0.\dot{0}\dot{8}$ (5) $3.\dot{1}3\dot{2}$

(2) $0.\dot{1}2\dot{3}$ (6) $2.3\dot{4}\dot{5}$

(3) $0.\dot{1}0\dot{4}$ (7) $0.12\dot{7}$

(4) $2.\dot{1}\dot{3}$ (8) $2.35\dot{7}$

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Exercise 40 **Date:…………………..**

Write each of recurring decimals in the form $\frac{a}{b}$, where $a, b, \in \mathbb{Z}$ and $b \neq 0$.

1. $0.\dot{6}$
2. $0.\dot{4}$
3. $0.\dot{5}$
4. $0.\dot{2}\dot{7}$
5. $0.\dot{8}\dot{1}$
6. $0.\dot{2}\dot{4}$
7. $0.\dot{5}\dot{4}$
8. $0.\dot{4}8\dot{6}$
9. $0.\dot{8}1\dot{0}$
10. $0.9\dot{3}$
11. $0.\dot{9}7\dot{2}$
12. $0.6\dot{3}$
13. $0.31\dot{6}$
14. $0.28\dot{3}$

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STANDARD FORM

A number is said to be in standard form, if it can be written in the form $A \times 10^n$, where $1 \leq A < 10$ and $n \in \mathbb{Z}$.

NOTE:

1. For a whole number, the decimal place is after the last digit.
2. If you move the decimal place to the left, the power is positive.
3. If you move the decimal place to the right, the power is negative.

Example 1
Write 7400 in standard form.

Solution…
$7{,}400 = 7.400 \times 10^3$

Exercise 1 **Date:....................**
Express the following in standard form
(i) 196650 (iv) 580,000,000
(ii) 98000 (v) 73×89
(iii) 128000

Example 2
Express 0.0059613 in standard form.

Solution…
$0.0059613 = 5.9613 \times 10^{-3}$

Exercise 2 **Date:.................**
Express the following in standard form.
(i) 0.000834 (iv) 0.00096
(ii) 0.0000816 (v) 0.0462
(iii) 0.0000163

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Exercise 3 **Date:………………**

Express the following in standard form.

(i) 0.04945

(ii) 0.000344

(iii) 0.000316×10^{-7}

(iv) $3.0 \times 10^{1} - 2.8 \times 10^{-1}$

(v) $(0.3 \times 10^{2}) \times (0.5 \times 10^{3})$

$a^m \times a^n = a^{m+n}$

Example 3

Evaluate $\frac{9.6\times10^{18}}{0.24\times10^{5}}$, leaving your answer in standard form.

$\frac{a^m}{a^n} = a^{m-n}$

Solution…

$\frac{9.6\times10^{18}}{0.24\times10^{5}} = \frac{96\times10^{-1}\times10^{18}}{24\times10^{-2}\times10^{5}}$

$= \frac{4\times10^{-1+18}}{10^{-2+5}}$

$= \frac{4\times10^{17}}{10^{3}}$

$= 4 \times 10^{17-3}$

$= 4 \times 10^{14}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Exercise 4 **Date:……………………..**

Without using calculator or table, evaluate the following leaving your answer in standard form.

(i) $\frac{0.0048\times0.81}{0.0027\times0.004}$

(ii) $\frac{0.6\times32\times0.004}{1.2\times0.008\times0.16}$

(iii) $\frac{0.015\times0.567}{0.05\times0.189}$

(iv) $\frac{0.0125\times0.00576}{0.0015\times0.32}$

(v) $\frac{0.0245\times1.2}{0.08\times1.75}$

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Exercise 5 **Date:.......................**

1. Express the product of 0.6 and 0.09 in standard form.

2. Simplify $0.2 \times 0.03 \times 0.004$ and express your answer in standard form.

3. Express the sum of 200.2 and 4.27×10^3 in standard form.

4. Without using mathematical table or calculator, evaluate $\sqrt{\left(\frac{P}{Q}\right)}$ where $P = 3.6 \times 10^{-3}$ and $Q = 2.25 \times 10^6$, leaving your answer in standard form.

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Exercise 6 **Date:..........................**

1. Given that $a = 4.0 \times 10^{-2}$, $b = 3.0 \times 10^{-2}$ and $c = 100$, evaluate without using table or calculator $\sqrt{\frac{a^2+b^2}{c}}$, leaving your answer in standard form.

2. Without using mathematical table or calculator, evaluate $\sqrt{\frac{0.0048\times0.81\times10^{-7}}{0.027\times0.04\times10^{6}}}$.

3.
   (a) Use your calculator to work out $\frac{1-(\tan 40)^2}{2(\tan 40)}$.
   (b) Write your answer in standard form.
4. Without using tables or calculators, express $\frac{(0.00042\times10^{-8})(15{,}000)}{(5000\times10^{7})(0.00021\times10^{14})}$ in the form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

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Exercise 7 **Date:……………………..**

Solve for n in the following.

1. $0.0000568 = 5.68 \times 10^n$
2. $5(n + 3 \times 10^6) = 4 \times 10^7$
3. $\frac{7\times5^4}{2^3\times2^{-4}} = 8.75 \times 10^n$
4. $4.95 \times 10^n = 0.0495$
5. $\frac{2.34\times(981)^2\times10^{-3}}{4.29\times10^5} = 5.2492 \times 10^n$
6. $\frac{3\times10^4}{8\times10^{-2}} = 3.75 \times 10^n$

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BINARY OPERATION

A binary operation $*$ on a non – empty set $\mathbb{R}$ of real numbers is a rule which combines two elements $a, b \in S$, to produce a new element which may or not be in S.

Addition $(+)$, subtraction $(-)$, multiplication $(\times)$ and division $(\div)$ are the most familiar examples of binary operations on the set of real numbers. Intersection $(\cap)$ and union $(\cup)$ of sets are other familiar examples of binary operations. We can represent binary operations by the symbols $*, \Delta, \circ, \sim$ and so on.

Example 1

A binary operation $*$ on the set of real numbers is defined by $a * b = a + b - 3$

Evaluate

(i) $2 * 3$ (iii) $(1 * 2) * 3$

(ii) $\frac{1}{2} * \frac{1}{3}$ (iv) $4 * (3 * 2)$

Solution…

$a * b = a + b - 3$

(i) $2 * 3 = 2 + 3 - 3 = 2$

(ii) $\frac{1}{2} * \frac{1}{3} = \frac{1}{2} + \frac{1}{3} - 3 = -\frac{13}{6}$

(iii) Recall **BODMAS**

$1 * 2 = 1 + 2 - 3 = 0$

Now, $(1 * 2) * 3 = 0 * 3$

$= 0 + 3 - 3 = 0$

(iv) Similarly, $3 * 2 = 3 + 2 - 3 = 2$

$\therefore 4 * (3 * 2) = 4 * 2$

$= 4 + 2 - 3 = 3$

Example 2

If $x \circ y = 2x + 3y$. Solve the equations

(i) $m \circ 4 = 20$ (iii) $m \circ 3 = 4 \circ m$

(ii) $5 \circ m = 11$ (iv) $m \circ m = 1$

Solution…

$x \circ y = 2x + 3y$

(i) $m \circ 4 = 20$

$2m + 3(4) = 20$

$2m = 20 - 12$

$2m = 8$ $\therefore m = 4$

(ii) $5 \circ m = 11$

$2(5) + 3m = 11$

$10 + 3m = 11$

$3m = 1$

$m = \frac{1}{3}$

(iii) $m \circ 3 = 4 \circ m$

$2m + 3(3) = 2(4) + 3m$

$2m + 9 = 8 + 3m$

$9 - 8 = 3m - 2m$

$1 = m$ $\therefore m = 1$

(iv) $m \circ m = 1$

$2m + 3m = 1$

$5m = 1$

$m = \frac{1}{5}$

Exercise 1 **Date:……………………..**

If $a * b = a + 3b$, evaluate

(i) $1 * 2$ (iv) $5 * 3$

(ii) $2 * 1$ (v) $3 * (-1)$

(iii) $3 * 5$ (vi) $2 * \frac{1}{3}$

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Exercise 2 **Date:..........................**

If $x \circ y = 2x - y$, evaluate

(i) $4 \circ 1$
(ii) $8 \circ 0$
(iii) $3 \circ (-1)$
(iv) $(-2) \circ (-4)$
(v) $(4 \circ 1) \circ 2$
(vi) $(3 \circ 2) \circ 1$

Exercise 3 **Date:..........................**

If $a * b = a^2 + b^2$, evaluate

(i) $3 * 2$
(ii) $4 * 5$
(iii) $2 * 1$
(iv) $(3 * 2) * 4$
(v) $1 * (3 * 2)$
(vi) $(2 * 1) * (3 * 2)$

Exercise 4 **Date:..........................**

If $a * b = 3a - b$, solve the equations

(i) $x * 2 = 7$
(ii) $x * 5 = 2$
(iii) $x * 1 = 5$
(iv) $3 * x = 2$
(v) $x * 4 = 10$
(vi) $4 * x = 5$
(vii) $x * x = 12$

Exercise 5 **Date:..........................**

If $a * b = a + 2b + 2$, solve the equations

(i) $x * 3 = 12$
(ii) $2 * x = 14$
(iii) $x * 6 = 24$
(iv) $x * x = 8$
(v) $3 * x = 3$
(vi) $x * 3x = 23$

Exercise 6 **Date:.........................**

1. If $a * b = \frac{ab}{a+1}$
   (i) Evaluate $2 * 3$
   (ii) Evaluate $\frac{1}{2} * \frac{1}{3}$
   (iii) Solve the equation $x * 5 = 2$

2. The operation $*$ is defined by $a * b = a(b + 2)$
   (i) Evaluate $2 * 5$
   (ii) Evaluate $1 * (2 * 3)$
   (iii) Solve the equation $4 * x = 16$

Exercise 7 **Date:..........................**

1. The operation $*$ is defined by
   $p * q = p + q + pq$
   (a) Evaluate (i) $3 * 5$ (ii) $5 * -2$
   (b) Find n, when $7 * n = 23$
   (c) Find y when $\left(1 * -\frac{1}{2}\right) * y = 5$

2. If $a * b = \frac{a+b}{ab}$
   (a) Evaluate (i) $1 * 2$ (ii) $5 * 2$
   (iii) $(1 * 2) * \frac{1}{2}$
   (b) Solve the equations
   (i) $x * 3 = 2$
   (ii) $5 * x = 1$
   (c) Simplify $2x * 3x$.

COMMUTATIVE PROPERTY

A binary operation $*$ on the set S is commutative if for all $a, b \in S$, $a * b = b * a$

Example 3

If $x * y = x + y + 4$, when $x, y \in \mathbb{R}$. If $-1, -2 \in \mathbb{R}$, show whether or not $*$ is commutative.

Solution…

$x * y = x + y + 4$

$-1 * -2 = -1 + (-2) + 4 = 1$

$-2 * -1 = (-2) + (-1) + 4 = 1$

Since $-1 * -2 = -2 * -1$, $\therefore$ $*$ is commutative.

Example 4

A binary operation $*$ is defined on the set $\mathbb{R}$ of real numbers by $p * q = p^2 + q^2 - pq$, show that $*$ commuatative.

Solution…

$*$ is commutative if for all $p, q \in \mathbb{R}$, $p * q = q * p$.

Given, $p * q = p^2 + q^2 - pq$ ……………… (1)

Also $q * p = q^2 + p^2 - qp$

$= p^2 + q^2 - pq$ ………………... (2)

Since $(1) = (2)$

i.e. $p * q = q * p$

Hence $*$ is commutative.

Example 5

If $x \Delta y = x^2 - y^2$, where $x, y \in \mathbb{R}$.
If $4, 5 \in \mathbb{R}$, show whether or not $*$ is commutative.

Solution…

$x \Delta y = x^2 - y^2$

$4 \Delta 5 = 4^2 - 5^2 = -9$

$5 \Delta 4 = 5^2 - 4^2 = 9$

Clearly $4 \Delta 5 \neq 5 \Delta 4$

$\therefore$ Δ is not commutative.

Exercise 8 **Date:……………………..**

1. If $a * b = a + 2b + 2$, where $a, b \in \mathbb{R}$. If $3 * -2 \in \mathbb{R}$, show whether or not $*$ is commutative.

2. If $m \Delta n = \frac{m+n}{mn}$, where $m, n \in \mathbb{R}$.
   If $-3 \Delta - 5 \in \mathbb{R}$, show whether or not Δ is commutative.

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Exercise 9 **Date:……………………..**

1. An operation $\circledast$ is defined on the set of real numbers $m \circledast n = \frac{-n}{m^2+1}$ where $m, n \in \mathbb{R}$. If $-3, -10 \in \mathbb{R}$, show whether or not $\circledast$ is commutative.
2. A binary operation $\circledast$ is defined on the set $\mathbb{Q}$, of rational numbers $\frac{a}{b} \circledast \frac{c}{d} = \frac{a+c}{bd}$.
   Evaluate
   (i) $\frac{2}{3} \circledast -\frac{3}{5}$
   (ii) $\frac{1}{2} \circledast \frac{3}{4}$
   (iii) $\frac{1}{3} \circledast \left(\frac{1}{5} \circledast \frac{2}{3}\right)$

Exercise 10 **Date:……………………..**

1. $x * y = \frac{x}{y} + 3$
   (a) Is operation $*$ commutative?
   (b) Find z if $z * 2 = 7$
   (c) Find m if $m * 5 = -4$
   (d) Find k if $4 * k = 5$

2. $m \circ n = m^2 + 2n + 3.$
   Solve the equations
   (i) $3 \circ x = 22$
   (ii) $x \circ 5 = 22$
   (iii) $x \circ x = 2$
   (iv) $3 \circ (x \circ 1) = 24$

Exercise 11 **Date:..........................**

The operation $*$ is defined on the set of real numbers, $\mathbb{R}$ by: $x * y = \frac{x+y}{2}, x, y \in \mathbb{R}$

(i) Evaluate $3 * \frac{2}{5}$

(ii) If $8 * y = 8\frac{1}{4}$, find the value of y.

Exercise 12 **Date:..........................**

An operation $\circledast$ is defined on the set $X = \{1, 3, 5, 6\}$ by $m \circledast n = m + n + 2(\text{mod}7)$, where $m, n \in X$.

(i) Draw a table for the operation

(ii) Using the table, find the truth set of

(α) $3 \circledast n = 3$

(β) $n \circledast n = n$

(γ) $n \circledast n = 3$

Exercise 13 **Date:..........................**

1. A binary operation $*$ is defined on $\mathbb{R}^+$, the set of positive real numbers, by $a * b = a + b + \frac{1}{2}ab$
   (a) Find
      (i) $(5 * 2) * 3$
      (ii) The value of x for which $4 * x = 19$
      (iii) The value of n for which $n * n = 30$
   (b) Determine whether or not $*$ is commutative.

2. If $a * b = 2a - 3b$
   (i) Evaluate $-1 * -3$
   (ii) Solve the equation $x * 2 = 2 * x$
   (iii) Is the operation $*$ commutative or not?

Exercise 14 **Date:..........................**

The operation Δ is defined on the set $T = \{2, 3, 5, 7\}$ by $x \, \Delta \, y = (x + y + xy)\text{mod}8$

(i) Construct modulo 8 table for the operation Δ on the set T.

(ii) Use the table to find

(a) $2 \, \Delta \, (5 \, \Delta 7)$

(b) n such that $2 \, \Delta \, n = 5 \, \Delta \, 7$

Exercise 15 **Date:..........................**

1. A binary $\otimes$ is defined on the set of real numbers, $\mathbb{R}$, by $m \otimes n = mn - n - 2m$, where $m, n \in \mathbb{R}$. If $5 \otimes x = 22$, find the value of x.

2. Given that $x * y = 2x - y$, where x and y are real numbers, find the value of
   (i) $5 * (4 * 5)$
   (ii) y if $y * (3 * y) = 6$

3. The binary operation $*$ is defined on the set $\mathbb{R}$ of real numbers by $m * n = \frac{m-n}{n}$; $n \neq 0$
   (a) Evaluate $3 * (5 * 2)$
   (b) Find the truth set of $8 * k = 12 * 3$

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Exercise 16 **Date:..........................**

1. The operation $*$ is defined on the set $\{0, 1, 2, 3, 4\}$ by the relation $a * b =$ the remainder when $a \times b$ is divided by 3. E.g. $3 * 2 = 0$, since $3 \times 2 = 6$ and $\frac{6}{3} = 2$ remainder 0.
   (a) Construct an operation table for the operation $*$ on the set given.
   (b) Is there an identity element?
   (c) Find x if $2 * x = 1$.
   (d) Find y if $y * 1 = 2$.

2. The operation table $\circ$ is shown.

$\circ$	w	x	y	z
w	x	z	w	y
x	z	y	x	w
y	w	x	y	z
z	y	w	z	x

   Find
   (a) the identity element
   (b) the inverse of w
   (c) the inverse of z
   (d) the inverse of x

3. The operation table for $*$ is shown.

$*$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

   Find.
   (a) the identity element
   (b) $b * d$
   (c) $(b * d) * b$
   (d) the inverse of d
   (e) x if $a * x = c$
   (f) y if $y * c = d$

4. The operation table for $\circledast$ is shown.

$\circledast$	A	B	C	D	E
A	C	D	E	A	B
B	D	E	A	B	C
C	E	A	B	C	D
D	A	B	C	D	E
E	B	C	D	E	A

   Find
   (a) the identity element
   (b) the inverses of A and C
   (c) A $\circledast$ C
   (d) (A $\circledast$ C) $\circledast$ D
   (e) (B $\circledast$ E) $\circledast *$ (D $\circledast$ A)
   (f) X if X $\circledast$ C = A
   (g) Y if Y $\circledast$ Y = B
   (h) Z if (Z $\circledast$ B) $\circledast$ C = A
   Is the set {A, B, C, D, E} closed under $\circledast$?

5. Construct an operation table for the operation $\circ$ on the set $\{1, 2, 3, 4\}$, where $a \circ b =$ the remainder when $a \times b$ is divided by 5.
   (a) What is the identity element?

   Solve the equations
   (b) $x \circ 3 = 1$
   (c) $x \circ x = 4$
   (d) $(4 \circ 2) \circ x = 1$
   (e) $x \circ (3 \circ 3) = 3$
   (f) $x \circ (x \circ 2) = 3$
   (g) $(x \circ 3) \circ x = 3$

6. The operation table for the set $\mathrm{X} = \{a, b, c, d, e, f\}$ under the operation $*$ is shown.

$*$	a	b	c	d	e	f
a				c		
b	a					
c		d		a		
d		b				
e		e	c			
f	d	f	b	e	a	

   Complete the table, given that
   (a) the operation $*$ is commutative,
   (b) each element of X appears just once in each row and column,
   (c) the set X is closed under $*$.

(There may be more than one solution)

7. Repeat question **7** for the operation tables below which are subject to the same conditions.

(a)

$*$	a	b	c	d	e	f
a		a	b			
b			d		e	
c					c	
d	c			b		e
e	f					
f		b				

(b)

$*$	1	2	3	4	5	6
1		3	6			
2			5		4	
3					1	
4					5	4
5	2					
6		6				

Exercise 17 **Date:..........................**

1. A binary operation $*$ is defined on the set of rational numbers by $m * n = \frac{m^2-n^2}{2mn}$.
   (a) Find $-3 * 2$.
   (b) Show whether or not $*$ is associative.

2. The binary operation $*$ is defined on the set of real numbers, $\mathbb{R}$, by $a * b = \frac{a}{b} + \frac{b}{a}$. If $(\sqrt{x}+1) * (\sqrt{x}-1) = 4$, find the value of x.

3. Two binary operations $*$ and ∇ are defined as follows $p * q = \frac{1}{p} + \frac{1}{q}$ and $p \nabla q = \frac{1}{p} - \frac{1}{q}$ where $p \neq 0$ and $q \neq 0$. If $p = \frac{3}{5}$ and $q = \frac{1}{3}$, evaluate
   (i) $p * q$
   (ii) $p \nabla q$
   (iii) $\frac{p * q}{p \nabla q}$
   (iv) If $p * q = \frac{1}{p \nabla q}$, evaluate $p^2 \nabla q^2$.

4. A binary operation $*$ is defined on the set $\mathbb{R}$ of real numbers by $a * b = \frac{1+ab}{a+b}$, $a \neq -b$.
   (i) Is the operation closed?
   (ii) Find the identity element e under the operation.
   (iii) Find the inverse under the operation.
   (iv) Is the operation associative?

5. A binary operation $\otimes$ is defined on the set $\mathbb{R}$ of real numbers by $x \otimes y = \frac{x+y}{1+xy}$, where $x, y \in \mathbb{R}, xy \neq -1$.
   Determine whether or not the operation $\otimes$ is
   (i) commutative.
   (ii) associative.

6.
   (a) Let $*$ be a binary operation on a non – empty set S. State what it means for
   (i) $*$ to be associative;
   (ii) $*$ to be commutative;
   (iii) e to be an identity element of S.

   (b) Let $S = \{a, b, c, d\}$ and define a binary operation $*$ on S by the multiplication table.

$*$	a	b	c	d
a	d	a	b	b
b	a	b	c	d
c	a	c	d	b
d	b	d	a	c

   (i) Find an identity element of S with respect to $*$.
   (ii) Determine which (if any) elements have an inverse, in each case giving the inverse.
   (iii) Determine whether $*$ is commutative, briefly justifying your answer.
   (iv) Show that $*$ is not associative.

ALGEBRAIC EXPRESSIONS

In algebra, letters can be used to stand for numbers. A pronumeral is a letter that stands for a number. If a pronumeral could represent any number rather than just one, it is also called a variable.

- The parts of an algebraic expression are called terms. Terms are separated from each other by + or − signs. So $a - b$ is an expression with two terms, but ab is an expression with only one term and $3 + \frac{a}{b} - \frac{b}{4}$ is an expression with three terms.

ADDITION AND SUBTRACTION OF LIKE TERMS

Algebraic terms that have the same variable factors are called like terms. $3x$ and $4x$ are like terms, x^2y and $5x^2y$ are like terms.
The variables and any indices attached to them have to be identical for them to be like terms.

NOTE: Variables in a different order mean the same thing, so xy and yx are like terms.
When the algebraic terms have different variable factors, they are called unlike terms so $2x$ and $3y$ are unlike terms. So x^2y and xy^2 are unlike terms.

Like terms can be added or subtracted to simplify algebraic expressions.
The distributive properties of arithmetic are use to simplify algebraic sums of like terms.

If a, b and c are rational numbers then,
$a(b + c) = ab + ac$

$(a + b)c = ac + bc$

$a(b - c) = ab - ac$

$(a - b)c = ac - bc$

Example 1
Simplify the following.
1. $7a + 10a$
2. $8x - 5x - 2x$
3. $3t + 4t - t$
4. $3a + 5b + 2a - b$
5. $13a + 8b + 2a - 5b - 4a$

Solution…
1. $7a + 10a = (7 + 10)a = 17a$

2. $8x - 5x - 2x = (8 - 5 - 2)x$
$= x$

3. $3t + 4t - t = (3 + 4 - 1)t$
$= 6t$

4. $3a + 5b + 2a - b = 3a + 2a + 5b - b$
$= (3 + 2)a + (5 - 1)b$
$= 5a + 4b$

5. $13a + 8b + 2a - 5b - 4a$
$= 13a + 2a - 4a + 8b - 5b$
$= (13 + 2 - 4)a + (8 - 5)b$
$= 11a + 3b$

Exercise 1 **Date:……………………..**
Simplify the following.
1. $3x + 4x$
2. $5x + 3x + x$
3. $7x - 3x$
4. $4y - 8y - 16y$
5. $3a + 8 + 6a$
6. $4x + 5y + 6x + 7y$
7. $20y - 32x - 36x + 8y$
8. $5a + 2a + b + 8b$
9. $3x + 7x + 3y - 4x + y$
10. $10 + 7y - 3x + 5x + 2y$

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Exercise 2 **Date:……………………..**

Simplify the following.

1. $2p - q - 3q - 5p$
2. $2x - 6y + 3x + 2y$
3. $-8a + 7b - a - 2b$
4. $4c + 2b - c + 6b$
5. $4x^2 - 2x^2$
6. $10xy^2 - 8xy^2$
7. $13x^2y - 4x^2y$
8. $12x^2 - 4x + 2x^2$
9. $5x^2 + 3x^2 - 2xy + 3xy$
10. $3xy + 4xy + 5xy$
11. $5uv + 12v + 4uv - 5v$
12. $5w + 7p^2 - 4w + 3p^2$
13. $3x^2 + 4x + 12x^2 - 5x$

Exercise 3 **Date:……………………..**

Simplify the following.

1. $4f^2 - 3y + 4y - 9f^2$
2. $3x^2 + 6xy - 3y^2 + 4x^2 - 8xy + 2y^2$
3. $2x - 6y + 3x + 2y$
4. $5x^3 - 3x^3 + 7x^3$
5. $5a + 4b - 2a - b + 3a - 2b$
6. $6a + 5h - 4a - 8h$
7. $7e - 5f + 4e - f$
8. $2a - b + 5a - 3b$
9. $5j + 2k + j - 3k$
10. $10x - 15 - 6x + 8$
11. $5t - 2t + 4t$

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Exercise 4 **Date:……………………..**

Simplify the following.

1. $4 + x + 4xy + 2xy + 5x$
2. $10 - 3xy + 8x + 4 + 9xy - 7$
3. $12mn + 3m + 2n + 5nm$
4. $7u + 3v + 2uv - 3u$
5. $12b + 4a + 2b + 3a + 4$
6. $xy - 2xz + 7xy$
7. $12x^2 - 4x^2 + 2x^2$
8. $4xyz - 3xy + 2xz - xyz$
9. $5xy - 4 + 3yx - 6$
10. $9xy + 3x + 6xy - 2x$

MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

Example 2

Simplify the following.

1. $2b \times b$
2. $6a \times 3b$
3. $3xy \times 7xy$

Solution…

1. $2b \times b = 2 \times b \times b = 2b^2$

2. $6a \times 3b = 6 \times 3 \times a \times b = 18ab$

3. $3xy \times 7xy = 3 \times 7 \times x \times x \times y \times y$
 $= 21x^2y^2$

Exercise 5 **Date:……………………..**

Simplify the following.

1. $3 \times a \times b$
2. $4m \times m$
3. $3x \times 5x$
4. $2x \times 3y \times 2$
5. $8abc \times 2ab$
6. $4xy^2 \times 2x^2y$
7. $3 \times 2x \times 5$
8. $2x \times 3y \times 2xy$
9. $9 \times x^2 \times xy$
10. $2cb \times 3a \times 4d$

EXPANDING BRACKETS

The distributive properties are also used here:

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

Example 3

Expand each of the following.

1. $3(x + y)$
2. $2b(a - b)$
3. $\frac{1}{4}(16y - 20x)$

Solution…

1. $3(x + y) = 3x + 3y$

2. $2b(ab - b) = 2ab - 2b^2$

3. $\frac{1}{4}(16y - 20x) = \frac{1}{4} \times 16y - \frac{1}{4} \times 20x$
 $= 4y - 5x$

Exercise 6 **Date:……………………..**

Expand the following expressions.

1. $8(a + 4)$
2. $9(e - 7)$
3. $7(6 - x)$
4. $5(2x + 4y + 3z)$
5. $-3(y - 3t + 4m)$

Exercise 7 **Date:…………………….**

Expand and simplify the following.

1. $2(5 + x) + 3x$
2. $3(y - 2) + 4y$
3. $2z + 3(z - 4)$
4. $3 - 2(x - 1)$
5. $5a - 4(a - 2b)$
6. $2(x - 2) + 2(x + 3)$
7. $3(x + 2) + 4(x + 5)$
8. $4(2r - 3) - 5(6r + 9)$
9. $6(2y - 3) - 5(y + 1)$
10. $5(x + 3) + 4(2x - 6)$

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Exercise 8 **Date:……………………..**

Expand and simplify the following.

1. $3(2x-1)-4(x-5)$
2. $4(x-2)+3x(4-y)$
3. $x(2x+3)+3(5-2x)$
4. $x(2x+3)+5(x-7)$
5. $3x(x-2)-2x(3x-5)$
6. $x(x+3)+4x(x-1)$
7. $3(4xy-2x)+5(3x-xy)$
8. $3x(4-8y)+3(2xy-5x)$
9. $3x^2(4-x)+2(5x^2-2x^3)$
10. $5x(x+7y)-2x(2x-y)$

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Exercise 9 **Date:……………………..**

Expand and simplify

1. $4(2x + 1) - 3x$
2. $8x - 3(2 + x) + 4$
3. $2(3x - 1) - (5 - 2x)$
4. $x(2x + 5) - 9$
5. $5(7y + 3) - 18y$
6. $6(3 - x) - (x + 1)$
7. $3x(x - 7) - 8(x - 5)$
8. $15 - 3(4x - 5)$
9. $7x + 5 - 3(x - 4)$

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Exercise 10 **Date:……………………..**

Expand and simplify

1. $2(a+3)+3(a+1)$
2. $5(2x+1)-2(x-1)$
3. $5(2x+5)-4(x+2)$
4. $2(a-7)-3(a-9)$
5. $-3(w-5)+3(w+5)$
6. $-5(3h+1)-(2h-5)$
7. $-2a(3a+b)+b(a+4)$
8. $-(x+1)-4(x-1)$
9. $7(x^2-x+1)-3(2x-1)$
10. $x-\left(x-\left(x-(x-1)\right)\right)$

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Exercise 11 **Date:……………………..**

Simplify as far as possible.

1. $7(3x - 4y) - 3(5x + 2y)$
2. $3x - 5(4x - 2)$
3. $2(3x + 4) - 2(2 - x)$
4. $2(3x - 4) - 5(x - 3)$
5. $(-2p - 3q + 5) + (3p - 4q - 6)$
6. $(4x + 3y - 5z) - (2x - 3y + 4z)$
7. $2(3x - x^2) + 2(x^3 - 3x + 4)$
8. $2(6x - 5z + 3y) - (5y - 2x + z)$
9. $3x - (p - x) - (r - p)$
10. $x(y + z) + y(x + z) - z(x + y)$
11. $3[4x - 5(3x - 5)] - 1$

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ALGEBRAIC FRACTIONS

CANCELLATION

Number fractions can be simplified by cancelling common factors.

For example, $\frac{12}{20} = \frac{4\times3}{4\times5} = \frac{3}{5}$, where the common factor 4 was cancelled.

The same principle can be used to algebraic fractions:

> If the numerator and denominator of an algebraic fraction are both written in factored form and the common factors are found, we can simplify by cancelling the common factors.

For example, $\frac{8xy}{2x} = \frac{2\times2\times2\times x\times y}{2\times x} = \frac{4y}{1} = 4y$

ILLEGAL CANCELLATION

A fraction such as: $\frac{x+2}{2}$.

The expression in the numerator, $x + 2$, cannot be written as the product of factors other than $1 \times (x + 2)$. x and 2 are terms of the expression, not factors.

Error in cancellation: $\frac{x+2}{2} = \frac{x+1}{1} = x + 1$.

NOTE:

When cancelling in algebraic fractions, only factors can be cancelled, not terms.

Exercise 12 **Date:……………………..**

Simplify.

1. $\frac{2x}{5x}$
2. $\frac{2x}{4}$
3. $\frac{21x}{7x}$
4. $\frac{9x}{3xy}$
5. $\frac{2ab}{40y}$

Exercise 13 **Date:……………………..**

Simplify if possible.

1. $\frac{15x^2y^3}{3xy^4}$
2. $\frac{8abc^2}{4bc}$
3. $\frac{(2a)^2}{a}$
4. $\frac{(2a)^2}{4a^2}$
5. $\frac{(3a^2)^2}{18a^3}$

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Exercise 14 **Date:……………………..**

Simplify if possible.

1. $\frac{2(t-1)}{2}$
2. $\frac{4(t+1)}{2}$
3. $\frac{(x+2)(x+5)}{5(x+5)}$
4. $\frac{(x+5)(2x-1)}{3(2x-1)}$
5. $\frac{x^2(x+1)}{x(x+1)(x-1)}$
6. $\frac{(x+3)(x+1)}{4(x+2)}$
7. $\frac{(t+2)^2(x-1)^2}{(x-1)^2x^2}$

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WORD PROBLEM

If x represents an unknown number, then:

1. 3 more than the number: $x + 3$.
2. 4 less than the number: $x - 4$.
3. 8 times the number: $8x$
4. Half the number $\frac{x}{2}$.
5. 3 times the number is subtracted from 2 and the result is multiplied by 9: $9(3 - 3x)$
6. 5 less than the number and result is 3 times the number: $x - 5 = 3x$.
7. 6 more than $\frac{1}{3}$ the number and the result is 4: $\frac{1}{3}x + 6 = 4$.
8. 1 more than 3 times the number is 4 less than 5 times the number: $3x + 1 = 5x - 4$.

Exercise 15 **Date:……………………..**

Write an expression for each of the following.

1. 6 more than p.
2. 8 less than m.
3. The sum of t and q.
4. Four times q.
5. The product of p and q.

Exercise 16 **Date:……………………..**

Write an expression for each of the following.

1. 2 more than x
2. The sum of p and 7
3. Double the value of m
4. 8 lots of q
5. Half of x

Exercise 17 **Date:……………………..**

Write an expression for each of the following

1. 21 less than t
2. The product of v and 6
3. 9 less than p
4. One third of m
5. q is subtracted from 3

Exercise 18 **Date:……………………..**

Write an expression for each of the following:

1. 7 is added to x, then the result is doubled.
2. x is tripled, then 5 is added.
3. y is multiplied by 6, then 7 is subtracted.
4. 2 is subtracted from p, then the result is multiplied by 7.
5. The product of m and n is subtracted from 13.

Exercise 19 **Date:……………………..**

1.
   (a) Write an expression in terms of n for:
   - (i) the sum of a number and 12
   - (ii) twice a number minus four
   - (iii) a number multiplied by x and then squared
   - (iv) the square of a number cubed.

   (b) Two positive whole number p and q are such that p is greater than q and the sum is equal to three times their difference
   - (i) express p in terms of q
   - (ii) hence, evaluate $\frac{p^2+q^2}{pq}$.

2.
   - (i) What is the result of subtracting $3x^2 - 4x - 1$ from $4x^2 + x + 1$?
   - (ii) Subtract $(-y + 3x + 5z)$ from $(4y - x - 2z)$.
   - (iii) Subtract $\frac{1}{2}(a - b - c)$ from the sum of $\frac{1}{2}(a - b - c)$ and $\frac{1}{2}(a - b - c)$.
   - (iv) By how much is the sum of $3x, (6x - 5), 9x$ and $(4x + 1)$ less than $30x$.
   - (v) Simplify
     α) $3a^2b^3 \times 4a^3b$
     β) $3x^2 + 6xy - 3y^2 + 4x^2 - 8xy + 2y^2$.

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MULTIPLICATION OF TWO BINOMIALS

$$(a+b)(c+d) = a(c+d) + b(c+d)$$
$$= ac + ad + bc + bd$$

Example 4

Expand and simplify the following

1. $(x+1)(x+2)$
2. $(x+3)(3x-2)$

Solution…

1. $(x+1)(x+2) = x(x+2) + 1(x+2)$
   $= x^2 + 2x + x + 2$
   $= x^2 + 3x + 2$

2. $(x+3)(3x-2) = x(3x-2) + 3(3x-2)$
   $= 3x^2 - 2x + 9x - 6$
   $= 3x^2 + 7x - 6$

Exercise 20 **Date:……………………..**

Expand and simplify the following.

1. $(x+2)(x+3)$
2. $(x-2)(x+3)$
3. $(x-2)(x-3)$
4. $(2x+5)(3x-4)$

Exercise 21 **Date:……………………..**

Expand and simplify the following.

1. $(2x - 3y)(3x + 4y)$
2. $(2p + 3)(3p - 2)$
3. $(3a - 2b)(4a + 3b)$
4. $(3x + 2)(2x - 4)$

Exercise 22 **Date:……………………..**

Expand and simplify

1. $(m + 5)(m - 1)$
2. $(a + 4)(a - 3)$
3. $(m - n)(m + n)$
4. $(m - 5)(m - 1)$
5. $(x + 3)(x + 2) + x^2 - 1$
6. $(x + 4)^2 + 2x - 3$
7. $(x + 4)(x + 1) - (x + 1)$
8. $(x + 1)(x + 2) - (x - 1)(x + 1)$
9. $x^2 - (x - 1)^2$

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Exercise 23 **Date:……………………..**

Expand and simplify the following.

1. $(2t-5)(t-4)$
2. $(2a-1)(3a+2)$
3. $(3x-2y)(x-3y)$
4. $(1-2x)(4x+1)$
5. $(x^2+2x+3)(x-1)$
6. $(2x^2-3x+1)(2-3x)$
7. $(3x^2-2x-4)(3-5x)$
8. $(x^2+2x+1)(x-2)$
9. $(x-1)(2x+3)(x+4)$
10. $\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)$

Example 5

Expand and simplify.

1. $(x+2)^2$
2. $(4x+1)^2$
3. $(3-2x)^2$

Solution…

1. $(x+2)^2 = (x+2)(x+2)$
$= x(x+2) + 2(x+2)$
$= x^2 + 2x + 2x + 4$
$= x^2 + 4x + 4$

2. $(4x+1)^2 = (4x+1)(4x+1)$
$= 4x(4x+1) + 1(4x+1)$
$= 16x^2 + 4x + 4x + 1$
$= 16x^2 + 8x + 1$

3. $(3-2x)^2 = (3-2x)(3-2x)$
$= 3(3-2x) - 2x(3-2x)$
$= 9 - 6x - 6x + 4x^2$
$= 9 - 12x + 4x^2$

Exercise 24 **Date:……………………..**

Expand and simplify.

1. $(x+3)^2$
2. $(x-5)^2$
3. $(4-y)^2$
4. $(3x+4)^2$
5. $(1-2x)^2$

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Exercise 25 **Date:……………………..**

Expand and simplify.

1. $3x + 1 + (x - 2)^2$
2. $5x + 2 - (x + 2)^2$
3. $3(x - 1)^2 + (x - 1)$
4. $(2x - 3)^2 - 3x(x - 4)$
5. $16 - 4(3x - 2)^2$
6. $(1 - x)^2 + (x + 2)^2$
7. $(2x + 3)(2x - 3) - (x + 1)^2$
8. $2(x - 3)^2 - (2x - 3)^2$

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Exercise 26 **Date:……………………..**

1. Expand and simplify the following.
   (i) $(3x+1)^2 - 2(2x-3)^2$
   (ii) $(2x+5)^2 - (x-3)^2$
   (iii) $(x+4)(5x-3) - 3(x-2)^2$
   (iv) $(2x+1)(x-3)(x+4)$
   (v) $(x-5)(x^2+3) - (x+4)(x-1)$

2.
   (i) Find the coefficient of x^3 in the expansion of $(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1)$
   (ii) Find the coefficient of x^4 in the expansion of $x(x^2 + 2x + 3)(x^2 + 7x - 2)$
   (iii) The coefficient of x^2 in the expansion of $(x+3)(x+k)(2x-5)$ is -3. Find the value of the constant k
   (iv) Expand $(x-2)^2(x+1)$, simplifying your answer.

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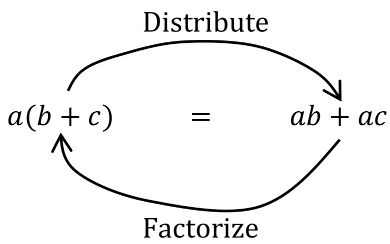
FACTORIZATION



Example 1

Factorize each of the following

1. $2x+4$
2. $3a^2+a$
3. $4xy+8y^2$

Solution…

1. $2x+4=2(x+2)$
2. $3a^2+a=a(3a+1)$
3. $4xy+8y^2=4y(x+2y)$

Exercise 27 **Date:……………………..**

Factorize the following

1. $12m-36$
2. $3x+6$
3. $13v-26t$
4. $3p-15q$
5. $3uv+9vw$
6. $15pq+21p$
7. $2xy-4yz$

Exercise 28 **Date:……………………..**

Factorize fully

1. x^2+x
2. $4x^2+4$
3. $9m^2-18m$
4. a^2-ab
5. $3p^2-6pq$
6. $12ax^3+18xa^3$
7. $20p+25p^2$
8. $15c^2-5c$
9. $5a^2b+ab^2$
10. ax^2+bx^2
11. $7x^7+14x^{14}$
12. $6x^2+6x$

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Exercise 29 **Date:……………………..**

Factorize the following expressions

1. $3k^2 + 5klm$
2. $5mp - 3mnq$
3. $2a^2b^2 - 3b^2c^2$
4. $\frac{1}{4}ax^2 + \frac{1}{8}ax$
5. $\frac{1}{81}v - \frac{1}{27}w$
6. $4y^2 - 10xy$

Example 2
Factorize the following
1. $3x(4x+2)-7(4x+2)$
2. $2x(x-1)+5(x-1)$

Solution…
1. $3x(4x+2)-7(4x+2)=(4x+2)(3x-7)$
2. $2x(x-1)+5(x-1)=(x-1)(2x+5)$

NOTE: $b-a=-1(a-b)$

Exercise 30 **Date:……………………..**
Factorize the following
1. $x(x+1)-2(x+1)$
2. $3x(1-x)+2(1-x)$
3. $(m+n)(2x-y)-x(m+n)$
4. $m(2a-b)-2n(b-2a)$
5. $x(a-c)+y(c-a)$
6. $(m-n)(3m+2n)-(m-n)^2$

Example 3
Factorize completely
1. $ax+ay+bx+by$
2. $am-bm-an+bn$
3. $m^2+nt-mt-mn$
4. t^3+t^2+t+1

Solution…
1. $ax+ay+bx+by=a(x+y)+b(x+y)$
$=(x+y)(a+b)$

2. $am-bm-an+bn=m(a-b)-n(a-b)$
$=(a-b)(m-n)$
3. $m^2+nt-mt-mn=m^2-mn-mt+nt$
$=m(m-n)-t(m-n)$
$=(m-n)(m-t)$
4. $t^3+t^2+t+1=t^2(t+1)+1(t+1)$
$=(t+1)(t^2+1)$

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Exercise 31 **Date:……………………..**

Factorize the following completely

1. $2x + 2y + 3x + 3y$
2. $pm - qm - pn + qn$
3. $2kp - km + 6p - 3m$
4. $px - 2qx - 4qy + 2py$
5. $bx - ax + x^2 - ab$
6. $2xy - 6pq - 3py + 4qx$

Exercise 32 **Date:……………………..**

Factorize the following completely

1. $xy - 18 + 3y - 6x$
2. $ac - bc - ad + bd$
3. $pr + 3ps + 2qr + 6qs$
4. $4xy + 8y^2 - 8xd - 16yd$
5. $2xy - 6mn - 3my + 4nx$
6. $xy + xw + 2ay + 2aw$

Exercise 33 **Date:……………………..**

Factorize the following completely

1. $6pq - 3rs + 3qs - 6pr$
2. $tr - ps - pt + rs$
3. $x + y - ax - ay$
4. $x(x - 2) - 3xy + 6y$
5. $x^2 - 2x - 3xy + 6y$
6. $am + bn - an - bm$
7. $6ax - 12by - 9ay + 8bx$

FACTORIZATION OF QUADRATIC EXPRESSION

A quadratic expression in one variable is of the form $ax^2 + bx + c$, where a, b and c are constant and $a \neq 0$. a is the coefficient of x^2, b is the coefficient of x.

Case 1

When the coefficient of x^2 is 1, i.e. $a = 1$, then the quadratic expression becomes $x^2 + bx + c$.

To factorize $x^2 + bx + c$, find two integers p and q such that: the sum $p + q = b$ and the product $p \times q = c$.

Hence $x^2 + bx + c = x^2 + (p + q)x + pq$

$$= x^2 + px + qx + pq$$
$$= x(x + p) + q(x + p)$$
$$= (x + p)(x + q)$$

Example 4

Factorize

1. $x^2 + 3x + 2$
2. $x^2 + x - 6$
3. $x^2 - 7x - 18$

Solution…

1. $x^2 + 3x + 2$
   Two integers such that the sum is 3 and the product is 2 are 2 and 1.
   $\therefore x^2 + 3x + 2 = x^2 + (2 + 1)x + 2$
   $= x^2 + 2x + x + 2$
   $= x(x + 2) + 1(x + 2)$
   $= (x + 1)(x + 2)$

2. $x^2 + x - 6$
   Two integers such that the sum is 1 and the product is -6 are 3 and -2.
   $\therefore x^2 + x - 6 = x^2 + (3 - 2)x - 6$
   $= x^2 + 3x - 2x - 6$
   $= x(x + 3) - 2(x + 3)$
   $= (x + 3)(x - 2)$

3. $x^2 - 7x - 18$
   Two integers such that the sum is -7 and the product is -18 are 2 and -9
   $\therefore x^2 - 7x - 18 = x^2 + (2 - 9)x - 18$

$= x^2 + 2x - 9x - 18$
$= x(x + 2) - 9(x + 2)$
$= (x + 2)(x - 9)$

Exercise 34 **Date:……………………..**

Factorize completely

1. $x^2 + 5x - 6$
2. $x^2 + 7x + 12$
3. $x^2 - x - 6$
4. $x^2 - 12x - 45$
5. $a^2 - 2a - 24$
6. $x^2 + 3x - 154$
7. $x^2 - 2x - 35$
8. $x^2 - 12x + 36$

Case 2
When $a \neq 1$, i.e. a is any integer than one then the quadratic expression becomes $ax^2 + bx + c$.
To factorize, find the two integers p and q such that their sum $p + q = b$ and product $p \times q = ac$.
Hence; $ax^2 + bx + c = ax^2 + (p + q)x + c$
$= ax^2 + px + qx + c$
to be factorized.

Example 5
Factorize completely
(1) $2x^2 + 5x + 3$ (2) $6m^2 + 7m - 3$

Solution…
1. $2x^2 + 5x + 3$
   Two integers such that their sum is 5 and product 6 are 3 and 2
   $\therefore 2x^2 + 5x + 3 = 2x^2 + (3 + 2)x + 3$
   $= 2x^2 + 3x + 2x + 3$
   $= x(2x + 3) + 1(2x + 3)$
   $= (2x + 3)(x + 1)$

2. $6m^2 + 7m - 3$
   Two integers such that their sum is 7 and product -18 are 9 and -2.
   $\therefore 6m^2 + 7m - 3 = 6m^2 + (9 - 2)m - 3$
   $= 6m^2 + 9m - 2m - 3$
   $= 3m(2m + 3) - 1(2m + 3)$
   $= (2m + 3)(3m - 1)$

Exercise 35 **Date:……………………**
Factorize the following
1. $5x^2 + 13x - 6$
2. $2t^2 - 3t - 2$
3. $6x^2 + 11x + 4$
4. $3a^2 - 11a + 6$
5. $2x^2 - 21x + 45$
6. $2x^2 + x - 15$
7. $3y^2 + 2y - 8$
8. $2y^2 - 5y + 2$
9. $35x^2 + 12x + 1$
10. $6x^2 - 5x - 1$
11. $12x^2 - x - 1$

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Exercise 36 **Date:……………………..**

Factorize the following

1. $35 - 2b - b^2$
2. $9 + 35x - 4x^2$
3. $7 - 6x - x^2$
4. $3 - 2x - x^2$
5. $m^2n - 2mn^2 + n^3$
6. $x^2 + 4x + 3 + mx + 3m$

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Exercise 37 **Date:……………………..**

Factorize the following completely

1. $6x^2 + 5xy - 6y^2$
2. $2y^2 + xy - 3x^2$
3. $2s^2 - 3st - 2t^2$
4. $6x^2 + 7xy - 5y^2$
5. $10 - 3x - x^2$
6. $5y^2 + 2ay - 3a^2$
7. $x^3 - 5x^2y + 6xy^2$

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DIFFERENCE OF TWO SQUARES

$$a^2 - b^2 = (a-b)(a+b)$$

Memorize this result.

Example 5
Factorize the following
1. $m^2 - n^2$
2. $4t^2 - 1$
3. $25x^2 - 4y^2$
4. $x^2 - \frac{1}{4}$
5. $x^3 - 5x$

Solution…
1. $m^2 - n^2 = (m-n)(m+n)$

2. $4t^2 - 1 = 2^2t^2 - 1^2$
$= (2t)^2 - 1^2$
$= (2t-1)(2t+1)$

NB: $(ab)^n = a^nb^n$
$\therefore 4t^2 = 2^2t^2 = (2t)^2$

3. Similarly, $25x^2 - 4y^2 = 5^2x^2 - 2^2y^2$
$= (5x)^2 - (2y)^2$
$= (5x-2y)(5x+2y)$

4. $x^2 - \frac{1}{4} = x^2 - \left(\frac{1}{2}\right)^2$
$= \left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$

NB: $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
i.e. $\frac{1}{4} = \frac{1^2}{2^2} = \left(\frac{1}{2}\right)^2$

5. $x^3 - 5x = x(x^2 - 5)$
$= x\left(x^2 - \left(\sqrt{5}\right)^2\right)$
$= x(x-\sqrt{5})(x+\sqrt{5})$

Exercise 38 **Date:……………………..**
Factorize completely
1. $t^2 - 4$
2. $x^2 - 16$
3. $v^2 - 49$
4. $y^2 - 1$
5. $k^2 - 121$

Exercise 39 **Date:……………………..**

Factorize completely

1. $4x^2 - 1$
2. $9m^2 - 1$
3. $9 - 4t^2$
4. $x^2 - 4y^2$
5. $a^2 - 9b^2$
6. $4a^2 - 16$

Exercise 40 **Date:……………………..**

Factorize the following completely

1. $(x + 3y)^2 - (2y + x)^2$
2. $(x - 3y)^2 - (x + 3y)^2$
3. $\left(2a + \frac{1}{3}\right)^2 - \left(2a - \frac{1}{3}\right)^2$
4. $(2a + b)^2 - (b - 2a)^2$

Exercise 41 **Date:……………………..**

Factorize the following expressions completely

1. $3x^3 - 12x$
2. $8x^2 - 72y^2$
3. $32x^3 - 8xy^2$
4. $32x^2y - 48x^3y^3$
5. $8y^2 - 18$
6. $18p^2 - 2$

Exercise 42 **Date:……………………..**

Factorize the following expression completely

1. $36p^2 - 49q^2$
2. $9a^2t^2 - 1$
3. $a^2 - (a - b)^2$
4. $h^2 - k^2 - p(h - k)$
5. $(x + 6)^2 - 36x^2$
6. $9a^2 - 4(a - b)^2$
7. $4b^2 - ab + (a + 9b)^2 - a^2$
8. $m^2 - 2mn + n^2 - 9r^2$

Exercise 43 **Date:……………………..**

1. Without using mathematical tables or calculator, evaluate the following
   (i) $103^2 - 97^2$
   (ii) $6.4^2 - 3.6^2$
   (iii) $88.23^2 - 11.77^2$
   (iv) $61.5^2 - 38.5^2$
   (v) $(73.8)^2 - (26.2)^2$
   (vi) $53.8^2 - 46.2^2$

2. Find R, if $5R^2 + (22.55)^2 = (27.45)^2$

3. Without using mathematical tables or calculators, find the value of y if $13y = 187^2 - 174^2$.

Exercise 44 **Date:……………………..**

1. If $(3.46)^2 - (1.54)^2 = 10x$, find the value of x.

2. Without using calculator, find Q if $3Q + 13^2 = 16^2$.

3. If $y = m^2 - 4n^2$
   a) (i) Factorize $m^2 - 4n^2$
      (ii) Find the value of y when $m = 4.4$ and $n = 2.8$

      (iii) $m = 2x + 3$ and $n = x - 1$, find y in terms of x, in its simplest form.

      (iv) Make n the subject of the formula $y = m^2 - 4n^2$.

   b) (i) $m^4 - 16n^4$ can be written as $(m^2 - kn^2)(m^2 + kn^2)$, write down the value of k.

      (ii) Factorize completely $m^2n - 16n^5$.

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FACTORIZATION AND SIMPLIFICATION

NOTE:

(i) $$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

(ii) $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

(iii) $$b - a = -1(a - b)$$

Exercise 45 **Date:……………………..**

Simplify the following.

(i) $\frac{3+x}{9-x^2}$ (vi) $\frac{(x-5)(x+4)}{x^2+x-30}$

(ii) $\frac{x^2-6x}{x^2-36}$ (vii) $\frac{x^2-7x-8}{x^2-64}$

(iii) $\frac{x^2-3x}{x^2-9}$ (viii) $\frac{x^2-16}{x^2-3x-4}$

(iv) $\frac{x^2+x}{3x+3}$ (ix) $\frac{4(x-6)^2}{(x-6)}$

(v) $\frac{2x^2+5x-3}{2x^2-18}$ (x) $\frac{2x^2-18}{x^2+7x-30}$

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Exercise 46 **Date:……………………..**

Simplify the following

(i) $\frac{7x+21}{2x^2+9x+9}$

(ii) $\frac{h^2-h-20}{h^2-25}$

(iii) $\frac{x^2+6x-7}{3x+21}$

(iv) $\frac{4x^2-16x}{2x^2+6x-56}$

(v) $\frac{x^2-x-20}{x^3-10x^2+25x}$

(vi) $\frac{42np-7n}{12pt-2t+18mp-3m}$

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Exercise 47 **Date:……………………..**

Simplify the following.

1. $\frac{1}{y-1}-\frac{1}{y}$
2. $\frac{x+1}{x}-\frac{y-1}{y}$
3. $\frac{3}{x-4}+\frac{2}{2x+5}$
4. $\frac{1-x}{x}-\frac{2+x}{1-2x}$
5. $\frac{2}{x-2}+\frac{3}{x+2}$
6. $\frac{4}{x+3}+\frac{2x-1}{3}$
7. $2x-\frac{10x}{5-x}$
8. $\frac{5}{x-3}+\frac{3}{x+7}+\frac{1}{2}$
9. $\frac{3x}{x+2}-\frac{5x}{3x-1}+\frac{1}{3}$

Exercise 48 **Date:……………………..**

Simplify the following

1. $\frac{1}{3(x-y)} - \frac{1}{4(x-y)}$
2. $\frac{1}{c} + \frac{1}{d} - \frac{c-d}{cd}$
3. $\frac{6ab+3a^2}{21a^2-9ab^2}$
4. $\frac{1}{x+5} - \frac{2(x+2)}{x^2-25}$
5. $\frac{x^2-y^2}{(x+y)^2} \div \frac{(x-y)^2}{(3x+3y)}$
6. $\frac{1}{x+1} - \frac{2}{x-1} + \frac{3}{x^2-1}$
7. $\frac{2x}{(x-3)^2} - \frac{5}{x-3}$
8. $\frac{1-\frac{4}{x^2}}{x+\frac{2}{x}}$
9. $\frac{x^2-25}{x^2+5x} \div \frac{x^2-2x-15}{x^2-9}$
10. $\frac{r^2+t^2}{2(r+t)^3} - \frac{r-rt}{2(r+t)^2}$

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Exercise 49 **Date:……………………..**

Simplify the following

1. $\frac{4x-4y}{y-x}$
2. $\frac{xy^2-xy}{1-y}$
3. $\frac{x^2-y^2}{3x+3y}$
4. $\frac{m^2-n^2}{n-m}$
5. $\frac{a^2-4b^2}{a^2+4ab+4b^2}$
6. $\frac{4ab^2-2a^2b}{ab}$
7. $\frac{x^2-8x+16}{x^2-7x+12}$
8. $\frac{x^2-4}{4-x^2}$
9. $\frac{3x^2-6x}{4-x^2}$

Exercise 50 **Date:……………………..**

Simplify the following

1. $\frac{54k^2-6}{3k+1}$
2. $\frac{2x^2-5x-12}{4x^2-9}$
3. $\left(\frac{x^2}{2}-x+\frac{1}{2}\right)\left(\frac{1}{x-1}\right)$
4. $\frac{a^2b^4-b^2a^4}{ab(a+b)}$
5. $\frac{\frac{1}{4}m^2-9n^2}{\frac{1}{4}\left(\frac{1}{2}m-3n\right)}$

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Exercise 51 **Date:……………………..**

1. Divide $\frac{x^2-4}{x^2+x}$ by $\frac{x^2-4x+4}{x+1}$
2. Simplify $\frac{x^2+x-6}{x^2-3x+2} \times \frac{x^2-x}{x^2-9}$
3. If $k = \frac{m-1}{m+1}$, find the value of $\frac{k-1}{k} + \frac{2}{m-1}$

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Exercise 52 **Date:……………………..**

Simplify

1. $\frac{(p-r)^2-r^2}{2p^2-4pr}$

2. $\frac{\frac{1}{3}c^2-\frac{2}{3}cd}{\frac{1}{2}d^2-\frac{1}{4}cd}$

3. $\frac{x+\frac{1}{x}+2}{x^2-1}$

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Exercise 53 **Date:……………………..**

1. Simplify $\frac{2-18m^2}{1+3m}$

2. Given that $x = \frac{2m}{1-m^2}$ and $y = \frac{2m}{1+m}$. Express $2x - y$ in terms of m in the simplest form.

3. Simplify $\frac{4}{x^2-4} - \frac{1}{x^2-3x+2}$

4. Simplify $\frac{4a^2}{a^2-b^2} - \frac{2a}{a+b} - \frac{2b}{a-b}$

5. Simplify $\frac{m+1}{m-1} - \frac{m-1}{m+1} + \frac{4}{m^2-1}$

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Exercise 54 **Date:……………………..**

Simplify the following.

1. $\frac{1}{x^2-5x+6}-\frac{2}{x^2-2x}+\frac{2}{x^2-2x}+\frac{3}{x^2-3x}$
2. $\frac{3a}{x^2+ax-2a^2}-\frac{a}{x^2+3ax+2a^2}+\frac{2x}{x^2-a^2}$

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Zeros and Undefined Algebraic Fractions

The algebraic fraction $\frac{f(x)}{g(x)}$ is said to be undefined or not exist if $g(x) = 0$ and it is said to be zero if $f(x) = 0$.

Example 6

For what value of x is $\frac{1-3x}{x^2+4x+6}$ equal to zero?

Solution…

For zeros;

$1 - 3x = 0$

$1 = 3x$

$x = \frac{1}{3}$

Example 7

Find the values of x for which the expression $\frac{3-x-2x^2}{x^2(x+2)}$ is zero?

Solution…

For zeros;

$$3 - x - 2x^2 = 0$$
$$2x^2 + x - 3 = 0$$
$$2x^2 + 3x - 2x - 3 = 0$$
$$x(2x + 3) - 1(2x + 3) = 0$$
$$(2x + 3)(x - 1) = 0$$

$2x + 3 = 0$ or $x - 1 = 0$

$2x = -3$ or $x = 1$

$x = -\frac{3}{2}$

∴ The expression is zero if $x = -\frac{3}{2}$ or $x = 1$.

Example 8

Find the value of x for which $\frac{1}{2x-1}$ does not exist.

Solution…

The expression does not exist when the denominator is equal to zero.

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

∴ The expression does not exist if $x = \frac{1}{2}$.

Example 9

Find the values of x for which the function $f : x \longrightarrow \frac{x+2}{6x^2-5x-1}$ is undefined.

Solution…
$f(x)$ is undefined if the denominator is equal to zero.

$$6x^2 - 5x - 1 = 0$$
$$6x^2 - 6x + x - 1 = 0$$
$$6x(x-1) + 1(x-1) = 0$$
$$(x-1)(6x+1) = 0$$
$$x - 1 = 0 \quad \text{or} \quad 6x + 1 = 0$$
$$x = 1 \quad \text{or} \quad 6x = -1$$
$$x = -\frac{1}{6}$$

The function is undefined if $x = 1$ or $-\frac{1}{6}$.

Exercise 55 **Date:……………………..**

What are the values of x for which the following are not defined?

(i) $\frac{2x-1}{x-2}$

(ii) $\frac{x}{3x-2}$

(iii) $\frac{x}{2x-1}$

(iv) $\frac{x^2-7x+8}{x^2+2x-8}$

(v) $\frac{x-4}{x^2-2x-3}$

(vi) $\frac{3x-3}{\frac{1}{2}x(x+1)}$

(vii) $\frac{1+x}{4-x^2}$

(viii) $\frac{5x+3}{6x(x+1)}$

Exercise 56 **Date:……………………..**

For what value(s) of x is the following equal to zero?

(i) $\frac{x-3}{x^2+5x-6}$

(ii) $\frac{2x}{1-x^2}$

(iii) $\frac{6x^2+x-12}{6x^2-17x+12}$

(iv) $\frac{(2x+3)(1-x)}{x(x+1)}$

(v) $\frac{2-x-x^2}{1+x}$

(vi) $\frac{7-6x-x^2}{1-x}$

(vii) $\frac{9+35x-4x^2}{x^2+2x+1}$

Exercise 57 **Date:……………………..**

What are the values of x for which the following are undefined?

(i) $\frac{1+x}{9-x^2}$

(ii) $\frac{(2x+3)(1-x)}{x(x+1)}$

(iii) $\frac{1}{x}+\frac{1}{x-1}$

(iv) $\frac{2x}{2x^2-8}$

(v) $\frac{5}{x^2+2x-8}$

(vi) $\frac{1}{2}-\frac{3}{x-2}+\frac{2x}{x+3}$

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Exercise 58 **Date:……………………..**

1.

a) Simplify

(i) $\frac{2x+1}{2}-\frac{3x-7}{9}-\frac{5}{18}$

(ii) $\frac{2x}{1-x^2}+\frac{1}{1+x}$

b) If $\frac{p}{q}=\frac{r}{s}$, then $\frac{p+q}{r+s}$ is equal to?

c) If $p=\frac{x+3}{2x-1}$, express $\frac{2p+5}{3p+1}$ in terms of x

2. Simplify the following

(i) $\frac{2}{x-1}-\frac{1}{2(x+1)}$

(ii) $\frac{x+2}{x^2-9}-\frac{3}{x-3}$

(iii) $\left(x-\frac{27}{x^2}\right)\div\left(x-\frac{9}{x}\right)$

(iv) $\frac{1}{x+1}-\frac{2}{x-1}+\frac{2}{x^2-1}$

(v) $6(x-y)^2\div 4(y^3-x^3)$

(vi) $\frac{\frac{2}{x+3}-\frac{3}{x-2}}{\frac{5}{x^2+x-6}+\frac{4}{x-2}}$

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3. Factorize completely
   (a) $3x^2 + xy - 2y^2$
   (b) $x^2 - 4y^2 - 2x - 4y$
   (c) $(x^2 + 1)^2 - 4x^2$
   (d) $p^2 - q^2 - p + q$
   (e) $27 - 3y^2$
   (f) $(x + 5)^2 - (x - 1)^2$
   (g) $xy + 20 + 4y + 5x$
   (h) $2x^2 - 21x + 10$
   (i) $(2p + 3q)^2 - 9q^2$
   (j) $7x^2 + 33x - 10$
   (k) $xy - ys - y^2 + xs$
   (l) $p^2q^2 - 6pqr + 9r^2$
   (m) $(x + y - 1)^2 - (x - y + 1)^2$
   (n) $x^2 + x^2y + 3x - 10y + 3xy - 10$

4. Simplify
   (i) $\frac{a+2b}{a-b} \times \frac{a^2-b^2}{(a+2b)^2}$
   (ii) $\frac{2a}{(a-b)^2} \div \frac{a+b}{a-b}$
   (iii) $\frac{49x^2-25y^2}{5xy} \div \frac{7x+5y}{x^2y^2}$
   (iv) $\frac{2u^2-uv-v^2}{u^2+2uv+v^2} \times \frac{u^2-v^2}{2u+v}$
   (v) $\frac{5abc^2}{a(b+c)} \div \frac{10ac}{(a+c)(b+c)}$
   (vi) $\frac{2a^2+7ab+3b^2}{a^2+ba} \times \frac{a^3-b^2}{a+3b}$

5. Simplify
   (i) $\frac{4s+2t}{2s^2+3st+t^2} \div \frac{2s^2-st-t^2}{s+t}$
   (ii) $\frac{3x+y}{2x^2+5xy+3y^2} \times \frac{2x+3y}{3x^2+4xy+y^2}$
   (iii) $\frac{3xy}{2x^2+5xy+3y^2} \times \frac{2x+3y}{3x^2+4xy+y^2}$
   (iv) $\frac{2x+6y}{x^2-y^2} \times \frac{x+y}{2x^2+8xy+6y^2}$

6. Simplify the following
   (i) $\frac{x+2y}{20} - \frac{3x-2y}{12} - (y - x)$
   (ii) $\frac{1}{x-2} + \frac{2x-1}{2x^2-3x-2} - \frac{2}{2x+1}$

7.
   (a) Given that $xy = a^2$, show that $\frac{1}{a+x} + \frac{1}{a+y} = \frac{1}{a}$
   (b) Show that $\left(\frac{1+x^2}{1-x^2}\right)^2 - \left(\frac{2x}{1-x^2}\right)^2$ has the same numerical value for all $(x \neq \pm 1)$ and determine the value.
   (c) Express $\frac{2a}{x-a} + \frac{3a}{x+a} - \frac{6a^2}{a^2-x^2}$ as a single fraction in its lowest terms. What is the value of the above expression when$x = 6a$?
   (d) Prove that $\frac{\left(x^{\frac{3}{2}}+x^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)}{\left(x^{\frac{3}{2}}-x^{\frac{1}{2}}\right)^2} = \frac{x+1}{x(x-1)}$

8. Simplify the following
   (i) $\frac{x-2}{x^2+5x+6} + \frac{x+2}{x^2+7x+12} + \frac{x+3}{x^2+6x+8}$
   (ii) $\frac{2x}{x^2-y^2} - \frac{3x}{x^2-2xy+y^2}$
   (iii) $\frac{a+b}{a-b} + \frac{a-b}{a+b} - \frac{2ab}{a^2-b^2}$
   (iv) $\frac{3x}{x^2-y^2} - \frac{2x}{x^2-2xy+y^2} - \frac{4xy}{x^3-x^2y-xy^2+y^3}$
   (v) $\frac{x^2+2x-3}{(x-7)^2} \div \frac{x^2+x-6}{x^2-5x-14}$
   (vi) $\frac{x^3-8}{9-x^2} \times \frac{x^2+2x-3}{x^2+2x+4}$
   (vii) $\frac{2x^2+5x+3}{5x^2-24x-5} \times \frac{3x^2-20x+12}{x^2+3x+2} \div \frac{6x^2+5x-6}{4x^2+9x+2}$
   (viii) $\frac{x^2-(2x-3z)^2}{(x-3z)^2-4y^2} \div \left[\frac{4x^2-(3z-x)^2}{(2y-x)^2-9z^2} \times \frac{9z^2-(x-2y)^2}{(3z-2y)^2-x^2}\right]$
   (ix) $\left[\frac{x^2-2x-15}{4y^6z^9} \div \frac{9-4x^2}{36y^7z^7}\right] \times \frac{4x^2+12x+9}{48x^2y^3+60y^3}$
   (x) $\frac{\frac{x+2y}{x+1}\left[1-\frac{2y}{2y-x}\right]}{1-\frac{x+4y^2}{x^2-1}+\frac{1}{x-1}}$
   (xi) $\left(\frac{x+8}{x-1} - x\right)\left(\frac{x}{7x-4} - \frac{1}{x+2}\right) \div \left(x - 6 + \frac{4}{x-2}\right)$
   (xii) $\left(4y - \frac{x^2}{x-y}\right)\left(y - \frac{xy-x^2-y^2}{x-2y}\right) \div \left(2 - \frac{3x}{x+y}\right)$
   (xiii) $\frac{\frac{x+y}{x-y}-\frac{x-y}{x+y}}{\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}}$
   (xiv) $\frac{x+\frac{1}{x}}{1-\frac{1}{1+\frac{1}{x-1}}}$
   (xv) $\frac{\frac{x^2+y^2}{y}-1}{\frac{1}{x}-\frac{1}{y}} \div \frac{x^3+y^3}{x^2-y^2}$

SURDS

We know that $\sqrt{25} = 5$ and that $\sqrt{\frac{1}{36}} = \frac{1}{6}$. These are examples of rational numbers. Rational numbers are numbers that can be expressed as a ratio of two integers, i.e. $\frac{a}{b}$ where $a, b \in \mathbb{Z}$ and $b \neq 0$. However, $\sqrt{2}$ cannot be expressed as a ratio of two integers. It is for this reason that $\sqrt{2}$ is an example of an irrational number.

Roots such as $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \ldots$ are called surds.

NOTE

Roots such as $\sqrt{4}, \sqrt{9}, \sqrt{16}$ etc are called perfect since their result gives a rational number. The sign "$\sqrt{}$" means the positive square root.

$\left[\text{Although } x^2 = 9 \implies x = I3, \text{but } \sqrt{9} = 3\right]$

$\sqrt{3}$ means the positive square root of 3.

Rule 1

$$\sqrt{m} \times \sqrt{m} = \left(\sqrt{m}\right)^2 = m$$

i.e. $\sqrt{2} \times \sqrt{2} = 2$

Rule 2

$$m \times \sqrt{n} = m\sqrt{n}$$

For example $3 \times \sqrt{2} = 3\sqrt{2}$

Rule 3

$$\sqrt{m} \times \sqrt{n} = \sqrt{mn}$$

For example, $\sqrt{3} \times \sqrt{2} = \sqrt{6}$

Rule 4

$$\left(m\sqrt{n}\right)\left(p\sqrt{q}\right) = mp\sqrt{nq}$$

For example $2\sqrt{2} \times 3\sqrt{5} = 6\sqrt{10}$

Rule 5

$$\frac{\sqrt{m}}{\sqrt{n}} = \sqrt{\frac{m}{n}}$$

For example, $\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$.

Exercise 1 **Date:.....................**

Evaluate the following

1. $\sqrt{5} \times \sqrt{5}$
2. $5 \times \sqrt{7}$
3. $\sqrt{3} \times \sqrt{11}$
4. $\left(3\sqrt{5}\right) \times \left(2\sqrt{3}\right)$
5. $\frac{\sqrt{63}}{\sqrt{7}}$

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REDUCIBLE SURDS

If $\sqrt{m}$ is such that m can be expressed as a product of a perfect square and another integer, then $\sqrt{m}$ is said to be reducible surd.

Example 1

Evaluate the following

1. $\sqrt{12}$ 2. $\sqrt{200}$ 3. $\sqrt{27}$

Solution…

1. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
2. $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
3. $\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

Exercise 2 **Date:…………………**

Simplify each of the following

1. $\sqrt{8}$ 3. $\sqrt{112}$ 5. $\sqrt{175}$
2. $\sqrt{50}$ 4. $\sqrt{108}$

Exercise 3 **Date:…………………**

Simplify each of the following.

1. $\sqrt{147}$ 3. $(3\sqrt{3})^2$ 5. $\sqrt{20} \times (\sqrt{5})^3$
2. $\sqrt{363}$ 4. $\sqrt{1\frac{19}{81}}$

ADDITION & SUBTRACTION OF SURDS

We can only add or subtract surds which are similar. Two surds are said to be similar if they have the same number under the root sign, example:

$\sqrt{n}, m\sqrt{n}, p\sqrt{n}$ etc while surds like $\sqrt{n}, p\sqrt{m}$, $q\sqrt{x}$ are not similar surds. We cannot add or subtract such surds.

i.e. 1. $a\sqrt{m} + b\sqrt{m} = (a+b)\sqrt{m}$

Example

(i) $2\sqrt{3} + 5\sqrt{3} = (2+5)\sqrt{3} = 7\sqrt{3}$

(ii) $\sqrt{2} + 4\sqrt{2} = (1+4)\sqrt{2} = 5\sqrt{2}$

2 . $a\sqrt{m} - b\sqrt{m}(a-b)\sqrt{m}$

Example

(i) $6\sqrt{7} - 2\sqrt{7} = (6-2)\sqrt{7} = 4\sqrt{7}$

(ii) $3\sqrt{3} - \sqrt{3} = (3-1)\sqrt{3} = 2\sqrt{3}$

WORTHY OF NOTE

1. $\sqrt{m} + \sqrt{n} \neq \sqrt{m+n}$

2. $\sqrt{m} - \sqrt{n} \neq \sqrt{m-n}$

Exercise 4 **Date:.....................**

Evaluate the following

1. $\sqrt{2} + 5\sqrt{2}$
2. $\sqrt{3} + \sqrt{3} + 4\sqrt{3} + 7\sqrt{3}$
3. $2\sqrt{7} + 3\sqrt{7} + 5\sqrt{7} - \sqrt{7}$
4. $\sqrt{3} - 4\sqrt{2} + 3\sqrt{3} + 5\sqrt{2}$

Example 2

Evaluate the following

1. $\sqrt{12} + 3\sqrt{75}$
2. $5\sqrt{6} - \sqrt{24} + \sqrt{294}$

Solution…

1. $\sqrt{12} + 3\sqrt{75}$
$= \sqrt{4 \times 3} + 3\sqrt{25 \times 3}$
$= \sqrt{4} \times \sqrt{3} + 3 \times \sqrt{25} \times \sqrt{3}$
$= 2\sqrt{3} + 3 \times 5 \times \sqrt{3}$
$= 2\sqrt{3} + 15\sqrt{3}$
$= 17\sqrt{3}$

2. $5\sqrt{6} - \sqrt{24} + \sqrt{294}$
$= 5\sqrt{6} - \sqrt{4 \times 6} + \sqrt{49 \times 6}$
$= 5\sqrt{6} + \sqrt{4} \times \sqrt{6} + \sqrt{49} \times \sqrt{6}$
$= 5\sqrt{6} - 2\sqrt{6} + 7\sqrt{6}$
$= 10\sqrt{6}$

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Exercise 5 **Date:…………………**

Evaluate the following

1. $3\sqrt{10} \times \sqrt{2}$
2. $\sqrt{500} + \sqrt{125}$
3. $\sqrt{300} - \sqrt{48}$
4. $\sqrt{18} - \sqrt{2}$
5. $5\sqrt{8} - 3\sqrt{2}$

Exercise 6 **Date:…………………**

Evaluate the following.

1. $4\sqrt{15} \times \sqrt{3}$
2. $5\sqrt{20} + 2\sqrt{45}$
3. $\sqrt{32} + \sqrt{128} - \sqrt{200}$
4. $2\sqrt{8} + \sqrt{200} - 4\sqrt{18}$
5. $7\sqrt{5} + 3\sqrt{20} - \sqrt{80}$

Exercise 7 **Date:…………………**

Simplify the following

1. $3\sqrt{45} - 12\sqrt{5} + 16\sqrt{20}$
2. $3\sqrt{75} - \sqrt{12} + \sqrt{108}$
3. $\sqrt{147} - \sqrt{75} + \sqrt{27}$
4. $\sqrt{72} - \sqrt{50} + \sqrt{75}$
5. $3\sqrt{48} + 2\sqrt{12} - \sqrt{75}$
6. $\sqrt{8} - 40\sqrt{2} + 4\sqrt{200}$
7. $\frac{\sqrt{28}+\sqrt{343}}{2\sqrt{63}} + \frac{5}{3}$

Exercise 8 **Date:…………………**

Simplify the following.

1. $\left(\sqrt{3}\right)^4$
2. $\left(\sqrt{12}\right)^3$
3. $\left(\sqrt{6}\right)^5$
4. $\frac{\left(\sqrt{15}\right)^3}{\sqrt{3}}$
5. $\frac{\left(\sqrt{14}\right)^3}{\left(\sqrt{2}\right)^3}$
6. $\left(2\sqrt{3}\right)^3$
7. $\left(4\sqrt{2}\right)^3$
8. $\frac{\sqrt{35}}{\left(\sqrt{7}\right)^3}$

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Note

$$\sqrt{n} = m \Rightarrow (\sqrt{n})^2 = m^2$$
$$n = m^2$$

Exercise 9 **Date:....................**

Find p in the following.

1. $p\sqrt{28} + \sqrt{63} - \sqrt{7} = 0$
2. $\sqrt{128} + \sqrt{18} - \sqrt{p} = 7\sqrt{2}$
3. $\sqrt{3p} + \sqrt{18} - \sqrt{98} = \sqrt{50}$
4. $\sqrt{150} - \sqrt{12p} + \sqrt{54} = 0$
5. $\sqrt{75} - \sqrt{27} + \sqrt{48} = p\sqrt{3}$

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MULTIPLICATION OF SURDS

The usual method of multiplying and expanding applies as when:

(i) $a(m+n) = am + an$

(ii) $(a+b)(c+d) = a(c+d) + b(c+d)$
$= ac + ad + bc + bd$

Note:

The rules of surds are used.

Example 3

Expand and simplify the following.

1. $2\sqrt{3}(\sqrt{3} - 5\sqrt{2})$
2. $(\sqrt{2} + 3\sqrt{5})(2\sqrt{2} - \sqrt{5})$
3. $(5\sqrt{2} - \sqrt{3})^2$

Solution…

1. $2\sqrt{3}(\sqrt{3} - 5\sqrt{2})$
$= 2\sqrt{3} \times \sqrt{3} - 2\sqrt{3} \times 5\sqrt{2}$
$= 2(3) - 10\sqrt{6}$
$= 6 - 10\sqrt{6}$
2. $(\sqrt{2} + 3\sqrt{5})(2\sqrt{2} - \sqrt{5})$
$= \sqrt{2}(2\sqrt{2} - \sqrt{5}) + 3\sqrt{5}(2\sqrt{2} - \sqrt{5})$
$= 2(2) - \sqrt{10} + 6\sqrt{10} - 3(5)$
$= 4 - 15 - \sqrt{10} + 6\sqrt{10}$
$= -11 + 5\sqrt{10}$

3. $(5\sqrt{2} - \sqrt{3})^2$
$= (5\sqrt{2} - \sqrt{3})(5\sqrt{2} - \sqrt{3})$
$= 5\sqrt{2}(5\sqrt{2} - \sqrt{3}) - \sqrt{3}(5\sqrt{2} - \sqrt{3})$
$= 25(2) - 5\sqrt{6} - 5\sqrt{6} + 3$
$= 50 + 3 - 10\sqrt{6}$
$= 53 - 10\sqrt{6}$

Exercise 10 **Date:…………………**

Expand and simplify the following

1. $\sqrt{2}(\sqrt{6} - \sqrt{2})$
2. $\sqrt{3}(\sqrt{6} + \sqrt{3})$
3. $\sqrt{2}(\sqrt{10} - 3\sqrt{6} - \sqrt{8})$
4. $2\sqrt{5}(3\sqrt{5} - 2\sqrt{2})$
5. $3\sqrt{2}(2\sqrt{8} - \sqrt{18})$
6. $3\sqrt{15}(2\sqrt{3} + \sqrt{5})$

Exercise 11 **Date:....................**

Expand and simplify the following.

1. $\sqrt{3}(\sqrt{3}-\sqrt{2})+2(\sqrt{3}-\sqrt{2})$
2. $2\sqrt{5}(1+3\sqrt{5})-3(1+3\sqrt{5})$
3. $(\sqrt{2}+1)(\sqrt{2}-1)$
4. $(4\sqrt{3}+2\sqrt{5})(4\sqrt{3}-2\sqrt{5})$
5. $(3\sqrt{2}+\sqrt{5})(3\sqrt{2}-\sqrt{5})$

Exercise 12 **Date:....................**

Evaluate the following

1. $(\sqrt{3}+\sqrt{2})^2$
2. $(1-\sqrt{2})^2$
3. $(\sqrt{2}+1)^2$
4. $(\sqrt{3}+2\sqrt{2})^2$
5. $(\sqrt{5}-2\sqrt{10})^2$

Exercise 13 **Date:…………………..**

Simplify the following leaving the answer in surd form.

1. $\sqrt{12}\left(\sqrt{48}-\sqrt{3}\right)$
2. $\sqrt{50}-3\sqrt{2}\left(2\sqrt{2}-5\right)-5\sqrt{32}$
3. $\sqrt{2}\left(\sqrt{6}+2\sqrt{2}\right)-2\sqrt{3}$
4. $3\sqrt{27}-2\sqrt{3}\left(4\sqrt{3}-5\sqrt{12}\right)$
5. $\sqrt{1500}+3\sqrt{3}\times 5\sqrt{5}+2\sqrt{15}\left(\sqrt{13}-3\right)$

Exercise 14 **Date:…………………**

Evaluate the following

1. $\sqrt{7}\left(3\sqrt{7}+\frac{6}{\sqrt{7}}\right)$
2. $4\sqrt{3}\left(\frac{3}{\sqrt{3}}+\sqrt{3}\right)$
3. $\left(1-\sqrt{5}\right)\left(\frac{1}{5}+\sqrt{5}\right)$
4. $\left(3+3\sqrt{3}\right)\left(2+4\sqrt{3}\right)$
5. $\left(\sqrt{5}\right)^{-2}\times 75^{\frac{1}{2}}\times 12^{\frac{1}{2}}$

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Exercise 15 **Date:………………….**

Simplify the following leaving your answer in surd form where possible.

1. $\frac{3}{4}\sqrt{128} - \sqrt{50}$
2. $\frac{1}{2}\sqrt{32} - 18 + \sqrt{2}$
3. $\frac{(\sqrt{6})(2\sqrt{2})}{(\sqrt{3})^2}$
4. $(3\sqrt{2} - 2\sqrt{3})^2$
5. $(1 - \sqrt{3})^2 - (1 + \sqrt{3})^2$

CONJUGATE SURDS

Two surds are said to be conjugate of each other if their product gives rise to a rational number. Given the surd $\sqrt{m}+\sqrt{n}$, then $\sqrt{m}-\sqrt{n}$ is called its conjugate since their product give rise to a rational number.

i.e. $(\sqrt{m}+\sqrt{n})(\sqrt{m}-\sqrt{n}) = (\sqrt{m})^2-(\sqrt{n})^2$
$= m-n$

Note that: $m^2-n^2=(m-n)(m+n)$

Example 4

Write down the conjugate of each of the following

(i) $\sqrt{a}-\sqrt{b}$

(ii) $-p-m\sqrt{n}$

(iii) $3+2\sqrt{2}$

Solution….

(i) The conjugate of $\sqrt{a}-\sqrt{b}$ is $\sqrt{a}+\sqrt{b}$.

(ii) The conjugate of $-p-m\sqrt{n}$ is $-p+m\sqrt{n}$.

(iii) The conjugate of $3+2\sqrt{2}$ is $3-2\sqrt{2}$.

RATIONALIZATION OF THE DENOMINATOR

It is the process of making the denominator of fractional surds rational numbers.

Type 1

$\frac{1}{\sqrt{a}}$ can be rationalized by multiplying the numerator and the denominator by the denominator.

i.e. $\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{a}}\times\frac{\sqrt{a}}{\sqrt{a}}=\frac{\sqrt{a}}{a}$.

Type 2

Fractions with denominators as $a\sqrt{m}\pm b\sqrt{n}$, we rationalize by multiplying both the numerator and the denominator by the conjugate of the denominator given

(i) $\frac{1}{\sqrt{a}+\sqrt{b}}=\frac{1}{\sqrt{a}+\sqrt{b}}\times\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$
$=\frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a})^2-(\sqrt{b})^2}$
$=\frac{\sqrt{a}-\sqrt{b}}{a-b}$

(ii) $\frac{1}{\sqrt{a}-\sqrt{b}}=\frac{1}{\sqrt{a}-\sqrt{b}}\times\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$
$=\frac{\sqrt{a}+\sqrt{b}}{(\sqrt{a})^2-(\sqrt{b})^2}$
$=\frac{\sqrt{a}+\sqrt{b}}{a-b}$

Exercise 16 **Date:………………….**

Evaluate the following

1. $\frac{1}{\sqrt{7}}$
2. $\frac{10}{\sqrt{32}}$
3. $\frac{7\sqrt{5}}{\sqrt{7}}$
4. $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}}$
5. $\frac{3\sqrt{50}}{5\sqrt{27}}$
6. $\frac{3}{\sqrt{2}}-\frac{1}{\sqrt{2}}$

Exercise 17 **Date:………………….**

Evaluate the following

1. $\frac{1}{\sqrt{3}-\sqrt{2}}$
2. $\frac{1}{\sqrt{3}+\sqrt{5}}$
3. $\frac{2}{5-\sqrt{7}}$
4. $\frac{3+\sqrt{2}}{5+\sqrt{2}}$
5. $\frac{\sqrt{72}}{\sqrt{48}-\sqrt{3}}$
6. $\frac{\sqrt{8}+5\sqrt{32}-4\sqrt{50}}{1+\sqrt{18}}$
7. $\frac{3\sqrt{5}-4\sqrt{5}}{3\sqrt{5}+4\sqrt{5}}$

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Exercise 18 **Date:…………………..**

Simplify the following leaving your answer in surd form $a + b\sqrt{c}$.

1. $\frac{5}{\sqrt{7}-\sqrt{3}} + \frac{1}{\sqrt{7}+\sqrt{3}}$
2. $\frac{2+\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{2}-2}{\sqrt{2}}$
3. $\frac{2}{3+2\sqrt{2}} + \frac{1}{3-2\sqrt{2}}$
4. $\frac{5}{\sqrt{7}-\sqrt{3}} + \frac{1}{\sqrt{7}+\sqrt{3}}$
5. $\frac{\sqrt{3}}{\sqrt{3}-1} + \frac{\sqrt{3}}{\sqrt{3}+1}$

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Exercise 19 **Date:…………………..**

Without using table or calculator evaluate

(i) $3\sqrt{7}(7-2\sqrt{7})$ if $\sqrt{7}=2.646$

(ii) $2\sqrt{5}(6-2\sqrt{5})$ if $\sqrt{5}=2.236$

(iii) $\sqrt{0.0007}$ if $\sqrt{7}=2.646$

(iv) $2\sqrt{3}(2-\sqrt{3})+3\sqrt{2}(\sqrt{2}-1)$
if $\sqrt{2}=1.414$ and $\sqrt{3}=1.732$.

Exercise 20 **Date:…………………..**

Solve the following.

1. $\frac{\sqrt{2}}{k+\sqrt{2}} = \frac{1}{k-\sqrt{2}}$
2. $\frac{2+\sqrt{3}}{1-\sqrt{x}} = \frac{-4}{2-\sqrt{3}}$
3. $\frac{5}{\sqrt{2}} - \frac{\sqrt{8}}{8} = x\sqrt{2}$
4. $\frac{6}{5-\sqrt{x}} = \sqrt{x}$

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EQUALITY OF SURDS

Given two surds $a + m\sqrt{b}$ and $c + n\sqrt{d}$ then we can say that $a + m\sqrt{b} = c + n\sqrt{d}$ if and only if $a = c, m = n$ and $b = d$.

Exercise 21 **Date:…………………**

Find a such that $(3 + 4\sqrt{3})(2 - a\sqrt{3}) = -18 + 2\sqrt{3}$.

Exercise 22 **Date:…………………**

If $2\sqrt{5} + \sqrt{125} - \sqrt{45} = a + b\sqrt{5}$, evaluate $2a - b$.

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Exercise 23 **Date:....................**

Given that $(\sqrt{3}-5\sqrt{2})(\sqrt{3}+\sqrt{2})=a+b\sqrt{6}$. Find a and b.

Exercise 24 **Date:....................**

1. If $\frac{\sqrt{5}+4}{3-2\sqrt{5}}-\frac{2+\sqrt{5}}{4-2\sqrt{5}}=a+b\sqrt{5}$, find the values of a and b.

2. Given that $k=\frac{1}{\sqrt{3}}$ and that $p=\frac{1+k}{1-k}$, express in its simplest form,
   (i) p (ii) $p-\frac{1}{p}$

3. Given that $\sqrt{a+b\sqrt{3}}=\frac{13}{4+\sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and of b.

4. A cuboid has a square base of side $(2-\sqrt{3})$m and a volume of $(2\sqrt{3}-3)\text{m}^3$. Find the height of the cuboid in the form $(a+b\sqrt{3})$m, where a and b are integers.

5. Express $(2-\sqrt{5})^2-\frac{8}{3-\sqrt{5}}$ in the form $p+q\sqrt{5}$, where p and q are integers.

6.
   (a) Given that $p=\frac{\sqrt{3}+1}{\sqrt{3}-1}$, express in its simplest surd form,
   (i) p (ii) $p-\frac{1}{p}$

   (b) Show that $\frac{(4-\sqrt{x})^2}{\sqrt{x}}$ can be written in the form $px^{-\frac{1}{2}}+q+rx^{\frac{1}{2}}$, where p, q and r are integers to be found.

7. Simplify the following leaving your answer in surd form where possible.
   (i) $\sqrt{50}-\sqrt{18}$
   (ii) $(7+\sqrt{5})(3-\sqrt{5})$
   (iii) $\sqrt{75}-\sqrt{27}$
   (iv) $(5-\sqrt{8})(1+\sqrt{2})$

8. Evaluate the following your answer in surd where possible.
   (i) $\frac{2\sqrt{3}}{5-3\sqrt{3}}$ (vi) $\sqrt{80}+\frac{30}{\sqrt{5}}$
   (ii) $\frac{26}{4-\sqrt{3}}$ (vii) $\frac{5-2\sqrt{3}}{\sqrt{3}-1}$
   (iii) $\frac{8}{\sqrt{3}-1}$ (viii) $\frac{1+\sqrt{3}}{\sqrt{3}-1}$
   (iv) $\frac{2}{3-\sqrt{3}}$ (ix) $\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

(v) $\frac{\sqrt{50}+\sqrt{8}}{7\sqrt{2}}$ (x) $\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}$

9.
(a) Solve the following
 (i) $\sqrt{x+3} = x - 3$
 (ii) $(1-x)\sqrt{3} = 2(x+1)$

(b) Find the integers a and b such that $\frac{\sqrt{3}-2}{\sqrt{3}+2} = a\sqrt{3} + b$.

(c) Simplify the following
 (i) $\frac{7+\sqrt{5}}{\sqrt{5}-1}$ (iii) $\frac{7+\sqrt{5}}{3+\sqrt{5}}$
 (ii) $\frac{5-\sqrt{3}}{2+\sqrt{3}}$ (iv) $\frac{2+5\sqrt{7}}{4+\sqrt{7}}$

10.
(a) Find the prime numbers p and q such that $\sqrt{56} = 2\sqrt{p}\sqrt{q}$ where $p < q$.

(b) Simplify the following
 (i) $\sqrt{360} - \sqrt{2} \times (\sqrt{5})^2 - \frac{\sqrt{30}\times\sqrt{8}}{\sqrt{6}}$
 (ii) $\frac{4\sqrt{2}-\sqrt{11}}{3\sqrt{2}+\sqrt{11}}$
 (iii) $\frac{7}{2\sqrt{14}} + \left(\frac{\sqrt{14}}{2}\right)^3$

11.
(a) Evaluate the following equations leaving your answer in surd where possible.
 (i) $\frac{1+\sqrt{8}}{3-\sqrt{2}}$ (iii) $\frac{7\sqrt{2}+3\sqrt{3}}{4\sqrt{2}-2\sqrt{3}}$
 (ii) $\frac{5-2\sqrt{10}}{3\sqrt{5}+\sqrt{2}}$ (iv) $\frac{\sqrt{75}-3}{\sqrt{3}+1}$

(b) Evaluate the following and leave your answer in surd where possible.
 (i) $\frac{8-3\sqrt{6}}{2\sqrt{3}+3\sqrt{2}}$
 (ii) $\sqrt{24} \times \sqrt{27} + \frac{9\sqrt{30}}{\sqrt{16}}$
 (iii) $\frac{(2+\sqrt{5})^2}{\sqrt{5}-1}$
 (iv) $6(1+\sqrt{3})^{-2}$

(c) Express $\frac{5+\sqrt{2}}{3-2\sqrt{2}} - \frac{5-\sqrt{2}}{3+\sqrt{2}}$ in the form $a + b\sqrt{2}$.

(d) Evaluate the following
 (i) $\frac{2\sqrt{2}}{\sqrt{48}-\sqrt{8}-\sqrt{27}}$

(e) If $\frac{a}{\sqrt{3}+1} + \frac{b}{\sqrt{3}-1} = \sqrt{3} - 3$, find the possible values of a and b.

12.
(a) Rationalize $\frac{1}{\sqrt{2}+1}$
(b) Simplify $\frac{\sqrt{3}}{\sqrt{3}-1} + \frac{\sqrt{3}}{\sqrt{3}+1}$
(c) Simplify $\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} - \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}$
(d) Express $\frac{\sqrt{8}}{\sqrt{7}-\sqrt{5}}$ in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers.
(e) Given that $28 + p\sqrt{3} = (q + 2\sqrt{3})^2$, where p and q are integers, find the values of p and of q.

13. Solve the following simultaneous equations giving your answers for both x and y in the form $a + b\sqrt{3}$, where a and b are integers.
 $2x + y = 9$
 $\sqrt{3}x + 2y = 5$

NUMBER BASES

Decimal number system or base 10 uses the digits or numerals: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.

Binary number system, base 2, uses the digits 0 and 1.

Digits for base 3: $0, 1, 2$

Digits for base 7: $0, 1, 2, 3, 4, 5, 6$

Digits for Duodecimal number system, base 12: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T$ and E.

Here one the first few numbers in base five: $0, 1, 2, 3, 4, 10, 11, 13, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34$.

These are read zero, one, two, three, four, one – zero, one – one, one – two, one – three, one – four, two – zero, two – one, etc.

NOTE:
In each of these number systems, the number of digits contained within the system is the name of the system. Base 2 has two digits, Base 3 has 3 digits, …, base 10 has 10 digits etc. each system starts counting with 0.

CONVERTING FROM NUMBER BASE TEN (DECIMAL NUMERAL) TO OTHER BASES
To change from base 10 to a different base, the decimal numeral is divided repeatedly by the appropriate base number until there is nothing left to divide. The answer is obtained by reading the remainders upwards.

Example 1
Convert the following to base 7
(i) 243_{ten} (ii) 664_{ten}

Solution…
We divide repeatedly by 7
(i)

7	243
7	34 rem 5
7	4 rem 6
	0 rem 4

$\therefore 243_{\text{ten}} = 465_{\text{seven}}$

(ii)

7	664
7	94 rem 6
7	13 rem 3
7	1 rem 6
	0 rem 1

$\therefore 664_{\text{ten}} = 1636_{\text{seven}}$

Example 2
Express 267_{ten} as a number in base twelve.

Solution…
We divide repeatedly by 12

12	267
12	22 rem 3
12	1 rem 10
	0 rem 1

$\therefore 267_{\text{ten}} = 1T3_{\text{twelve}}$

Note that 10 is represented by T in base twelve.

Exercise 1 **Date:……………………**
1. Convert 35_{ten} to a binary number.
2. Convert 77_{ten} to a binary number
3. Convert 89_{ten} to a number in base two.
4. Write 37_{ten} as a number in base five.

Exercise 2 **Date:……………………..**

Write 138_{ten} as a number in each of the following bases.

(i) Twelve
(ii) Eight
(iii) Seven
(iv) Five
(v) Four

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Exercise 3 **Date:……………………**

Express 2138_{ten} as a number in each of the following bases.

(i) Twelve (iii) Seven

(ii) Eight (iv) Five

COVERTING FROM OTHER BASES TO A NUMBER BASE TEN

NOTE:

Numbers such as 222, 1006 can be written in expanded from using the appropriate column headings.

$222 = (2 \times 100) + (2 \times 10) + (2 \times 1)$ using exponents;

$222 = (2 \times 10^2) + (2 \times 10^1) + (2 \times 10^0)$

NOTE: $10^0 = 1$

Similarly,

$1006 = (1 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (6 \times 10^0)$

We also know that the column headings for base seven are ones, sevenths, 49s (7^2), 343s (7^3) and so on. This means that 54327 can be written in its expanded as:

5432_7
$= (5 \times 7^3) + (4 \times 7^2) + (3 \times 7^1) + (2 \times 7^0)$

NOTE: $7^0 = 1$.

In general any number zero numeral to the power zero is 1. i.e. $a^0 = 1$.

Example 3
Write 11001_{two} as a number in base tern.

Solution…
List the digits in order, and count them from right to left, starting with zero.

Digits: 1 1 0 0 1
Numbering: 4 3 2 1 0

Use this listing to convert each digit to the power of two that it represents:

11001_2
$= (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 1^0)$
$= (1 \times 16) + (1 \times 8(+(0 \times 4) + (0 \times 2) + (1 \times 1)$
$= 16 + 8 + 0 + 0 + 1$
$= 25_{\text{ten}}$

$\therefore 11001_2 = 25_{\text{ten}}$

Exercise 4 **Date:……………………**
Convert the following numbers to base ten.
1. 333_{four}
2. 564_{seven}
3. 7345_{eight}
4. 2213_{four}
5. 11010_{two}

Exercise 5 **Date:……………………**
Convert the following numbers to base ten.
1. 482_{twelve}
2. 6012_{eight}
3. $60T_{\text{twelve}}$
4. $3T0E_{\text{twelve}}$
5. 11111111_2

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Exercise 6 **Date:……………………**

Convert the following numbers to decimal numeral.

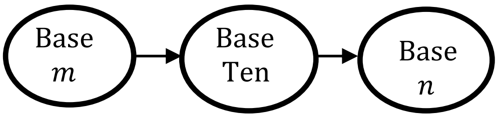
1. 101101_{two}
2. 10011_{two}
3. 1120_{five}
4. 1203_{five}
5. $\sqrt{11001_2}$
6. 11.011_{two}

CONVERSION BETWEEN NON – DECIMAL BASES

To convert between non – decimal bases.

1. Convert to base ten and then
2. Convert the result to the required base.

In other words, converting a number in base m to a number in base n (both being non – decimals) is done by first converting from base m to base ten (decimal) and finally from base 10 to base n.
i.e.



Example 4
Convert 42_{five} to a base three numeral.

Solution…
Step 1: Change 42_{five} to base ten.

$$42_5 = (4 \times 5^1) + (2 \times 5^0)$$
$$= 20 + 2 = 22$$

Step 2: We then convert 22_{ten} to a number base three.

3	22
3	7 rem 1
3	2 rem 1
	0 rem 2

$\therefore 42_{\text{five}} = 211_{\text{three}}$

Exercise 7 **Date:……………………**

1. Express 112_{seven} as a number in base four.
2. Express 441_{five} as a number in base four.
3. Convert 11010_{two} as a number in base five.
4. Convert 7345_{eight} as a number in base six.
5. Convert $3T0E_{\text{twelve}}$ as a number in base four.
6. $\sqrt{1001_{\text{two}}}$, leaving your answer in base two.

SOLVING EQUATIONS INVOLVING NUMBER BASES

More, the right hand side of the equation is already in base 10, so we only convert the left hand side of the equation to base ten:

Example 5

Solve $132_x = 42_{\text{ten}}$

Solution…

$$(1 \times x^2) + (3 \times x^2) + (2 \times x^0) = 42$$
$$x^2 + 3x + 2 = 42$$
$$x^2 + 3x - 40 = 0$$
$$x^2 + 8x - 5x - 40 = 0$$
$$x(x+8) - 5(x+8) = 0$$
$$(x+8)(x-5) = 0$$
$$x = -8 \text{ or } x = 5$$
$$\therefore x = 5 \text{ since } x > 0$$

Exercise 8 **Date:……………………**

Find the value of x in each of the following equations.

1. $133_x = 73_{\text{ten}}$
2. $123_x = 38_{\text{ten}}$
3. $23_x = 1111_{\text{two}}$
4. $57_{\text{eight}} = 233_x$
5. $110_x = 1020_{\text{four}}$

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Exercise 9 **Date:……………………**

Find the value of x in each of the following equations.

1. $x_{\text{four}} = 145_6$
2. $243_x = 73$
3. $233_x = 125_{\text{seven}}$
4. $211_x = 320_{\text{four}}$
5. $105_{\text{twelve}} = 302_x$

Exercise 10 **Date:……………………..**

Solve the following equations.

1. $312_{\text{four}} + 52_x = 96_{\text{ten}}$
2. $365_{\text{seven}} + 43_x = 217$
3. $3(14)_x = 45_x$
4. $124_x = 7(14_x)$
5. Given that $425_6 = 320_x$, evaluate $123_x - 34_x$.
6.
   (a) If 135 to the base n is equal to 76, find n.
   (b) A number is written as 37 in base x. Twice the number is written as 75 in base x. Find the value of x.

PERFORMING OPERATION ON NUMBER BASES

In base ten, we know that

$9 + 1 = 1 + 9 = 10$
$8 + 2 = 2 + 8 = 10$
$7 + 3 = 3 + 7 = 10$

∴ When we work out, for example $7 + 6$, we can think of it as:

$$\begin{aligned} 7 + (3 + 3) &= (7 + 3) + 3 \\ &= 10 + 3 \\ &= 13 \end{aligned}$$

Similarly in base five,

$4 + 1 = 1 + 4 = 10_{\text{five}}$

$$\begin{aligned} 4 + 3 = 4 + (1 + 2) &= (4 + 1) + 2 \\ &= 10_{\text{five}} + 2 \\ &= 12_{\text{five}} \end{aligned}$$

NOTE:

The principles and techniques developed for base five can be applied to other bases.

Exercise 11 **Date:……………………**

Add the following in base five.

1. $\begin{array}{r} 1\;0 \\ +\,1\;3 \\ \hline \\ \hline \end{array}$

2. $\begin{array}{r} 1\;1 \\ +\,3\;4 \\ \hline \\ \hline \end{array}$

3. $\begin{array}{r} 3\;3 \\ +\,2\;1 \\ \hline \\ \hline \end{array}$

4. $\begin{array}{r} 4\;3 \\ +\,3\;2 \\ \hline \\ \hline \end{array}$

5. $\begin{array}{r} 2\;3 \\ +\,2\;2 \\ \hline \\ \hline \end{array}$

6. $\begin{array}{r} 2\;3\;3 \\ +\;\;4\;4 \\ \hline \\ \hline \end{array}$

Exercise 12 **Date:……………………**

Add the following in base two.

1. $\begin{array}{r} 1\;0\;1 \\ +\;\;1\;0 \\ \hline \\ \hline \end{array}$

2. $\begin{array}{r} 1\;0\;1\;1 \\ +\;\;1\;0\;1 \\ \hline \\ \hline \end{array}$

3. $\begin{array}{r} 1\;1\;1 \\ +\;1\;1\;1 \\ \hline \\ \hline \end{array}$

4. $\begin{array}{r} 1\;0\;1\;1 \\ +\;1\;1\;0\;1 \\ \hline \\ \hline \end{array}$

5. $\begin{array}{r} 1\;1\;0\;1 \\ 1\;0\;1\;1 \\ +\;\;1\;0\;1 \\ \hline \\ \hline \end{array}$

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Exercise 13 **Date:........................**

The following numbers are in base four. Find the sums.

1. $$\begin{array}{r} 3\,3 \\ +\ \ 2\,2 \\ \hline \\ \hline \end{array}$$

2. $$\begin{array}{r} 1\,2\,3 \\ +\ \ 3\,1\,2 \\ \hline \\ \hline \end{array}$$

3. $$\begin{array}{r} 1\,2\,3 \\ 3\,1\,2 \\ +\ \ 2\,3\,1 \\ \hline \\ \hline \end{array}$$

4. $$\begin{array}{r} 1\,1\,1 \\ 2\,2\,2 \\ +\ \ 3\,3\,3 \\ \hline \\ \hline \end{array}$$

5. $$\begin{array}{r} 3\,2\,1 \\ 1\,0\,1 \\ +\ \ 1\,1\,2 \\ \hline \\ \hline \end{array}$$

6. $$\begin{array}{r} 2\,0\,2 \\ 1\,1\,2 \\ +\ \ 3\,0 \\ \hline \\ \hline \end{array}$$

Exercise 14 **Date:........................**

The following are base eight number. Find the sums.

1. $$\begin{array}{r} 4\,7 \\ +\ \ 2\,1 \\ \hline \\ \hline \end{array}$$

2. $$\begin{array}{r} 5\,4\,3 \\ +\ \ 5\,7\,7 \\ \hline \\ \hline \end{array}$$

3. $$\begin{array}{r} 7\,5\,7 \\ +\ \ 3\,0\,3 \\ \hline \\ \hline \end{array}$$

4. $$\begin{array}{r} 2\,7\,2 \\ 1\,7\,3 \\ +\ \ 4\,1\,4 \\ \hline \\ \hline \end{array}$$

5. $$\begin{array}{r} 7\,3\,2 \\ 4\,4 \\ +\ \ 1\,4\,6 \\ \hline \\ \hline \end{array}$$

6. $$\begin{array}{r} 2\,2\,2 \\ 3\,0\,3 \\ +\ \ 7\,5\,7 \\ \hline \\ \hline \end{array}$$

Exercise 15 **Date:........................**

Add in base twelve.

1. $$\begin{array}{r} 4\,8 \\ +\ \ 3\,4 \\ \hline \\ \hline \end{array}$$

2. $$\begin{array}{r} 5\,7 \\ +\ \ 6\,5 \\ \hline \\ \hline \end{array}$$

3. $$\begin{array}{r} 4\,3\,6 \\ +\ 1\,9\,4 \\ \hline \\ \hline \end{array}$$

4. $$\begin{array}{r} 5\,3\,4 \\ +\ 8\,7\,6 \\ \hline \\ \hline \end{array}$$

5. $$\begin{array}{r} \mathrm{T\,O\,E} \\ +\ \mathrm{T\,O\,T} \\ \hline \\ \hline \end{array}$$

6. $$\begin{array}{r} 2\,6\,2 \\ 1\,6\,3 \\ +\ \ 4\,1\,4 \\ \hline \\ \hline \end{array}$$

SUBTRACTION

Example 6

Evaluate

(i) $10110_2 - 1011_2$

(ii) $43_7 - 26_7$

Solution...

(i) $$\begin{array}{r} 1\,0\,1\,1\,0_2 \\ 1\,0\,1\,1_2 \\ \hline 1\,0\,1\,1_2 \\ \hline \end{array}$$

Starting from right, borrow 2 from the second column i.e. $2 + 0 - 1 = 1$. We again borrow 2 from the next column i.e. $2 + 0 - 1 = 1$. The next column becomes $0 - 0 = 0$. We borrow 2 from the fifth column i.e. $2 + 0 - 1 = 1$.

(ii) $$\begin{array}{r} 4\,3_7 \\ 2\,6_7 \\ \hline 1\,4_7 \\ \hline \end{array}$$

Starting from right we borrow 7 from the second column i.e. $7 + 3 - 6 = 4$. The next column becomes $3 - 2 = 1$.

Exercise 16 **Date:........................**

Perform the following operations.

1. $$\begin{array}{r} 1\,1\,0\,1_2 \\ -\ \ 1\,0\,1_2 \\ \hline \\ \hline \end{array}$$

2. $$\begin{array}{r} 1\,0\,1\,0\,1_2 \\ -\ \ 1\,1\,0_2 \\ \hline \\ \hline \end{array}$$

3. $$\begin{array}{r} 3\,1\,3_7 \\ -\ \ 1\,6\,1_7 \\ \hline \\ \hline \end{array}$$

4. $$\begin{array}{r} 2\,4\,2_7 \\ -\ \ 4\,5_7 \\ \hline \\ \hline \end{array}$$

5. $$\begin{array}{r} 4\,3 \\ -\ \ 2\,6 \\ \hline \\ \hline \end{array}$$

Exercise 17 **Date:........................**

Perform the following operations.

1. $67_8 - 46_8$
2. $313_8 - 161_8$
3. $2002_8 - 333_8$
4. $782_{12} - 2E9_{12}$
5. $TOE_{12} - 82T_{12}$

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Exercise 18 **Date:……………………**

Evaluate the following

(i) $11011_{\text{two}} - 101_{\text{two}}$

(ii) $11011_{\text{two}} - 1101_{\text{two}}$

(iii) $523_{\text{seven}} - 65_{\text{seven}}$

(iv) $1341_{\text{five}} - 242_{\text{five}}$

Exercise 19 **Date:……………………**

1. Subtract 122_3 from $1001_{3.}$
2. Subtract 436_8 from 1205_8.
3. What is the sum of 25_{eight}, 342_{eight} and 421_{eight}?
4. Find the sum of 322_{five}, 114_{five} and 324_{five}.
5. Simplify $10010 + 1101$ in base two.

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Exercise 20 **Date:……………………..**

1. Arrange the following numbers in ascending order of magnitude.
   (a) $21_8, 25_8, 30_6$
   (b) $11011_2, 130_4, 26_{10}$
   (c) $111_{\text{four}}, 10011_{\text{two}}$ and 20_{ten}

2. Arrange the following in descending order of magnitude.
   (a) $22_3, 34_5, 21_6$
   (b) $20_8, 30_6, 23_7, 19_{12}$

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Exercise 21 **Date:......................**

1. Subtract 65_7 from 523_7.
2. Evaluate $141_6 + 233_6 - 102_6$.
3. Find the base addition for $35 + 53 = 122$.
4. If $243_x + 452_x = 1135_x$, find the value of x.
5. In what number base was the addition $1 + mn = 100$, where $n > 0$, done?

Exercise 22 **Date:......................**

1. If $4m4_{\text{five}} = 119_{\text{ten}}$, what is the value of m?

2. Solve the equation $(100)_2Y - 11_2(Y - 1) = 100_2$

3. Which base five numeral is equivalent to $(4 \times 5^3) + (0 \times 5^2) + (2 \times 5^1) + (2 \times 5^0)$?

4.
   (i) The subtraction below is in base seven. Find the missing number.

$$\begin{array}{r} 5\,1\,6\,2 \\ -\quad 2\,6\,4\,4 \\ \hline 2 * 1\,5 \\ \hline \end{array}$$

   (ii) Find the missing number in the addition of the following numbers in base seven.

$$\begin{array}{r} 4\;3\;2\;1 \\ 1\;2\;3\;4 \\ +\quad *\;*\;*\;* \\ \hline 1\;2\;3\;4\;1 \\ \hline \end{array}$$

   (iii) The following subtraction was performed in base four. Find the missing number.

$$\begin{array}{r} 3\;0\;1\;2 \\ *\;*\;*\;* \\ \hline 1\;3\;3\;3 \\ \hline \end{array}$$

   (iv)

$$\begin{array}{r} 1\;0\;1\;0\;1 \\ 1\;0\;0\;1 \\ +\quad *\;*\;*\;*\;* \\ \hline 1\;1\;1\;0\;0\;1 \\ \hline \end{array}$$

   Find the missing number in the addition base 2.

MULTIPLICATION

Exercise 23 **Date:……………………**

Evaluate the following.

1. $\begin{array}{r} 312_4 \\ \times \quad 13_4 \\ \hline \end{array}$

2. $\begin{array}{r} 313_4 \\ \times \quad 23_4 \\ \hline \end{array}$

3. $\begin{array}{r} 313_5 \\ \times \quad 23_5 \\ \hline \end{array}$

4. $\begin{array}{r} 243_5 \\ \times \quad 32_5 \\ \hline \end{array}$

5. $\begin{array}{r} 542_7 \\ \times \quad 51_7 \\ \hline \end{array}$

6. $\begin{array}{r} 765_8 \\ \times \quad 41_8 \\ \hline \end{array}$

Exercise 24 **Date:……………………**

Evaluate the following.

1. $1011_2 \times 1101_2$
2. $21_3 \times 21_3$
3. $1011_2 \times 101_2$
4. $1101_2 \times 111_2$
5. $(111_2 + 101_2)(111_2 - 101_2)$

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Exercise 25 **Date:……………………..**

Evaluate the following

1. $2102_3 \times 122_3$
2. $(111_{two})^2$
3. $(20_3)^2 - (11_3)^2$
4. $(202_3)^2 - (112_3)^2$

Exercise 26 **Date:……………………**

1. Copy and complete the following multiplication table in base seven.

×	3	4	5
1	3		
2	6		
3	12		

2. Copy and complete the binary multiplication table.

×	10	11	100	101
10	100	−	1000	−
11	110	−	1100	−
100	−	−	10000	10100

3. If $124_n = 232_{\text{five}}$, find n.

Exercise 27 **Date:……………………**

1. If all numbers in the equation $\frac{y}{y+101} = \frac{11}{10010}$ are in base two, solve for y.

2. A number is written as 14_n. If three times the number is equal to 45_n, find the value of n.

3.
   (i) Draw the multiplication table ($\otimes$) in base 6 on the set $\{1, 2, 3, 4\}$
   (ii) From the table, solve:
   α) $2 \otimes x = 2$
   β) $m \otimes m = 4$

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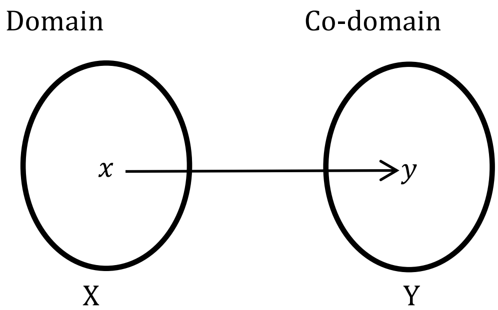
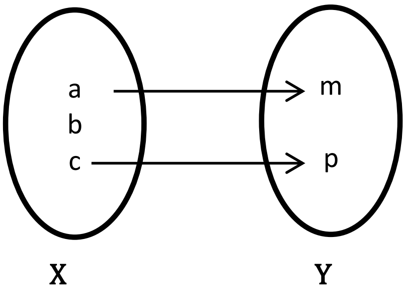
RELATIONS & FUNCTIONS

RELATIONS
A relation is an association between two non-empty sets. It may also be defined as a set of ordered pairs, (x, y). The set of first components x values, in the ordered pairs is the domain of the relation. The set of the second components, y —value is called the range of the relation.

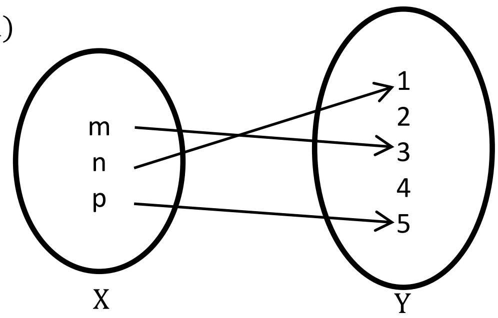


From the diagram, an element $x \in$ X (domain) is related to an element $y \in$ Y (co-domain), we say y is the image of x. The set of all images is called the range. The range is a subset of the co-domain.

NOTE:
A relation may exist between two sets and not all the elements of the domain may be associated with the elements of the co – domain and vice – versa.
For example,
(i)



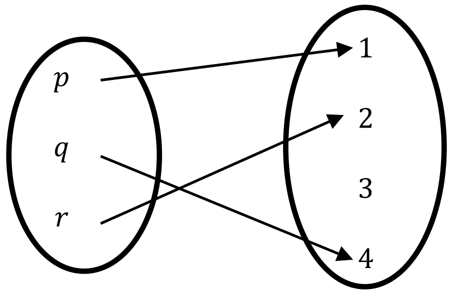
(ii)



From the arrow diagram,
(i) Domain = $\{a, b, c\}$
Co-domain= $\{m, p\}$
Range = $\{m, p\}$
Clearly the element $b \in$ X has no image in Y
(ii) Domain = $\{m, n, p\}$
Co – domain = {1,2,3,4,5}
Range = {1,3,5}
The elements 2,4 ∈ Y are not images of the members of the domain(X).

In both examples (i) and (ii) the range is a subset of the co-domain.

Exercise 1 **Date:……………………**
1.

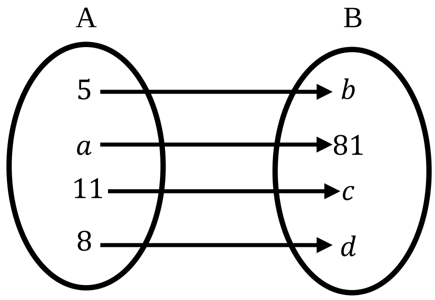


In the relation above, write down the:
(i) domain
(ii) co – domain
(iii) range

2. Determine the domain and range of the following relations.
(i) $(3, 2), (2, 3), (3, 3)$
(ii) $(-5, 2), (0, 2), (-5, 3)$.

Exercise 2 **Date:……………………**

1. $A \longrightarrow$ "is square root of"



   (a) Determine the value of
      (i) a (iii) c
      (ii) b (iv) d

   (b) Write down the
      (i) domain
      (ii) range
      (iii) co – domain

2. If set $A = \{$prime numbers greater than 1 but less than 14$\}$ and the relation is 'subtract from 14', find all the elements in the co – domain, B. Draw arrow diagram between A and B.

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Exercise 3 **Date:……………………**

State the relation from the first set to the second set and match the members in each pair of sets.

1. G = {beans, cereals, fruits, fish, palm oil}
   H = {carbohydrate, protein, fat}

2. J = {dog, bird, goat, lion}
   K = {nest, kennel, den, pen house}

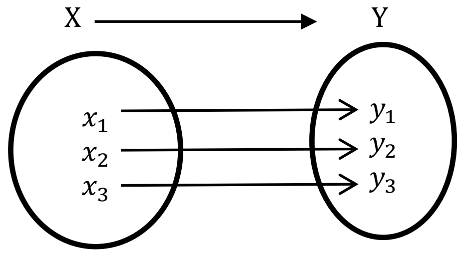
3. M = {hen, parrot, lizard, snake, cat, dog}
   N = {bird, mammal, reptile}

4. N = {aeroplane, car, canoe, train, ship}
   P = {rail, road, sea}

TYPES OF RELATIONS

(i) **One-to-one relation**

A relation is said to be the one-to-one if each element in the domain associates to a unique element in the co-domain.

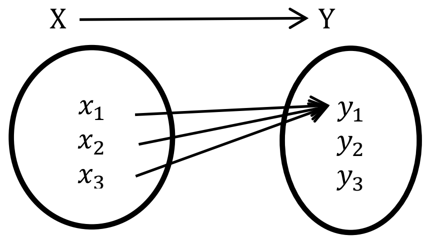


Clearly each element in the domain associates to a unique element in the co – domain.

i.e. x_1 associates to only y_1, x_2 associates to only y_2, x_3 associates to only y_3.

(ii) **Many-to-one relations**

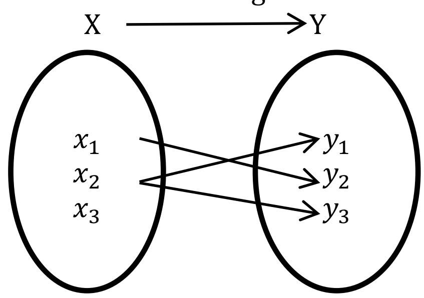
A relation is said to be many-to-one if at least two different elements of the domain (X) have the same image in the co-domain(Y).



Clearly from the diagram, there exist $x_{2,}x_3 \in$ X which associate to $y_1 \in$ Y.

(iii) **One-to-many relations**

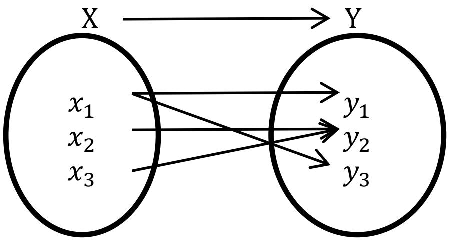
A relation is said to be one to many if at least one element in the domain (X) has more than one image in the co-domain (Y)



Clearly from the diagram, there exist $x_2 \in$ X which associates to $y_1, y_3 \in$ Y.

(iv) **Many-to-many relations**

A relation is said to be many-to-many if at least one element in the domain X associates to more than one image in the co-domain (Y) and at least two different elements of the domain(X) associate to the same image in the co-domain (Y).



Exercise 4 **Date:……………………**

For each of the following pairs of sets, the relation from the first set to the second set is stated. Match the members and identify the type of relation.

1. $A = \{\text{Ga, Ewe, Akan, Dagomba}\}$
   $B = \{\text{Nana, Nii, Na, Togbe}\}$
   Relation: 'is a Chief's title in'

2. $C = \{1, 2, 3, 12\}$
   $D = \{4, 6, 9, 24\}$
   Relation: 'is a multiple of'

3. $E = \{-3, -2, -1, 0, 1, 2\}$
   $F = \{1, 4, 9\}$
   Relation: 'is the square of'

4. $G = \{4, 6, 9, 10\}$
   $H = \{0, 1, 2, 3, 5\}$
   Relation: 'is a factor of'

5. $P = \{\text{pressure, rainfall, temperature, wind}\}$
   $L = \{\text{thermometer, barometer, rain guage, wind vane}\}$
   Relation: 'is an instrument for measuring'

MAPPINGS

A mapping is a relation in which each element in the domain (X) associates onto exactly one element in the co-domain (Y). From this definition, a one-to-one relation and many-to-one relations are mappings. However, many-to-one and one to many relations are not mappings since each element of their domain does not correspond to exactly one element in the co-domain.

Example

Make a table of the mapping defines by the rule $2x + 1$ on the domain.

Solution…

x :	-2	-1	0	1	2	3
↓	↓	↓	↓	↓	↓	↓
$2x+1$	-3	-1	1	3	5	7

Exercise 5 **Date:……………………**

Make a table of the mapping defined by each of the following rules in the domain $\{-2, -1, 0, 1, 2, 3, 4\}$.

1. $x \longrightarrow 3x - 1$
2. $x \longrightarrow \frac{1}{2}x + 1$
3. $x \longrightarrow x^2 + 2$
4. $x \longrightarrow 3x^2 - 2$

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Exercise 6 **Date:……………………**

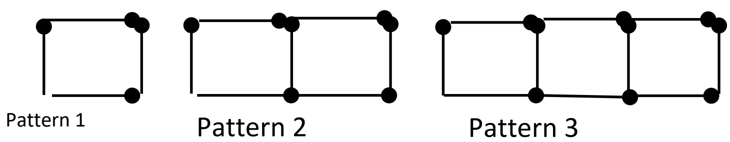
Make a table of the mapping for the first five terms of each sequence given by the following rules.

1. $r \longrightarrow 2r$
2. $r \longrightarrow 2 + r$
3. $r \longrightarrow r(3r - 2)$
4. $r \longrightarrow 4r^2 - 2$
5. $r \longrightarrow 5(2)^{r-1}$

Exercise 7 **Date:……………………**

1. A mapping is defined by $x \rightarrow \frac{x+3}{x}, x \neq 0$.
   (i) What is the image of $\frac{1}{4}$ under the mapping?
   (ii) Find the domain which associates the mapping to $\frac{1}{4}$.

2. Match sticks are used to make the following patterns.



Complete the table for the number of sticks in the perimeter and the pattern.

Pattern number 1 2 3 n	1	2	3	4	5	6	p
Sticks in the perimeter	4	6					
Total number of sticks	4	7					

Using your table find the number of match sticks

i. In the perimeter of the 20th pattern.
ii. In the 20th pattern.
iii. In the perimeter of the n^{th} pattern.
iv. In the n^{th} pattern.

RULE FOR MAPPING

Linear Mapping

The rule of a linear mapping is of the form:

$$y = ax + b$$

Where $\boldsymbol{a} = \frac{\text{constant difference of co−domain}}{\text{constant difference of domain}}$

And $\boldsymbol{b}$ can be determined by substituting an element of the domain and its image in the co – domain into the rule.

Example 1

Find the rule of the mapping

x	1	2	3	4	5
↓	↓	↓	↓	↓	↓
y	2	5	8	11	14

Solution…

There is a constant difference of 3 between consecutive elements of the co – domain.
(i.e. $14 - 11 = 11 - 8 = 8 - 5 = 5 - 2 = 3$)

There is also a common difference of 1 between consecutive terms of the domain.

$$a = \frac{\text{constant difference of co−domain}}{\text{constant difference of domain}}$$
$$= \frac{3}{1} = 3$$

$\therefore y = 3x + b$, where b is a constant to be determined.

When $x = 1, y = 2$

$\therefore 2 = 3(1) + b$

$\therefore b = -1$

$\therefore y = 3x - 1.$

EXPONENTIAL MAPPING

The rule of an exponential mapping is of the form:

$$y = b\frac{r^x}{r^a}$$

Where a is the first element in the domain, b is the first element in the co – domain and r is the constant ratio between consecutive elements of the co – domain.

Exercise 8 **Date:…………………..**

Find the rules for the following mapping.

1.

0	1	2	3	4
↓	↓	↓	↓	↓
0	$^1/_2$	1	$1\,^1/_2$	2

2.

−3	−1	1	3
↓	↓	↓	↓
−3	1	5	9

3.

0	1	2	3	4
↓	↓	↓	↓	↓
1	2	4	8	16

4.

2	3	4
↓	↓	↓
−2	−3	−4

Exercise 9 **Date:……………………..**

Find the rule for each of the following mappings.

1.

0	1	2	3	4	5
↓	↓	↓	↓	↓	↓
2	5	8	11	14	17

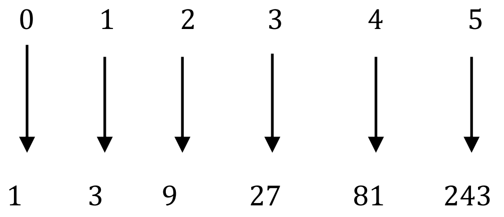
2.

0	1	2	3	4	5
↓	↓	↓	↓	↓	↓
-4	-6	-8	-10	-12	-14

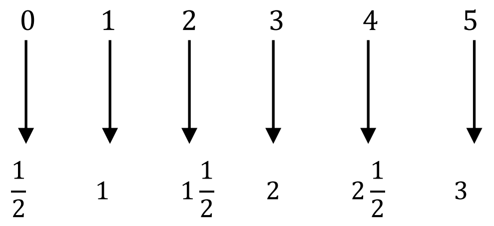
3.

0	1	2	3	4	5
↓	↓	↓	↓	↓	↓
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$

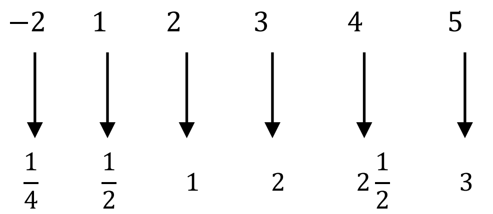
4.


0 1 2 3 4 5
1 3 9 27 81 243

5.


0 1 2 3 4 5
$\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 $2\frac{1}{2}$ 3

6.


−2 1 2 3 4 5
$\frac{1}{4}$ $\frac{1}{2}$ 1 2 $2\frac{1}{2}$ 3

Exercise 10 **Date:……………………**

Find the rule for each of the following mappings.

1.

1	2	3	4	5
↓	↓	↓	↓	↓
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

2.

-2	-1	0	1	2	3
↓	↓	↓	↓	↓	↓
7	5	3	1	-1	-3

3.

-2	-1	0	1	2	3
↓	↓	↓	↓	↓	↓
$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

4.

1	2	3	4	5	6
↓	↓	↓	↓	↓	↓
5	13	21	29	37	45

5.

0	1	2	3	4	5
↓	↓	↓	↓	↓	↓
1	1	4	27	256	3125

Exercise 11 **Date:……………………**

1. A car uses 10 litres of petrol for every 100km. make a table of the mappings: Distance travelled ⟶ petrol used (up to a distance of 500km)

2. It costs GH¢200.00 to hire a labourer to clear an acre of a cocoa farm. Make a table for the mapping for the cost of cleaning up to five acres of land.

3. A painter uses 5 litres of paint for a wall of area $4.5m^2$. Make a table for the wall painted to litres of paint used, up to 30 litres. Find the rule for this mapping. How many litres of paint are needed for the following area?
   (a) $10.5m^2$ (b) $12m^2$

4. A man wants to make a square enclosure in one of his fields. Make a table for the mapping of the area of field to the length of fencing for 80m, 100m, 120m, up to 200m. find the rule for this mapping. What length of fencing is required for the following areas of field?
   (i) $100m^2$ (iii) $169m^2$
   (ii) $121m^2$ (iv) $225m^2$

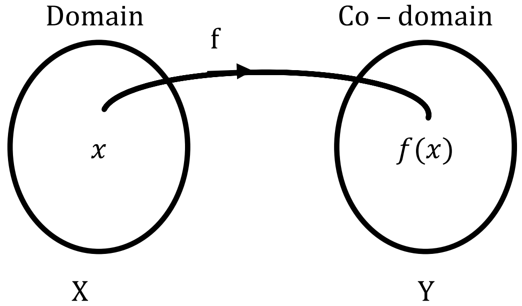
5. The basic cost of establishing a local soap factory is GH¢10,000.00. The cost of producing one bar of the soap is GH¢100.00. Make a table of the mapping for the number of soap bars (0, 5, 10, 15, 20) and the total cost.
   (a) What is the rule for this mapping?
   (b) Find the number of soap bars that can be made for a total production cost of
      (i) GH¢50,000.00
      (ii) GH¢75,000.00

(c) A bar of soap from this factory is sold for GH¢204.00. How many bars of soap should the factory produce to cover its cost?

FUNCTIONS

Let X and Y be non-empty sets. A function from X to Y is a rule which associates with each element of X a unique element of Y. X is called the domain of the function and Y the co-domain of the function. In this definition the sets X and Y may be equal.

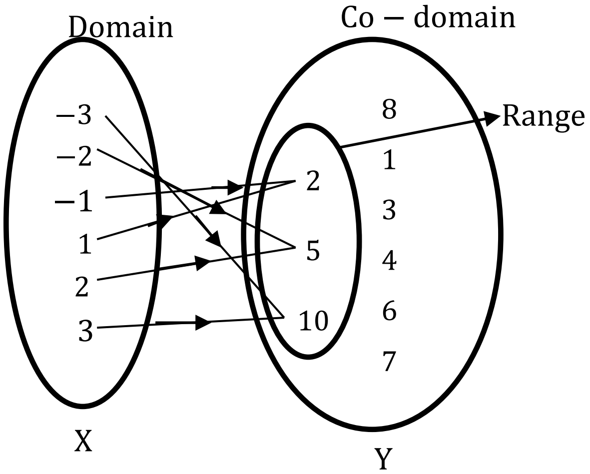
If we denote the function by f we write, $f : X \to Y$ to indicate that f is a function from X to Y. If $x \in X$, we denote the unique member of Y which f associates with x; $f(x)$ is called the image of x under f, or the value of f at x. We can represent the mapping diagrammatically as follows:



The subset of Y consisting of the members of Y that are images under f of element of X, i.e the subset $\{y \in Y: y = f(x) \text{for some } x \in X\}$ is called the image of X under f, or simply, the image of f. It is often denoted by $f(x)$. The word range is often used instead of image.

Exercise 12 **Date:……………………**

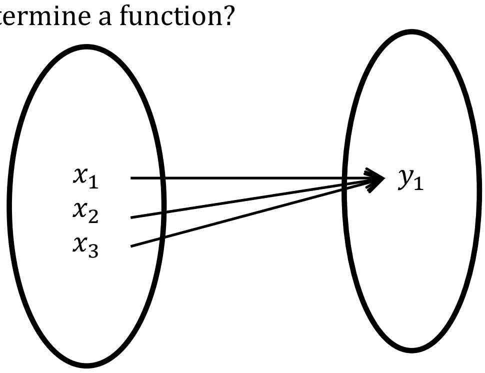
1.



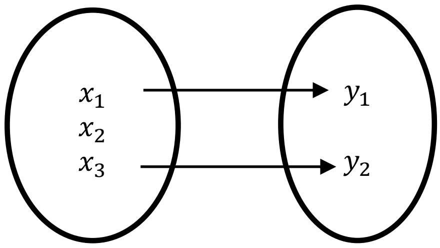
(i) What is the domain of the mapping?
(ii) What is the range of the mapping?
(iii) What is the rule of the mapping?

2. Which of the following mappings determine a function?

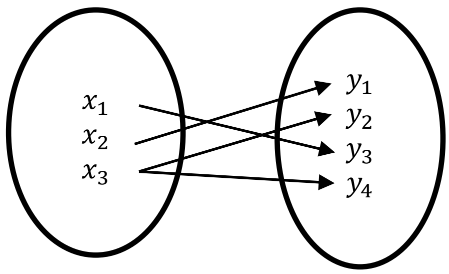
(i)



(ii)



(iii)



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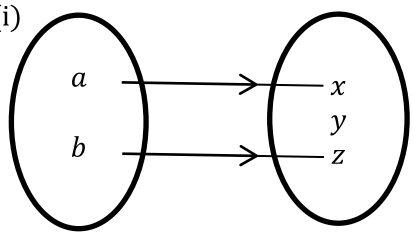
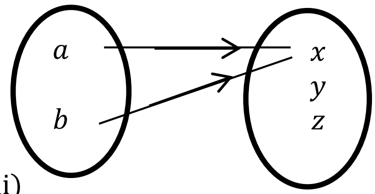
Exercise 13 **Date:……………………..**

1. Which of the following mappings is/are functions(s)?

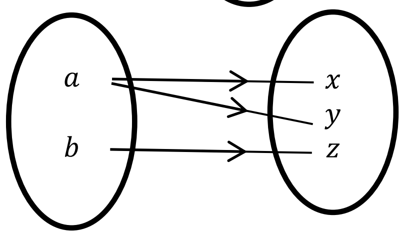
(i)



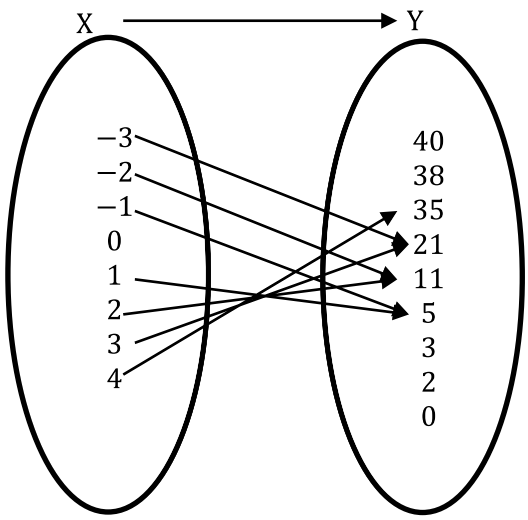
(ii)



(iii)



2.



(i) What is the domain of the mapping?
(ii) What is the range of the mapping?
(iii) What is the rule of the mapping?

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Example

A function f is defined by f: $x \longrightarrow 2x + 3$, find the image of

(i) 2 (ii) $-\frac{1}{2}$ under f.

Solution…

Given, $\mathrm{f}(x) = 2x + 3$

(i) When $x = 2$, $\mathrm{f}(2) = 2(2) + 3 = 7$

(ii) When $x = -\frac{1}{2}$, $\mathrm{f}\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) + 3$

$= -1 + 3$

$= 2$

Example

A function $g: x \longrightarrow \frac{3x+1}{x-1}, x \neq 1.$

Is defined on the set $T = \{-1,0,2,3,4,5\}$

(a) Find the range of g under the set T given

(b) Find the value of x for which the function has an image of 5

Solution…

Given $g: x \rightarrow \frac{3x+1}{x-1}, x \neq 1$

I.e. $g(x) = \frac{3x+1}{x-1}$

$T = \{-1,0,2,3,4,5\}$

When $x = -1$, $g(-1) = \frac{3(-1)+1}{-1-1} = 1$

$x = 0,\quad g(0) = \dfrac{3(0)+1}{0-1} = -1$

$x = 2,\quad g(2) = \dfrac{3(2)+1}{2-1} = 7$

$x = 3,\quad g(3) = \dfrac{3(3)+1}{3-1} = \dfrac{10}{2} = 5$

$x = 4\quad g(4) = \dfrac{3(4)+1}{4-1} = \dfrac{13}{3}$

$x = 5,\quad g(5) = \dfrac{3(5)+1}{5-1} = \dfrac{16}{4} = 4$

$\therefore$ The image under T is given as $\left\{1,-1,7,5,\frac{13}{3},4\right\}$

(b) Given image $= 5$

$\Rightarrow g(x) = 5$

$$\frac{3x+1}{x-1} = 5$$

$$3x+1 = 5(x-1)$$

$$2x = 6$$

$$x = 3$$

Hence the value of x that has an image of 5 is 3.

Exercise 14 **Date:……………………**

1. If $g(x) = \frac{x-2}{x+3}, x \in \mathbb{R}$ and $x \neq -3$.
   Find
   (i) $g(-2)$ (iii) $g\left(-\frac{3}{4}\right)$
   (ii) $g(1)$ (iv) $g(\sqrt{2})$

2. The function f is defined as $f: x \longrightarrow 3x^2 - 5x.$
   (i) Evaluate $f(-3)$
   (ii) Find the value of x for which $f(x) = -\frac{4}{3}$

Exercise 15 **Date:……………………..**

1. If $f(x) = x^3 - 3x^2 + 6x - 4$ and $g(x) = 2x - 1$, find
   (i) $f(-1)$ (ii) $g(-2)$ (iii) $f(\sqrt{2})$

2. The function f and g are defined as:
   $f: x \longrightarrow 2 - x^2$ and $g: x \longrightarrow \frac{1}{x-1}, x \neq 1.$
   Evaluate
   (i) $g\left(-\frac{1}{4}\right)$ (ii) $\frac{f(2)}{g(3)}$

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Exercise 16 **Date:……………………..**

1. Given that $f(x) = 2x - 1$ and $g(x) = x^2 + 1$.
   (i) Find $f(1 + x)$
   (ii) Find the range of value of x defined for which $f(x) < -3$.
   (iii) Simplify $f(x) - g(x)$.

2. Two functions f and g are defined as $f: x \longrightarrow \frac{x^2}{4} - 9$ and $g: x \longrightarrow \frac{1}{2x} (x \neq 0)$.
   (i) Evaluate $f(4) + g\left(-\frac{1}{3}\right)$
   (ii) If $f \times g = 2$, solve for x.

Exercise 17 **Date:……………………**

1. The set P $= \{-2, -1, 0, 1, 2\}$ map Q by the function $f(x) = x^2 - 2$, where $x \in$ P.
   (i) Find the elements of Q
   (ii) Draw a diagram showing the mapping between P and Q

2. The functions f and g are defined as follows:
   $f: x \longrightarrow \frac{x-1}{2}$ and $g: x \longrightarrow 3x + 1$
   (i) Evaluate $f\left(-\frac{1}{2}\right) + 1$
   (ii) Solve $f(x) = g(-2)$

3. The functions f and g are defined as $f: x \longrightarrow x - 2$ and $g: x \longrightarrow 2x^2 - 1$.
   Solve:
   (i) $f(x) = g\left(-\frac{1}{2}\right)$
   (ii) $f(x) + g(x) = 0$.

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Exercise 18 **Date:……………………**

1. Two functions f and g are defined by $f: x \longrightarrow 2x^2 - 1$ and $g: x \longrightarrow 3x + 2$, where x is a real number.
   (i) If $f(x-1) - 7 = 0$, find the values of x.
   (ii) Evaluate $\frac{f\left(-\frac{1}{2}\right) \cdot g(3)}{f(4) - g(5)}$.

2. B is the set $\{0, 1, 2, 3, 4, 5\}$ and a function $g: B \longrightarrow \mathbb{Z}$ is defined by $g(x) = 5x - 4$. Find the image of g.

3. A is the set $\{-3, -2, -1, 0, 1, 2\ 3\}$ and B is the interval $\{x \in \mathbb{R}: -3 \leq x \leq 3\}$. If $f: A \longrightarrow \mathbb{R}$ and $g: B \longrightarrow \mathbb{R}$ defined by $f(x) = 2x - 1$ and $g(x) = \frac{x}{3} + 1$. Find the image in each case.

4. If $f(x) = 2x - 1$, $g(x) = x^2 + 1$ and $h(x) = 2^x$.
   (a) Find the value of
      (i) $f\left(-\frac{1}{2}\right)$ (iii) $h(-3)$
      (ii) $g(-5)$

   (b) $g(x) = z$, find x in terms of z.

   (c) Find $g(f(x))$, in its simplest form.

   (d) $h(x) = 512$, find the value of x

   (e) solve the equation $2f(x) + g(x) = 0$, giving your answer correct to two decimal places.

   (f) Sketch the graph of
      (i) $y = f(x)$ (ii) $y = g(x)$

5. Given that $f: x \longrightarrow 2x^2 - 8x + 5$ and $g: x \longrightarrow x - 2$.
   (i) Calculate $f(-3)$
   (ii) Find the values of x such that $f(x) = g(x)$.

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Example

Given $g: x \longrightarrow ax^2 + b$, where a and b are constants. If $g(2) = 3$ and $g(-3) = 13$. Find the value of a and b and hence evaluate $g(-1)$.

Solution…

$g: x \longrightarrow ax^2 + b \Longrightarrow g(x) = ax^2 + b$

$g(2) = 3$

$\Longrightarrow a(2)^2 + b = 3$

$\therefore 4a + b = 3$………………….. (1)

Also,

$g(-3) = 13$

$a(-3)^2 + b = 13$

$9a + b = 13$ ……………………. (2)

$(2) - (1): 5a = 10 \quad \therefore a = 2.$

Put $a = 2$ into (1)

$4(2) + b = 3$

$b = -5$

Hence, $a = 2, b = -5$.

$\therefore g(x) = 2x^2 - 5$

$g(-1) = 2(-1)^2 - 5$

$= -3$

Exercise 19 **Date:……………………..**

1. Given that $\mathrm{f}(x) = px + q$, find the value of p and q if $\mathrm{f}(2) = 4$ and $\mathrm{f}(4) = 10$.

2. If $\mathrm{g}: x \longrightarrow \frac{9}{mx+n}$, where m and n are constants, $\mathrm{g}(2) = 3$ and $\mathrm{g}(-4) = -1$. Find the values of m and n.

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Exercise 20 **Date:…………………..**

1. If f: $x \longrightarrow m + nx$, where m and n are constants. Given also that f $(0) = 1$ and f $(4) = 21$, find the values of m and n.

2. A function f is defined by the set $\mathbb{R}$, of real number by f: $x \longrightarrow px^2 + qx + 2$, where p and q are constants. If $f(-2) = 0$ and $f(1) = 3$. Find $f(-4)$.

3. If $f(x) = ax^2 + bx + c$, where a, b and c are constants. If $f(0) = 5$.
   (i) What is the value of c?
   (ii) Given also that $f(1) = 6$ and $f(-1) = 8$, find the values of a and b
   (iii) Evaluate f(3).

4.
   (a) A function is defined by f: $x \longrightarrow \frac{x+1}{x-3}$, $x \neq 3$. Find the objects which have images $-4, 0, 1$ and 4.
   (b) If $f(x) = 5x^2 + 2x + 1$, find
   (i) f(2) (iii) $f(1 - a)$
   (ii) $f(-2)$ (iv) $f(x - 1)$

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Exercise 21 **Date:……………………**

1. A function is defined by f: $x \longrightarrow 3x - 2$ on the domain $\{-2, -1, 0, 1, 2, 3\}$.

   α) Find the images of the following elements in the domain.
   (a) -2 (b) 0 (c) 3

   β) What elements of the domain satisfy these equations?
   (a) f $(x) = 1$
   (b) f $(x) = 7$
   (c) f $(x) = -5$

2. A function is defined by f: $x \rightarrow \frac{1}{2}x$ on the domain $\{-2, -1, 0, 1, 2, 3\}$.

   α) Find the following elements of the range.
   (a) f(-1) (c) f (1)
   (b) f (3)

   β) What elements of the domain correspond to these images?
   (a) 0 (b) 1 (c) $1\frac{1}{2}$

3. A function g is defined by g: $x \rightarrow \frac{1}{x}$ on the domain $\{-2, -1, 0, 1, 2, 3\}$.

   α) Find the images of the following elements of the domain
   (a) -2 (b) 3 (c) 1

   β) Is the function g defined for all real numbers?

   γ) For what real number would g be undefined?

4. A function g is defined by g : $x \longrightarrow \frac{1}{2-x}$ on the domain $\{-3, -2, 1, 3, 5\}$.

   α) Find the images of the following elements of the domain.

   β) Is the function g defined for all real numbers?

   γ) For what real number would g be undefined?

5. (a) Find the range of the function $f : x \rightarrow 1 - \frac{1}{1-x}$ for the domain $A = \left\{-2, -1, 0, \frac{1}{2}, 2, 3\right\}$.

   (b) A function f is defined by
   $$f(x) = \begin{cases} x, \text{ when } x \in \mathbb{Z} \\ \frac{1}{x}, \text{ when } x \notin \mathbb{Z} \end{cases}$$
   Evaluate the following
   (i) $f(1)$ (ii) $f(0)$ (iii) $f\left(\frac{1}{3}\right)$

6. Given that $f(x) = \frac{ax+4}{ax+3b}$, find the values of a and b if $f(2) = 0$ and $f(3) = f(1)$.

7. Under the mapping $f(x) = ax^2 - bx + 2$, the image of 3 is 14 and the image of -2 is 24, find
   (i) the values of a and b
   (ii) the elements whose image is 4

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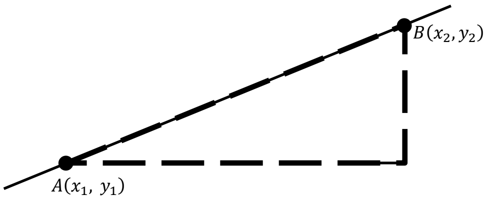
CO – ORDINATE GEOMETRY

GRADIENT (SLOPE) OF A STRAIGHT LINE
The gradient of a line is a measure of its steepness. The steeper the line, the larger the gradient (slope).



The gradient of the line joining $A(x_1, y_1)$ to $B(x_2, y_2)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1
Find the gradient of the line joining $A(3, -2)$ and $B(-5, -8)$.

Solution…
The gradient of AB, $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-8-(-2)}{-5-3}$
$= \frac{-6}{-8} = \frac{3}{4}$

Exercise 1 **Date:…………………**
Find the gradient of the line joining of the following pairs of points.
(i) $(1, 3)$ and $(2, 6)$
(ii) $(6, 3)$ and $(7, 4)$
(iii) $(-2, 5)$ and $(-3, 6)$
(iv) $(3, 2)$ and $(-5, 4)$
(v) $(-2, -1)$ and $(-6, -5)$

Exercise 2 **Date:…………………**
1.
(i) Find the gradient of the line joining the points $(2, -3)$ and $(-2, 5)$.
(ii) Find the gradient of the straight line which passes through the points $E(5, 0)$ and $F(0, 2)$.

2. The gradient of the line passing through the points $M(6, 7)$ and $N(y, 8)$ is $\frac{1}{3}$. Find the value of y.

3. The gradient of the line passing through the points $P(x, 5)$ and $R(4, -7)$ is $\frac{1}{2}$. Find the value of x.

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THE EQUATION OF A STRAIGHT LINE WITH GRADIENT M

CASE 1: $$y = mx + c$$

Where m is the gradient and c is a constant.

Example 2
Find the equation of a straight line which passes through the point $(2, 4)$ with gradient $\frac{1}{3}$.

Solution…
$y = mx + c$
Given $m = \frac{1}{3}$,

$\therefore y = \frac{1}{3}x + c$

$(x, y) = (2, 4)$
$\Rightarrow 4 = \frac{1}{3} \times 2 + c$
$\Rightarrow 12 = 2 + 3c$
$\Rightarrow 10 = 3c$
$\therefore c = \frac{10}{3}$

$\therefore y = \frac{1}{3}x + \frac{10}{3}$

Example 3
Find the equation of the line which passes through the points $(-2, 7)$ and $(2, -3)$.

Solution…
$y = mx + c$

$m = \frac{-3-7}{2-(-2)} = \frac{-10}{4} = -\frac{5}{2}$
$y = -\frac{5}{2}x + c$

Equation at $(-2, 7)$
$\Rightarrow 7 = -\frac{5}{2}(-2) + c$
$\Rightarrow 7 = 5 + c$
$\therefore c = 2$

$\therefore y = -\frac{5}{2}x + 2$

CASE II:

$$y - y_1 = m(x - x_1)$$

Where, (x, y) is a fixed point and m is the gradient.

Example 4
Find the equation of a straight line which passes through the point $(2, 4)$ with gradient $\frac{1}{3}$.

Solution…
$y - y_1 = m(x - x_1)$

$m = \frac{1}{3}, (x_1, y_1) = (2, 4)$
$\Rightarrow y - 4 = \frac{1}{3}(x - 2)$
$\Rightarrow y - 4 = \frac{1}{3}x - \frac{2}{3}$
$\Rightarrow y = \frac{1}{3}x - \frac{2}{3} + 4$
$\therefore y = \frac{1}{3}x + \frac{10}{3}$

Example 5
Find the equation of the line which passes through the points $(-2, 7)$ and $(2, -3)$.

Solution…
$y - y_1 = m(x - x_1)$

$m = \frac{-3-7}{2-(-2)} = -\frac{10}{4} = -\frac{5}{2}$

Let $(x_1, y_1) = (2, -3)$
$\Rightarrow y - (-3) = -\frac{5}{2}(x - 2)$
$\Rightarrow y + 3 = -\frac{5}{2}x + 5$
$\therefore y = -\frac{5}{2} + 2$

Exercise 3 **Date:....................**
Find the gradient of each of the following straight lines.
1. $y = 3x - 4$
2. $y = 4 - x$
3. $2y + 3x + 7 = 0$
4. $8y + 4x - 3 = 0$
5. $\frac{y}{2} + \frac{x}{3} = 7$

Exercise 4 **Date:....................**

Find the equation of the straight line that has the following properties.

(1) Gradient 3 and pass through $(4, 3)$

(2) Gradient -5 and passes through the point $(-1, -2)$

(3) Gradient $-\frac{1}{3}$ and passes through the point $(3, -2)$

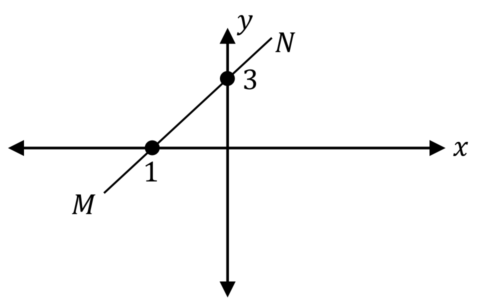
(4) Gradient $\frac{2}{3}$ and passes through the origin.

Exercise 5 **Date:…………………**

1. Find the equations of the straight lines joining each of the following pairs of points.
   (i) $(5, 6)$ and $(-1, -3)$
   (ii) $(-2, -4)$ and $(-3, -8)$
   (iii) $(2, -6)$ and $(0, -3)$
   (iv) $(0, -1)$ and $\left(\frac{2}{3}, 0\right)$

2. Find the equation of the straight line which intercepts -3 on the $x-$axis and -5 on the $y-$axis.

3. The diagram shows a straight line MN. Find the equation of MN.



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DISTANCE BETWEEN TWO POINTS

The general formula for the distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 6

Find the distance between the points $A(2, 7)$ and $B(4, 10)$.

Solution…

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(4 - 2)^2 + (10 - 7)^2}$

$|AB| = \sqrt{13}$ units

Example 7

Find the length between the points $P(-3, -5)$ and $Q(2, -4)$.

Solution…

$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|PQ| = \sqrt{\left(2 - (-3)\right)^2 + \left(-4 - (-5)\right)^2}$

$|PQ| = \sqrt{(2 + 3)^2 + (-4 + 5)^2}$

$|PQ| = \sqrt{26}$ units

Exercise 6 **Date:…………………**

Find the distance between each of the following pairs of points.

1. $A(-15, 12)$ and $B(-3, -4)$
2. $P(7, 7)$ and $Q(2, -3)$
3. $M(-1, 1)$ and $N(-7, 4)$
4. $P(3, 6)$ and $Q(8, 18)$
5. $C(-2, -6)$ and $D(2, 8)$
6. $M(2, 3)$ and $N(4, 7)$

Exercise 7 **Date:…………………**

Find the lengths between the points:

1. $D(-3, -5)$ and $Q(2, -4)$
2. $A(8, 7)$ and $B(2, 3)$
3. $A(-8, -2)$ and $B(-3, -4)$
4. $C(1, 3)$ and $D(4, -1)$
5. $K(-5, -2)$ and $L(-6, -1)$
6. $X(5, -3)$ and $Y(-1, -2)$

Exercise 8 **Date:....................**

1. Given that the distance between $A(\alpha, 4)$ and $B(2, 3)$ is equal to the distance between $C(3, -1)$ and $D(-2, 4)$, calculate the possible values of α.

2. If the distance between points $P(-1, 4)$ and $Q(2, k)$ is $\sqrt{58}$. Find the possible values of the constant k.

3. Find k given that $P(-1, 1)$ and $Q(k, -2)$ are 5 units apart.

4. The distance between $P(x, 7)$ and $Q(6, 19)$ is 13 units. Find the values of x

5. The distance between the points $(\alpha, 0)$ and $(0, \alpha)$ is equal to the distance between the points $(1, 2)$ and $(-1, 3)$. Find the value of α.

MIDPOINT OF TWO POINTS

The co – ordinates of the midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 8

Find the coordinates of the midpoint of the following points.

(i) $A(1, 2)$ and $B(4, 6)$

(ii) $P(-1, -4)$ and $Q(-3, -2)$

Solution...

(i) Midpt of $AB = \left(\frac{1+4}{2}, \frac{2+6}{2}\right) = \left(\frac{5}{2}, 4\right)$

(ii) Midpt of $PQ = \left(\frac{-1+(-3)}{2}, \frac{-4+(-2)}{2}\right)$
$= (-2, -3)$

Example 9

The points of A and B are $(x, -1)$ and $(-5, y)$. If the midpoint of AB is $\left(-\frac{1}{2}, 2\frac{1}{2}\right)$. Find the values of x and y.

Solution...

$\left(-\frac{1}{2}, 2\frac{1}{2}\right) = \left(\frac{x-5}{2}, \frac{-1+y}{2}\right)$

$\Rightarrow -\frac{1}{2} = \frac{x-5}{2}$

$\Rightarrow -1 = x - 5$

$\therefore x = 4$

Also,

$\Rightarrow \frac{5}{2} = \frac{-1+y}{2}$

$\Rightarrow 5 = -1 + y$

$\therefore y = 6$

$\therefore x = 4, y = 6$

Exercise 9 **Date:....................**

Find the co – ordinates of the midpoint of the following pairs of points:

1. $(1, 4)$ and $(3, 8)$
2. $(2, 6)$ and $(3, 7)$
3. $(-2, 5)$ and $(9, -4)$
4. $(-4, -3)$ and $(-6, -7)$
5. $(7, -5)$ and $(-2, -3)$

Exercise 10 **Date:....................**

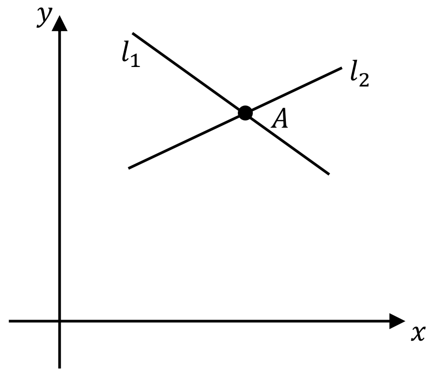
(a) Find the co – ordinates of the midpoint of the line segment joining the points.
  (i) $(8, 1)$ and $(2, 5)$
  (ii) $(5, -3)$ and $(-1, 0)$
  (iii) $(-2, 4)$ and $(4, -2)$

(b) If $P(x, -4)$ is the midpoint of point $Q(4, -1)$ and $R(x, y)$. Find the coordinates of R.

(c) The points P and Q are $(x, -3)$ and $(-7, y)$. If the midpoint of PQ is $\left(-\frac{1}{3}, 3\frac{1}{3}\right)$. Find the values of x and y.

THE POINT OF INTERSECTION OF TWO LINES

If two lines l_1 and l_2 meet at a point A, then A is called a point of intersection.



To find the points of intersection (A), we have to solve the equation simultaneously.

Example 10

The lines $2y + 3x - 16 = 0$ and $7y - 2x - 6 = 0$ intersect at point P. Find the co – ordinates of P.

Solution…

Since the two lines intersect, we solve them simultaneously.

$2y + 3x - 16 = 0$
$2y + 3x = -16$……………………… (1)

$7y - 2x - 6 = 0$
$7y - 2x = 6$……………………………… (2)

$2 \times (1) + 3 \times (2)$
$\Longrightarrow 25y = 50 \qquad \therefore y = 2$

Put $y = 2$ into (1)
$2(2) + 3x = 16$
$3x = 12$
$\therefore x = 4$
$\therefore P(x, y) = P(4, 2)$

Exercise 11 **Date:…………………**

Find the coordinates of the points of intersection of each of the following pairs of straight lines.

1. $2x + 5y + 6 = 0$ and $3x + 4y - 2 = 0$
2. $3x + 2y - 2 = 0$ and $5x + 3y - 8 = 0$
3. $y - 3 = -2x$ and $y - 8 = 3x$
4. $3y - 5x = 1$ and $y + 2x = 7$

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PARALLEL LINES

Two lines l_1 with gradient m_1 and l_2 with gradient m_2 are said to be parallel if and only if their gradients are the same.

i.e. $m_1 = m_2$

Example 11

If the line $y = 2kx + 5$ is parallel to $y = 10x - 6$, find the value of k.

Solution…

$y = 2kx + 5, \quad \therefore m_1 = 2k$
$y = 10x - 6, \quad \therefore m_2 = 10$

Since the lines are parallel

$\Rightarrow m_1 = m_2$
$\Rightarrow 2k = 10$
$\therefore k = 5$

Exercise 12 **Date:…………………**

1. If the line $y = (k + 2)x + 3$ is parallel to $y = (3k - 1)x + 5$, find the value of k.

2. The line $y = (k + 3)x + 4$ is parallel to $y = (4k - 9)x + 6$. Find the value of k.

3. If the line $ky + 2x + 7 = 0$ is parallel to the line $50y + kx - 19 = 0$, find the value of k. Where $k > 0$.

4. If the line $3x - 4y - 12 = 0$ and $px + 6y - 9 = 0$ are parallel, find the value of the constant p.

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Exercise 13 Date:....................

1. Write the equation of the line parallel to the straight line $y = \frac{3}{2}x + 5$ and passing through the point $(2, 3)$.

2. Find the equation of the line which passes through the point $L(1, 3)$ and is parallel to the line $y = \frac{2}{3}x + 1$.

Exercise 14 **Date:....................**

1. Find the equation of the straight line that has the following properties.
   (i) Pass through $(2, 3)$ and is parallel to $y = 2x + 3$.
   (ii) Passes through $(-3, -2)$ and is parallel to $y = 1 - 3x$.

2. Find the equation of the line passing through the point $\left(-\frac{1}{2}, 2\right)$ and parallel to the line $4x + 3y - 2 = 0$.

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PERPENDICULAR LINES

Two non – vertical lines l_1 and l_2 with (slopes) gradients m_1 and m_2 are perpendicular if and only if:

$$m_1 \times m_2 = -1$$

Example 12

The line $y = 2x + 1$ perpendicular to the line $3y + bx - 4 = 0$, find the value of b.

Solution…

$y = 2x + 1 \qquad \therefore m_1 = 2$

$3y + bx - 4 = 0$

$3y = -bx - 4$

$y = -\frac{b}{3}x - 4 \qquad \therefore m_2 = -\frac{b}{3}$

Since the lines are perpendicular,

$\Rightarrow m_1 \times m_2 = 1$

$2 \times -\frac{b}{3} = -1$

$-2b = -3$

$b = \frac{3}{2}$

Exercise 15 **Date:....................**

1. Find the equation of the line which is perpendicular to the line $y = 2x - 1$ and passes through the point $(2, 5)$.

2. Find the equation of the straight line that has the following properties.
   (i) Passes through $(-1, 4)$ and is perpendicular to $y = -5x + 3$.
   (ii) Passes through $\left(3, -\frac{2}{3}\right)$ and is perpendicular to $3y - 2x = 6$.
   (iii) Passes through $\left(-\frac{2}{3}, \frac{1}{5}\right)$ and is perpendicular to $2y + 9x = 7$.

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Exercise 16 **Date:....................**

1. Find the distance of the point $A(3, -2)$ from the mid – point of the line joining the points $B(4, -2)$ and $C(6, 6)$.

2. The line with equation $y = 2x - k$ passes through the point $(4, 0)$. Work out the value of k.

3. If the point $(p, 0)$ lies on the line $2y + 3x - 9 = 0$, find the value of p.

4. Find the point of intersection of $y = \frac{3}{x}$ and $y = (x + 2)$.

5.
   (a) The line $y = 4$ meets the line $2x + y = 8$ at the point A. Find the co – ordinates of A.
   (b) The line $3x + y = 18$ meets the x –axis at the point B. Find the co – ordinates of B.

   (c)
      (i) Find co – ordinates of the midpoint of the line joining A and B.
      (ii) Find the equation of the line through M parallel to $3x + y = 18$.

6. Find the perimeter of the triangle whose vertices are $P(6, 4)$, $Q(-3, 1)$ and $R(9, -5)$.

7. The lines $4x + 6y - 5 = 0$ and $2x + 4y - 3 = 0$ intersect at N. Find the equation of the line through N perpendicular to the line $x + 2y = 0$.

8. The lines $3x + 2y - 1 = 0$, $4x + 5y + 3 = 0$ intersect at M. Find the
   (a) Co – ordinates of M
   (b) Equation of the line through M parallel to the line $3x - 5y + 7 = 0$

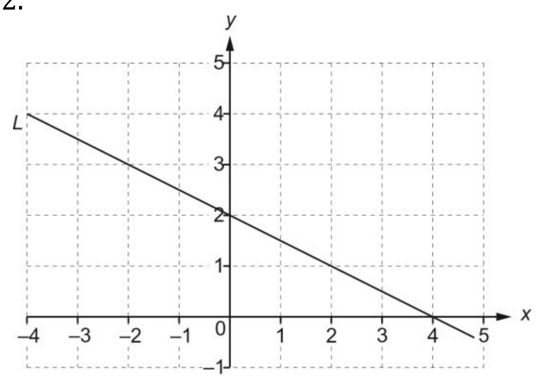
9. Find the equation of the line passing through the midpoint of the line joining $P(5, 1)$ and $Q(-1, -5)$ and perpendicular to PQ.

10. A straight line joins the point $(3, 2)$ to the point of intersection of the line $x - y + 4 = 0$ and $y - 2x - 5 = 0$. Find the equation.

Exercise 17 **Date:....................**

1. Find the equation of the straight line that is perpendicular to the line $y = \frac{1}{2}x + 1$ and passes through the point $(1, 3)$.

2.



Line L is drawn on the grid.
   (i) Find the gradient of line L
   (ii) Find the equation of line L in the form $y = mx + c$
   (iii) Line M is parallel to L. Line M passes through the point $(0, 3)$. Write down the equation of line M.

3. The lines whose equations are $3x + y = 2$ and $4x - 2y = 6$ intersect at (x, y). Find the point of intersection.

4. The line $4x + ky = 20$ passes through the points $A(8, -4)$ and $B(b, 2b)$, where k and b are constants.
   (i) Find the values of k
   (ii) Find the co – ordinates of the midpoint of AB

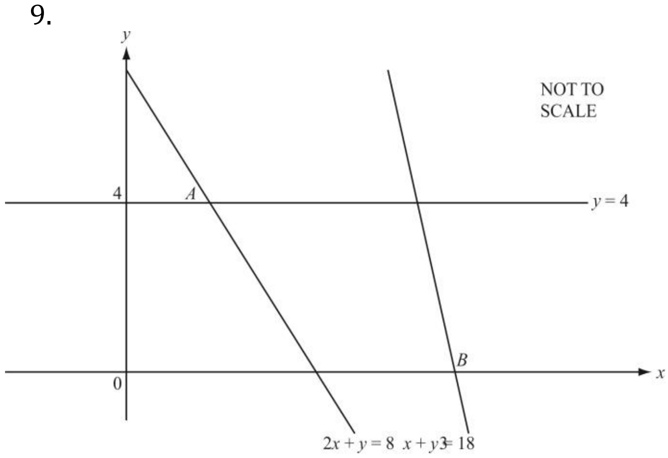
5. A straight line parallel to $2x + 3y = 6$, passes through the point $(-1, 2)$. Find the equation of the line.

6. The line joining the points $A(6, 2)$ and $B(-2, 6)$ is perpendicular to the line L. If L passes through the point $(0, 2)$, find the equation of the line L.

7.
   (a) The line joining $(3, a)$ to $(7, -4a)$ is parallel to the joining $(-1, -3)$ to $(3, 7)$. Find a.
   (b) The line joining $(-2, 1)$ to $(6, 4)$ is parallel to the line joining $(-a, 5)$ to $(4, a)$. Find a.

8. The points $(2, 5)$, $(3, 3)$ and $(k, 1)$ all lie in a straight line.
   (a) Find the value of k.
   (b) Find the equation of the line.

9.



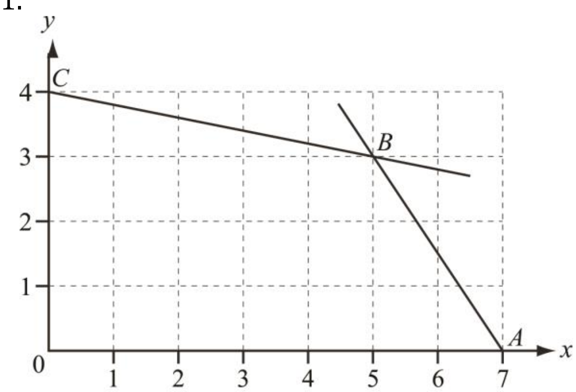
   (a) The line $y = 4$ meets the line $2x + y = 8$ at the point A. Find the co – ordinates of A.

   (b) The line $3x + y = 18$ meets the x –axis at the point B. Find the co – ordinates of B.

(c)
  (i) Find the co – ordinates of the midpoint M of the line joining A to B.
  (ii) Find the equation of the line through M parallel to $3x + y = 18$.

10.
  (a) Find the co – ordinates of the midpoint of the line joining $A(-8, 3)$ and $B(-2, -3)$.
  (b) The line $y = 4x + c$ passes through $(2, 6)$. Find the value of c.
  (c) The lines $5x = 4y + 10$ and $2y = kx - 4$ are parallel. Find the value of k.

11.



The lines AB and CD intersects at B.

  (a) Find the co – ordinates of the midpoint of AB.

  (b) Find the equation of the line CB.

12. The equation of a straight line can be written in the form $3x + 2y - 8 = 0$.
  (a) Rearrange the equation to make y the subject.

  (b) Write down the gradient of the line.

  (c) Write down the co – ordinates of the point where the line crosses the y –axis.

13.
  (a) A straight line passes through two points with co – ordinates $(6, 8)$ and $(0, 5)$. Work out the equation of the line.
  (b) Find the length of the straight line from $Q(-8, 1)$ to $R(4, 6)$.

14. Line A has equation $y = 5x - 4$.
Line B has equation $3x + 2y = 18$.
  (a) Find the gradient of the line.
    (i) A (ii) B

  (b) Write down the co – ordinates of the point where line A crosses the x –axis.

  (c) Find the equation of the line perpendicular to line A which passes through the point $(10, 9)$. Give your answer in the form $y = mx + c$.

15. Three points have co – ordinates $A(-8, 6)$, $B(4, 2)$ and $C(-1, 7)$. The line through C perpendicular to AB intersects AB at the point P.
  (i) Find the equation of the line AB
  (ii) Find the equation of the line CP
  (iii) Show that P is the midpoint of AB
  (iv) Calculate the length of CP
  (v) Hence find the area of the triangle ABC

16. The co – ordinates of three points are $A(-2, 6)$, $B(6, 0)$ and $C(p, 0)$.
  (i) Find the co – ordinates of M, the midpoint of AB.
  (ii) Given that CM is perpendicular to AB, find the value of the constant p.
  (iii) Find angle MCB.

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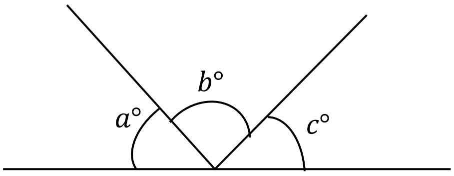
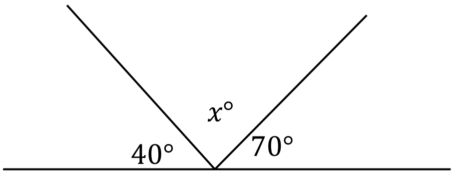
1. The angles on a straight line add up to 180°.



i.e. $a° + b° + c° = 180°$.

Example 1

Find the value of x.



Solution

$40° + x° + 70° = 180°$

$x + 110° = 180°$

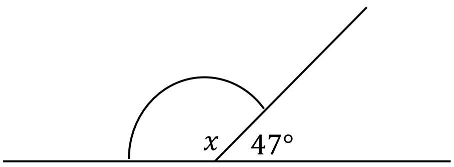
$x = 180° - 110°$

$x = 70°$

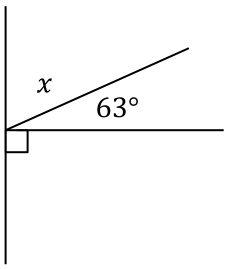
Exercise 1 **Date:……………………**

Find the value of x in the following.

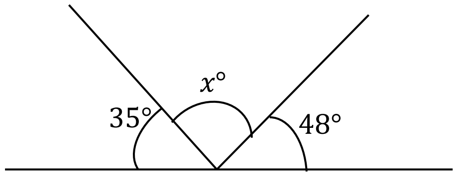
1.



2.



3.



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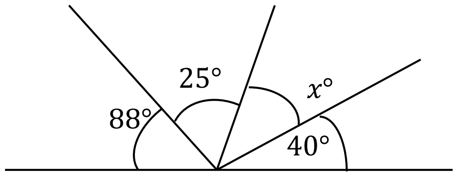

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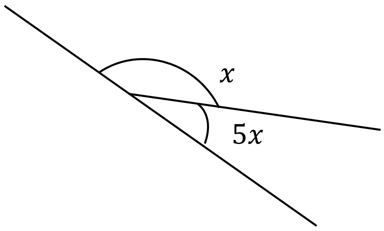

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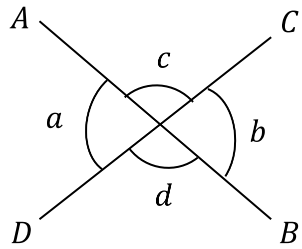
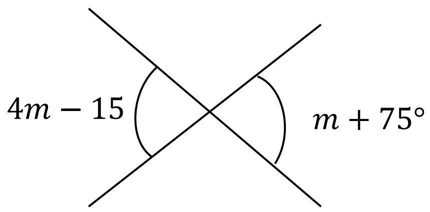
2. Vertically opposite angles are equal.

When two straight lines intersect at a point, the angles formed on the opposite sides of the point of intersection are called **vertically opposite angles**.



Angle c = Angle d
Angle a = Angle b

Example 2

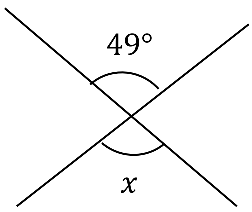


What is the value of m in the diagram?

Solution
Vertically opposite angles are equal

$$\Rightarrow 4m - 15° = m + 75°$$
$$4m - m = 75° + 15°$$
$$3m = 90°$$
$$m = 30°$$

Exercise 2 **Date:……………………..**
Find the values of the letters in the following diagrams.
1.



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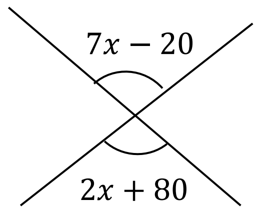
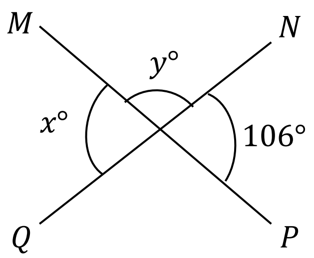
2.



3.



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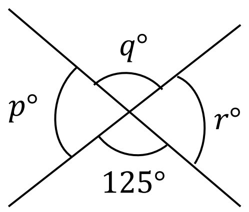

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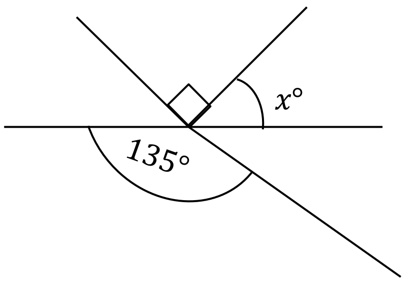
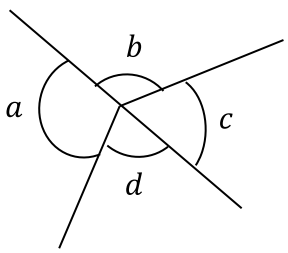


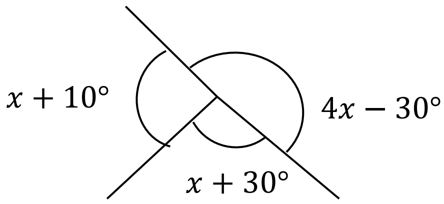
3. The angles of a point add up to 360°



$\hat{a} + \hat{b} + \hat{c} = 360°$

Example 3

Find the value of x in the diagram below.



Solution

Angles of a point add up to 360°

$x + 10° + 4x - 30° + 2x + 30° = 360°$

$7x + 10 = 360°$

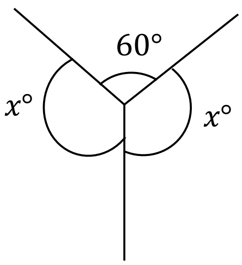
$7x = 350°$

$\therefore x = 50°$

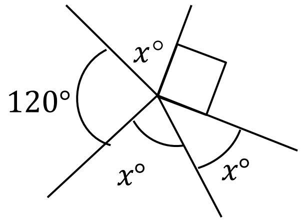
Exercise 3 **Date:……………………**

Find the values of the letters marked in the diagrams.

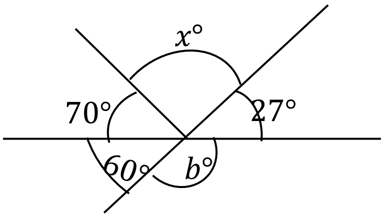
1.



2.



3.



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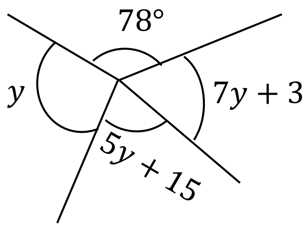

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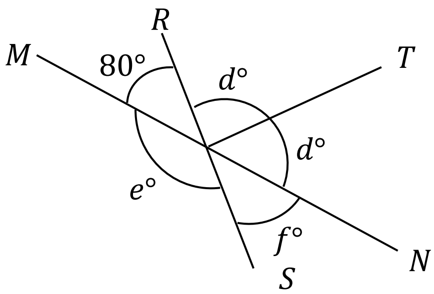

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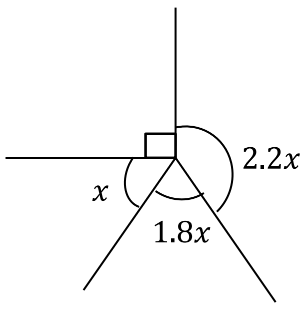

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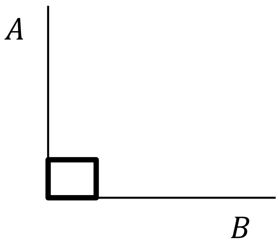
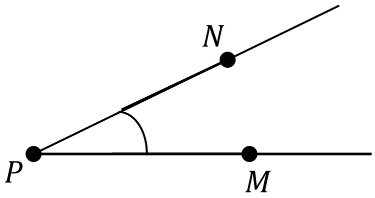
TYPES OF ANGLES

A full turn is made up to 360°

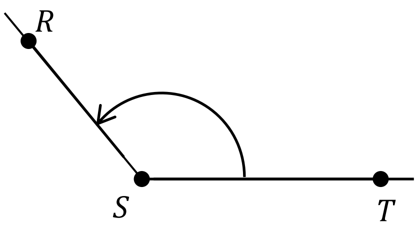
$\therefore$ a quarter turn has $\frac{360°}{4}$ or 90°. This is called a **right – angle** and it is represented as



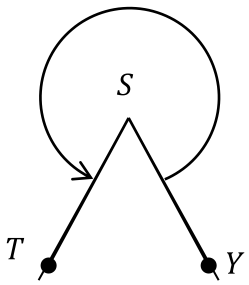
- An angle which is less than 90° is called **an acute angle**.



- An angle which is greater than 180° is called **an obtuse angle**.



- an angle that is greater than 180° but less than 360° is called a **reflex angle**.



Exercise 4 **Date:……………………**

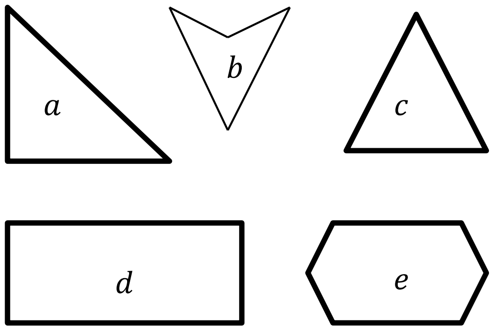
Which of the shapes below

i. have acute angle?

ii. have right angles?

iii. Have reflex angles?

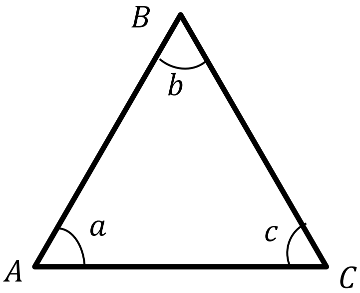
iv. are symmetrical



ANGLE PROPERTIES OF A TRIANGLE

A triangle is a closed figure formed by three line segments. There are various types of triangles depending on the angles or the lengths of the sides of the triangle.

The sum of the angles of a triangle is 180°



$$\hat{a} + \hat{b} + \hat{c} = 180°$$

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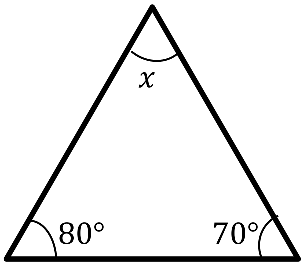
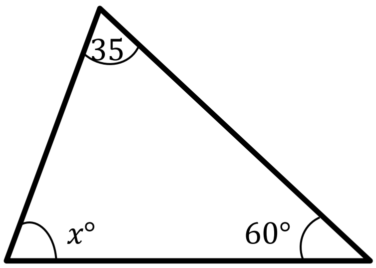
Exercise 5 **Date:……………………**

Find the value of x in the following diagrams.

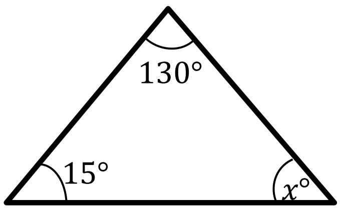
1.



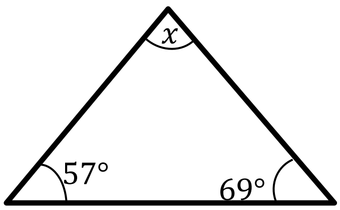
2.



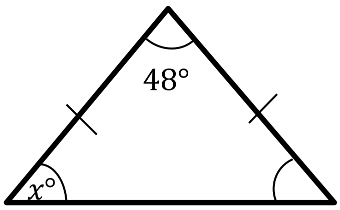
3.



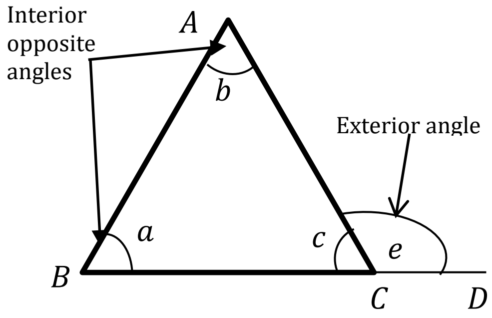
4.



5.



The exterior angle of a triangle is equal to the sum of the two interior opposite angles.



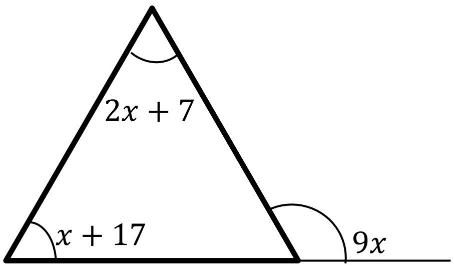
$a + b = e$

Example 4

The diagram is a triangle ABC with the side AC produced to D. Find

(i) the value of x.

(ii) angle ACB.



Solution

(i) The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

$\Rightarrow x + 17 + 2x + 7 = 9x$

$3x + 24 = 9x$

$24 = 9x - 3x$

$24 = 6x$

$\therefore x = 4°$

(ii) Angles on a straight line add up to 180°

$\Rightarrow \angle ABC + 9x = 140°$

But $x = 4°$

$\Rightarrow \angle ABC + 9(4) = 180°$

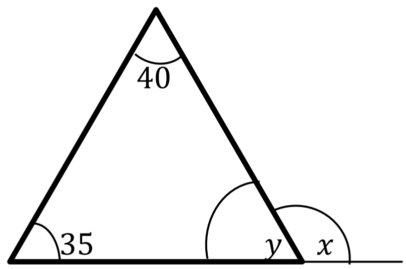
$\angle ABC = 180° - 36°$

$= 144°$

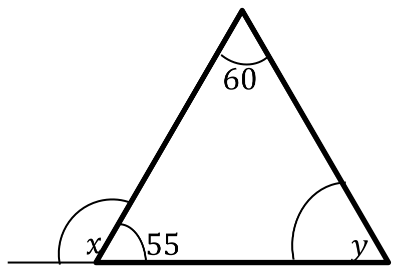
Exercise 6 **Date:……………………..**

Find x, y and z in the following diagrams.

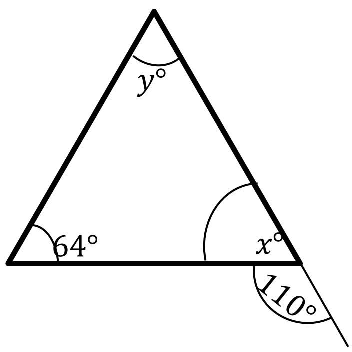
1.



2.



3.



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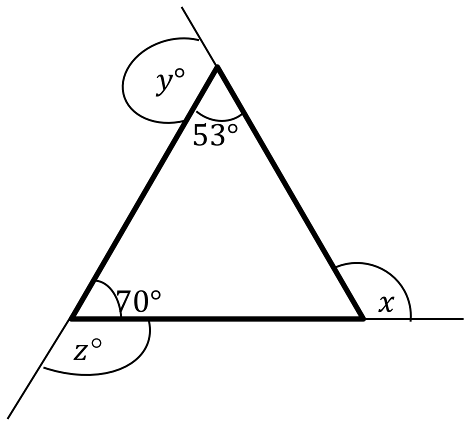

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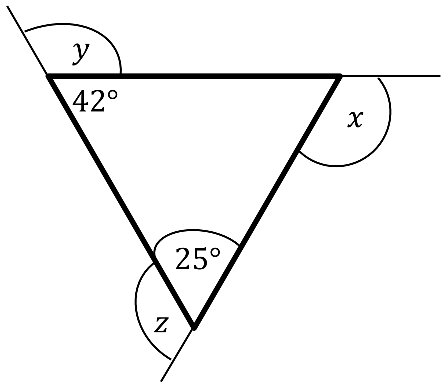
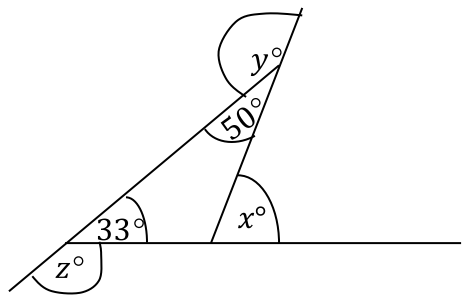
5.



6.



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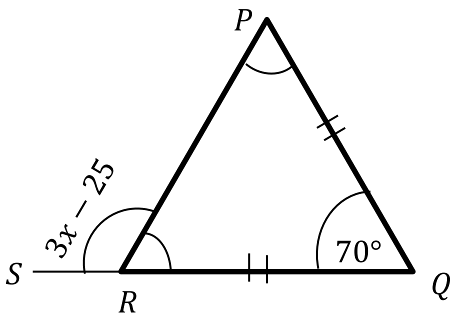
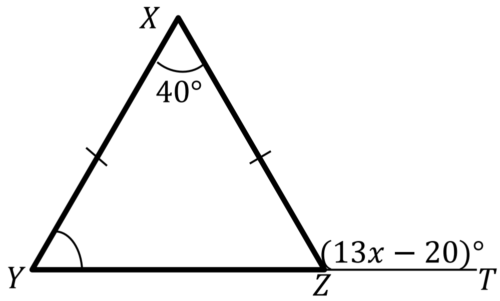
Exercise 7 **Date:……………………**

Find the value of x in the following diagrams.

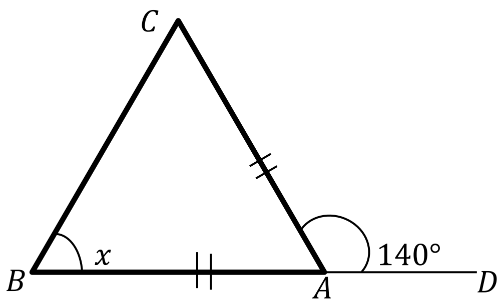
1.



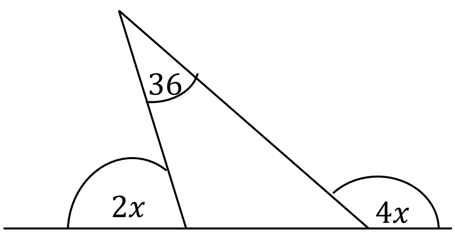
2.



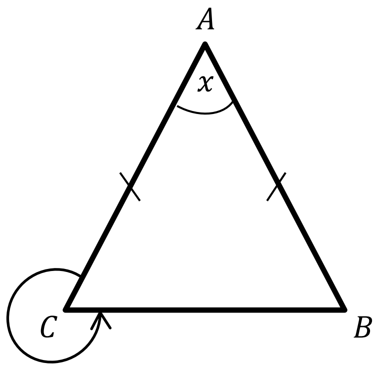
3.



4.

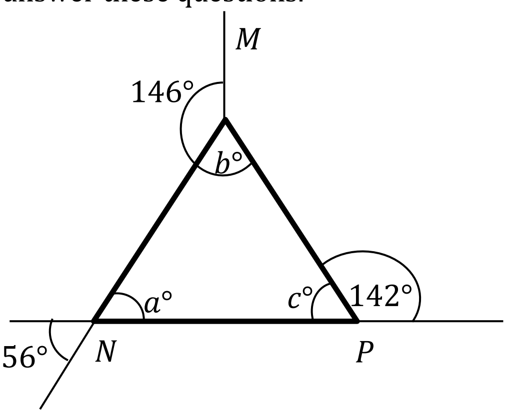


5.



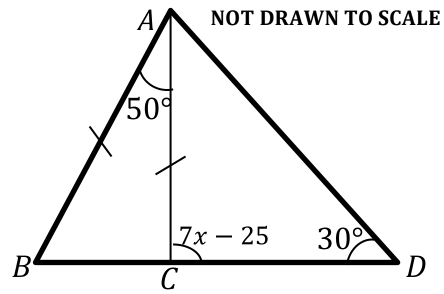
Exercise 8 **Date:……………………**

1. Consider the following diagram and answer these questions.



i) Find $\angle MNP$.
ii) What is the supplement of $142°$?
iii) Write down the value of $a + b + c$.

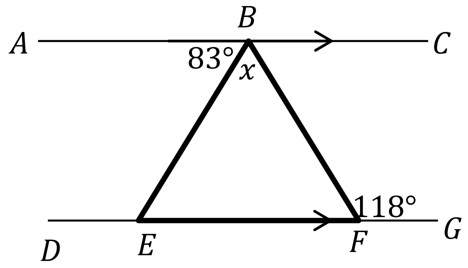
2.



In the diagram, $|AB| = |AC|$, angle $ADC = 30°$ and angle $ACD = 7x - 25°$, find
(i) the value of x.
(ii) angle DAC.
(iii) angle BAD.

i.e. corresponding angles are equal alternate angles are equal co – interior angles add up to 180°.

Example 5



In the diagram above, AC is parallel to DG angle $BFG = 118°$ and angle $ABE = 83°$. Find the value of

(i) angle CBF.

(ii) x.

Solution

(i) $\angle CBF$ and $\angle BFG$ are co – interior angles which add up to 180°.

$\Longrightarrow \angle CBF + \angle BFG = 180°$

$\angle CBF + 118° = 180°$

$\angle CBF = 180° - 118°$

$\angle CBF = 62°$

(ii) Alternate angles are the same

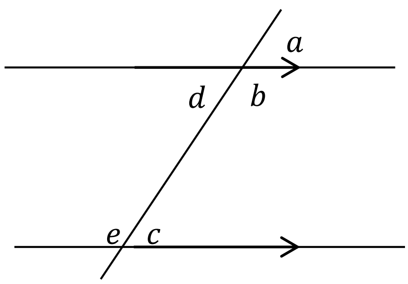
$\Longrightarrow \angle ABF = \angle BFG$

$\Longrightarrow 83° + x = 118°$

$x = 118° - 83°$

$x = 35°$

ANGLES FORMED WITHIN PARALLEL LINES



(i) $\hat{a} = \hat{d}$ (vertically opposite angles)

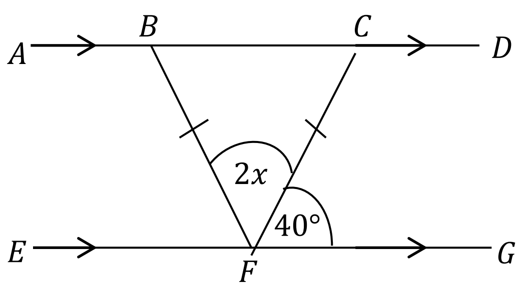
(ii) $\hat{a} = \hat{c}$ (corresponding angles)

(iii) $\hat{c} = \hat{d}$ (alternate angles)

$\hat{e} = \hat{d}$, (alternate angles)

(iv) $\hat{b} + \hat{c} = 180, \hat{d} + \hat{e} = 180°$ (co – interior angles)

Exercise 9 **Date:……………………**



NOT DRAWN TO SCALE

In the diagram, $\overline{AD}$ is parallel $\overline{EG}$, angle $CFG = 40°$ and triangle BCF is isosceles. Find the value of:

(i) angle CBF;

(ii) angle DCF

(iii) x

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Exercise 10 **Date:……………………**

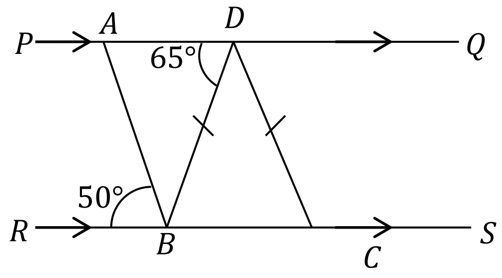
In the diagram $PADQ$ and $RBCS$ are parallel lines, $|BD| = |DC|$, angle $ADB = 65°$ and angle $ABR = 50°$.

(i) Calculate angle BDC

(ii) Calculate angle ABD

(iii) Calculate angle BAD

(iv) What type of triangle is triangle ABD?



NOT DRAWN TO SCALE

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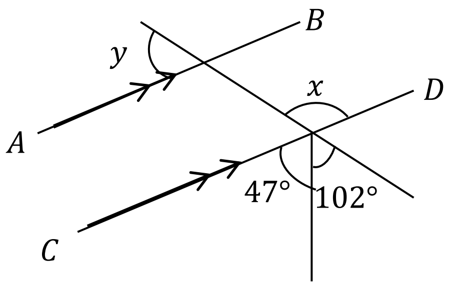
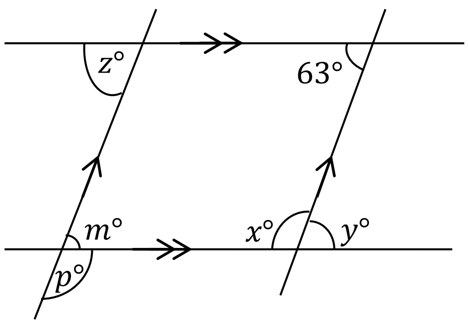
Exercise 11 **Date:……………………..**

(a) In the diagram, $\overline{AB}$ is parallel to $\overline{CD}$.
Find the value of :
(i) x (ii) y



NOT DRAWN TO SCALE

(b) Find the value of the letters marked in the diagram.

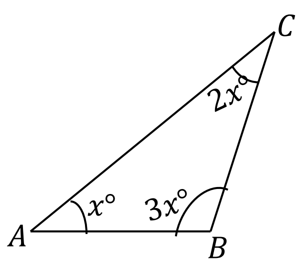


NOT DRAWN TO SCALE

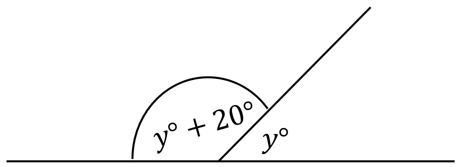
Exercise 12 **Date:……………………..**

For each of the following diagrams, write down an equation involving the given variable and solve for it.

1.



2.



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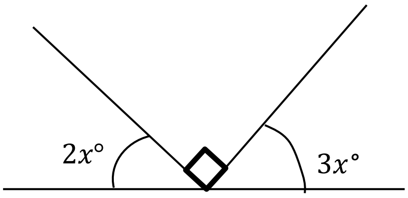
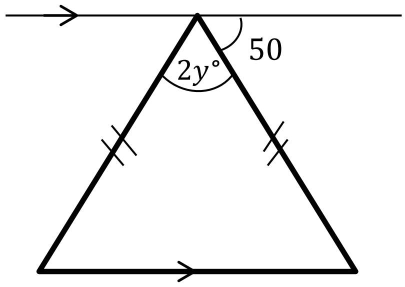
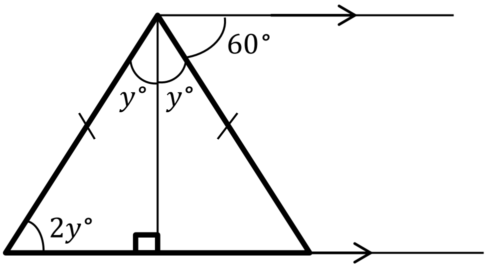
3.



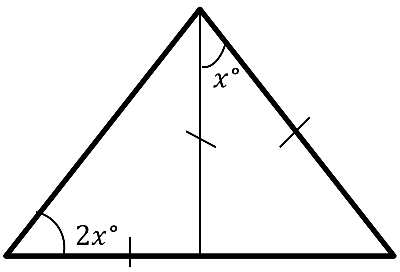
4.



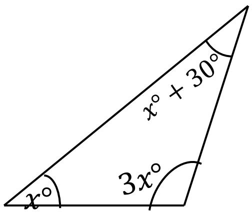
5.



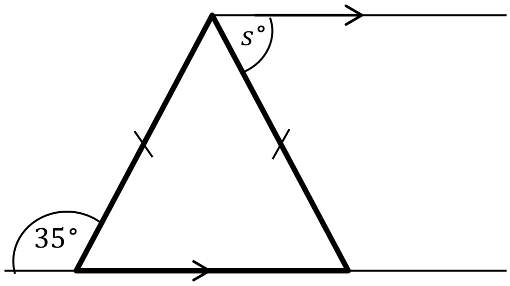
6.



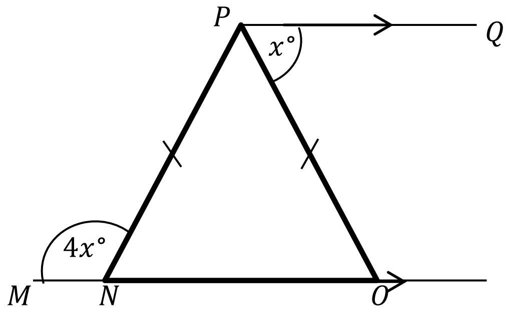
7.



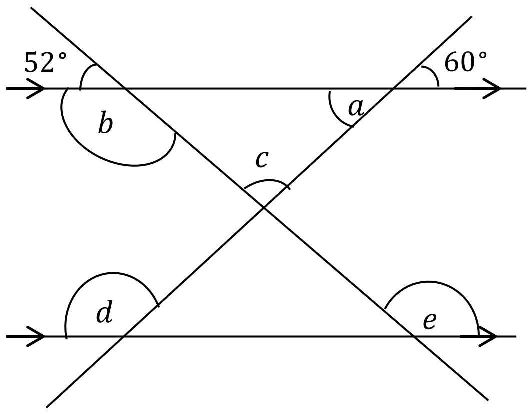
8.



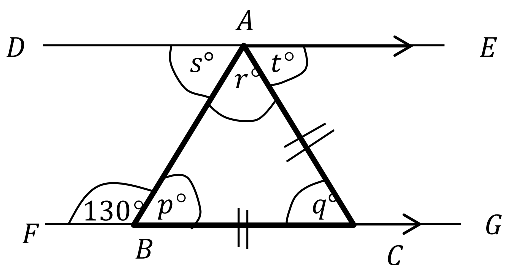
9.



10.



11.



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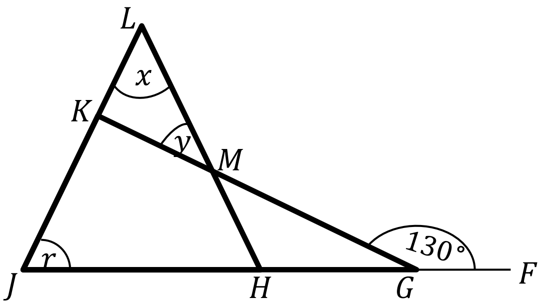
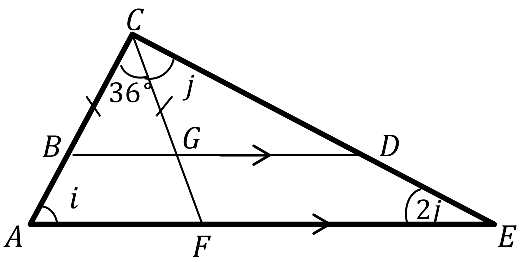
EXERCISE 13 **Date:……………………**



In the diagram, $\angle KLM = x$, $\angle LMK = y$, $\angle KJH = r$ and $\angle KGF = 130°$. If $2x = r = y$, find the value of x.

EXERCISE 14 **Date:……………………**



NOT DRAWN TO SCALE

In the diagram, ACE is a triangle, CF is a straight line, $BD//AE$ and $|BC| = |CG|$. If $\angle BCG = 36°$, $\angle BAF = i$, $\angle GCD = j$ and $\angle DEF = 2j$, find the values of i and j.

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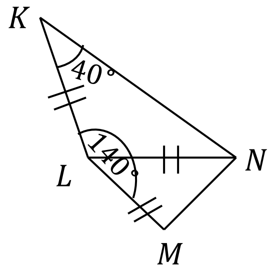
EXERCISE 15 **Date:……………………..**

In a quadrilateral $KLMN$, **not drawn to scale**, $LM = LN = LK$, $\angle KLM = 140°$ and $\angle LKN = 40°$.



NOT DRAWN TO SCALE

Giving the reson for each step of yor answer, calculate the size of

(i) $\angle LNK$ (ii) $\angle NLM$ (iii) $\angle KNM$

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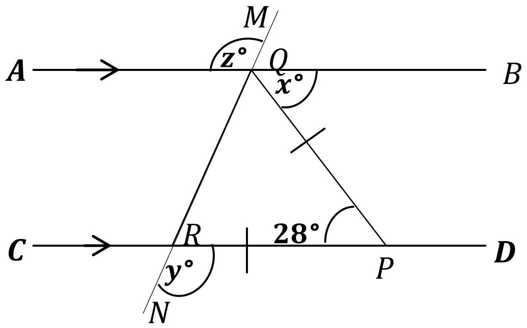
EXERCISE 16 **Date:……………………**

In the figure below, not drawn to scale, the lines AQB and $CRPD$ are parallel and $MQRN$ is a transversal. $PQ = PR$ and angle $QPR = 28°$.



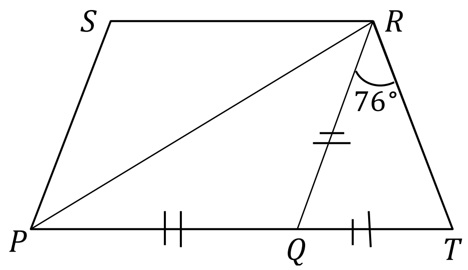
NOT DRAWN TO SCALE

Calculate, giving reasons for your answer, the value of

(i) x (ii) y (iii) z

EXERCISE 17 **Date:……………………**

$PTRS$, not drawn to scale, is a quadrilateral. Q is a point on PT such that $QT = QR = QP$. Angle $QRT = 76°$.



NOT DRAWN TO SCALE

Determine, giving a reason for each step of your answer, the measure of

(i) Angle RQT

(ii) Angle PRT

(iii) Angle SPT, given that angle $SRT = 145°$ and angle $PSR = 100°$.

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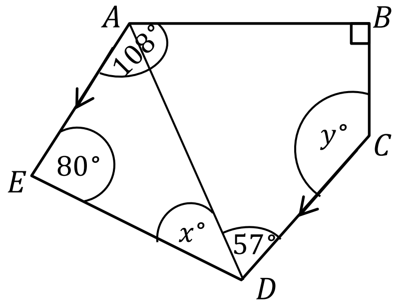
EXERCISE 18 **Date:……………………**

In the diagram shown below, $ABCDE$ is a pentagon. $\angle BAE = 108°$, $\angle ABC = 90°$, $\angle AED = 80°$, $\angle ADC = 57°$ and AE is parallel to CD.

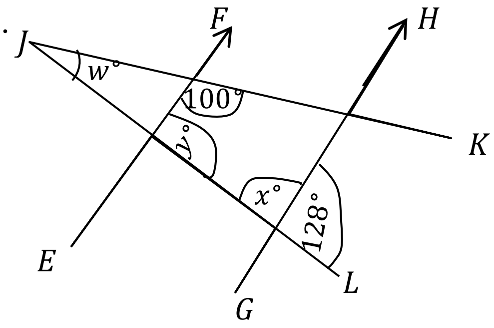


NOT DRAWN TO SCALE

Calculate the size of the angle marked
(i) $x°$ (ii) $y°$

EXERCISE 19 **Date:……………………**

1.



NOT DRAWN TO SCALE

Determine, giving reasons for EACH of your answers, the value of
(i) x (ii) y (iii) w

2. An angle is three times its supplement. Find the size of the angle.

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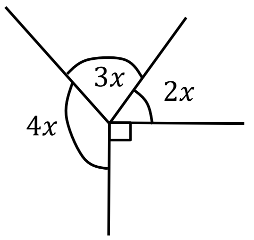
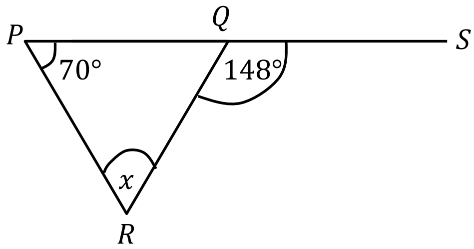
Exercise 20 **Date:……………………..**

For each of the following figures, write down an equation involving the given variable and solve it.

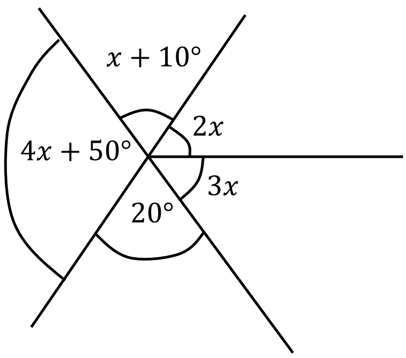
1.



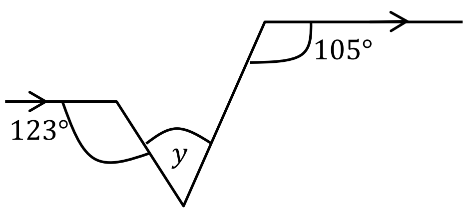
2.



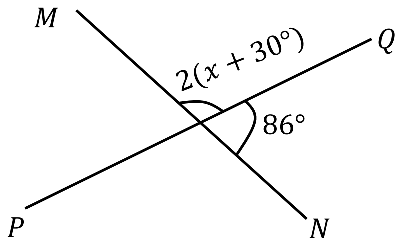
3.



4.



5.



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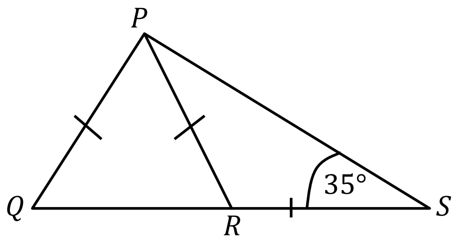
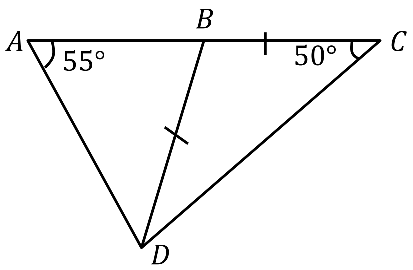
Exercise 21 **Date:……………………**

1. In the diagram below, $|PQ| = |PR| = |RS|$ and $< RSP = 35°$. Find $< QPR$.

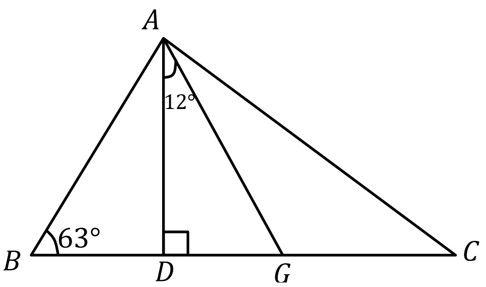


2. In ΔPQR, T is a point on QR such that $< QPT = 39°$ and $< PTR = 83°$, calculate $< PQT$.

3. In the diagram AC is a straight line. $|BC| = |BD|$, $< BCD = 50°$ and $< BAD = 55°$. Find $< BDA$.



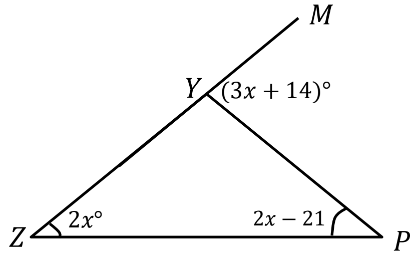
4. In the diagram, AD is perpendicular to BC. AG is the bisector of $< BAC$. $< ABC = 63°$, $< DAG = 12°$.



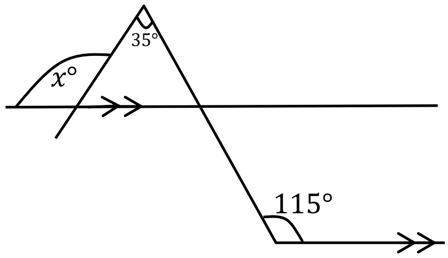
Show that triangle AGC is isosceles.

Exercise 22 **Date:……………………**

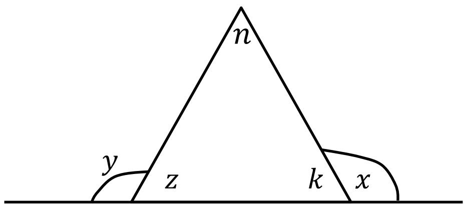
1. In the diagram, ZM is a straight line. Calculate the value of x.



2. In the diagram, calculate the value of x.

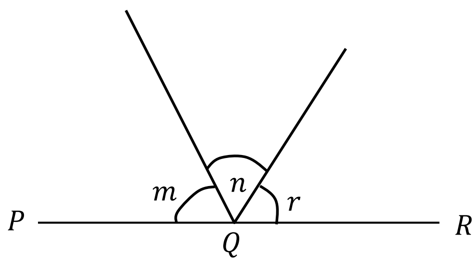


3.

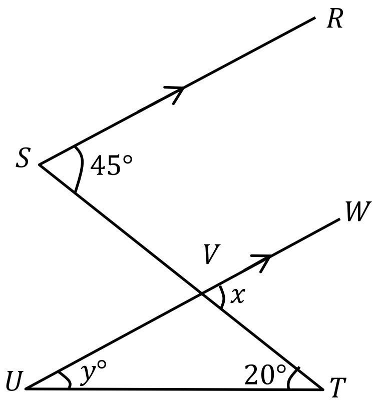


In the diagram, the value of $x + y = 220°$. Find the value of n.

4. In the diagram, PQR is a straight line. $(m + n) = 120°$ and $(n + r) = 100°$. Find $(m + r)$.



5.



In the diagram, $\overline{SR}$ is parallel to $\overline{UW}$.
$< WVT = x°, < VUT = y°, < RSV = 45°$ and $< VTU = 20°$.

(i) Find the value of x

(ii) Calculate the value of y

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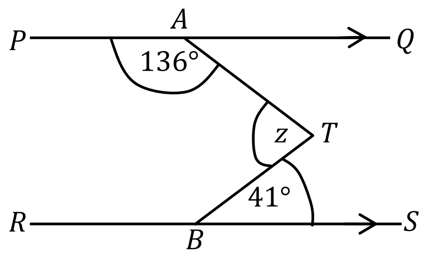
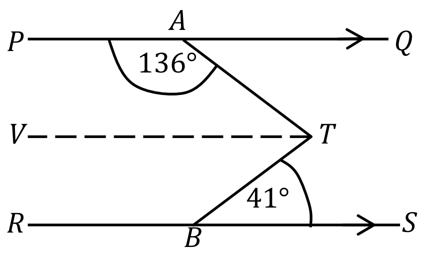
Example



In the diagram, $\overline{PQ}//\overline{RS}$. Find the value of z.

Solution…
Draw a line parallel to PQ and RS at the point T as shown in the diagram below.

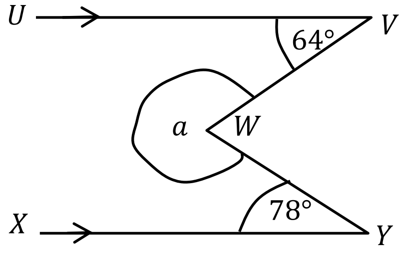


$< PAT + < QAT = 180°$
$136° + < QAT = 180°$
$< QAT = 180° - 136°$
$< QAT = 44°$
Alternate angles are equal
$\Rightarrow < VTB = < TBS = 41°$
$< QAT = < VTA = 44°$

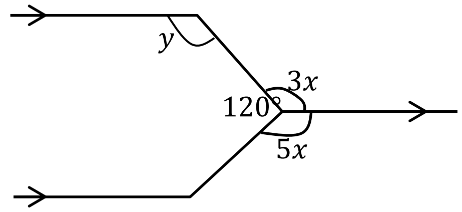
$z = < VTA + < VTB$
$z = 44° + 41°$
$z = 85°$

Exercise 23 **Date:……………………**

1. In the figure below $\overline{UV}$ is parallel to $\overline{XY}$, angle $UVW = 64°$ and angle $xyw = 78°$. Find the value of the angle marked a.

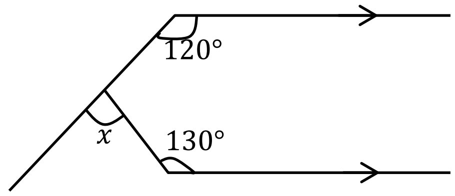


2.



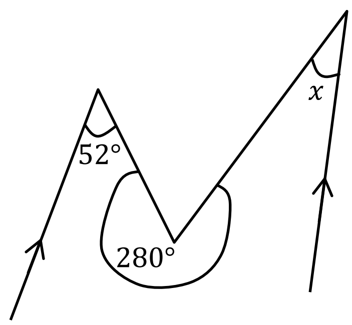
Find the value of the angle marked y in the diagram above.

3.



What is the value of x in the diagram above?

4.



Find the value of x in the diagram above.

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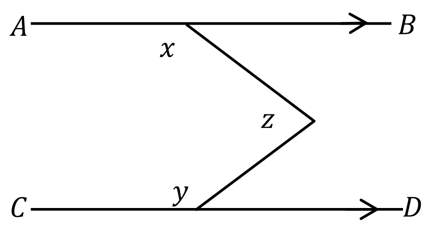
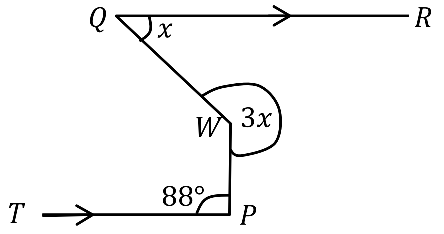
Exercise 24 **Date:……………………**

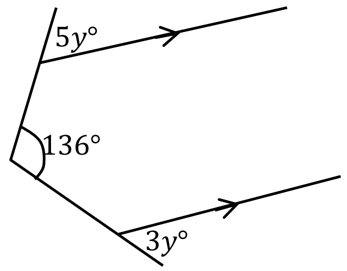
1. In the diagram below $\overline{AB}$ is parallel to $\overline{CD}$. Find the value of $x + y + z$.



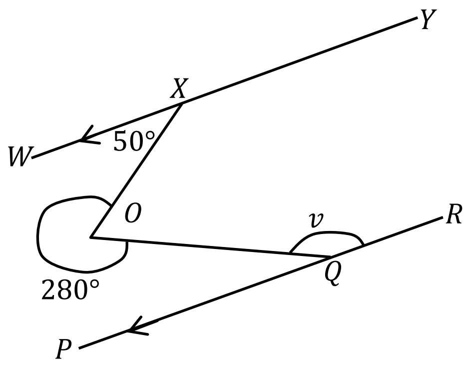
2. In the diagram, $QR//TP$ and $< WPT = 88°$. Find the value of x.



3. Calculate the value of y in the diagram.



4.

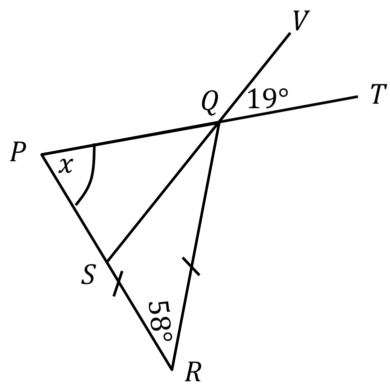


In the diagram, $WY//PR$, $< WXO = 50°$, reflex $< XOQ = 280°$ and $< OQR = v$. Find the value of v.

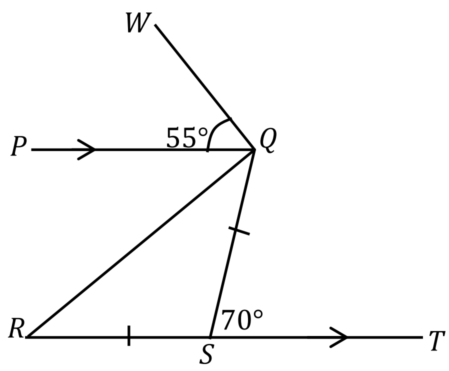
Exercise 25 **Date:……………………..**

Find the size of the angles marked with a letter.

1.

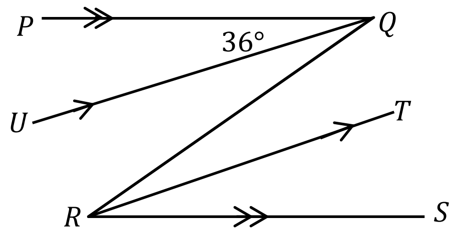


2.



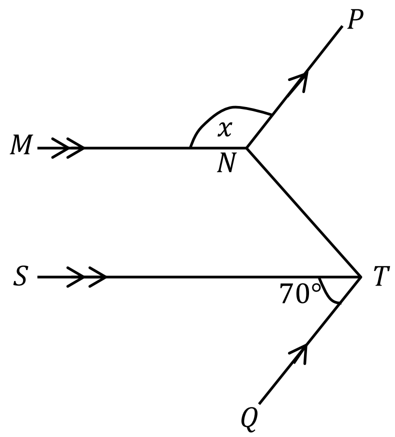
Find the size of reflex $< WQS$.

3.



In the diagram, $< PQU = 36°$, $< QRT = 29°$, $PQ//RS$ and $UQ//RT$. Find $< PQR$.

4.



In the diagram, $MN//ST$, $NP//QT$ and $< STQ = 70°$. Find x.

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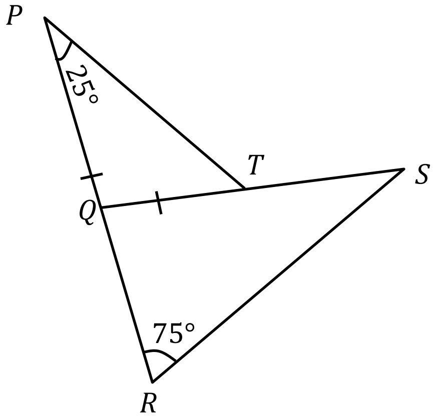
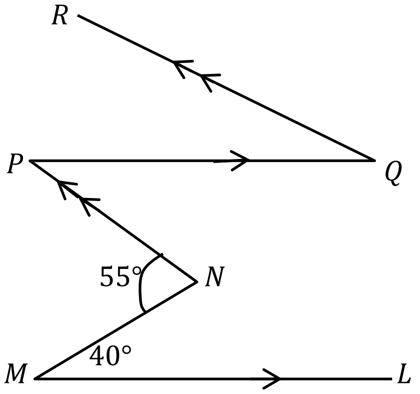
Exercise 26 **Date:……………………**

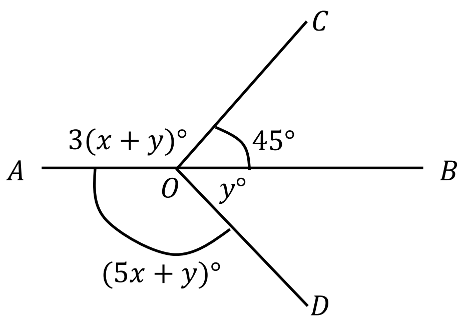
1. In the diagram below, PQT is an isosceles triangle; $|PQ| = |QT|$, $< SRQ = 75°, < QPT = 25°$ and PQR is a straight line. Find $< RST$.



2. In the diagram below, $ML//PQ$ and $NP//QR$. If $< LMN = 40°$ and $< MNP = 55°$, find $< PQR$.



3.



The diagram, AOB is a straight line, $< AOC = 3(x + y)°, < COB = 45°$, $< AOD = (5x + y)°$ and $< DOB = y°$. Find the value of x and y.

4. In the triangle PQR, M and N are points on the sides PQ and PR respectively such that MN is parallel to QR. If $< PRQ = 75°$, $|PN| = |QN|$ and $< PNQ = 125°$. Determine
(i) $< NOR$ (ii) $< NPM$

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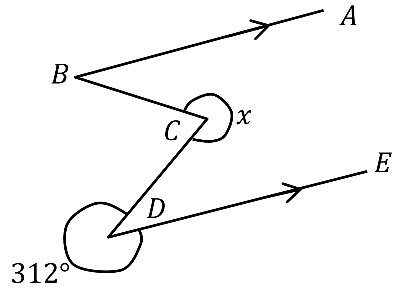
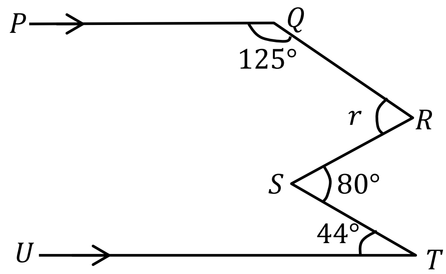
Exercise 27 **Date:……………………..**

1. In the diagram below, BA is parallel to DE. Find the value of x.

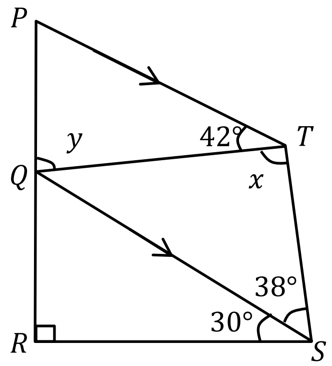


2. $ABCD$ is a trapezium with AB parallel to DC and $|AD| = |AB|$. If $< BAD = 106°$, find $< BDC$.

3. In the diagram, $< PQR = 125°$, $< QRS = r$, $< RST = 80°$ and $< STU = 44°$. Calculate the value of r.



4.



In the diagram, $PQRST$ is a quadrilateral. $PT//QS$, $< PTO = 42°$, $< TSQ = 38°$, and $< QSR = 30°$. If $< QTS = x$ and $< PQT = y$, find the value of

(i) x (ii) y

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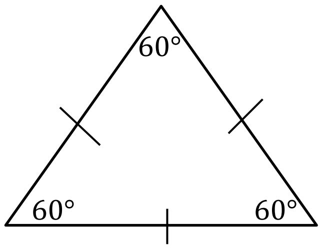
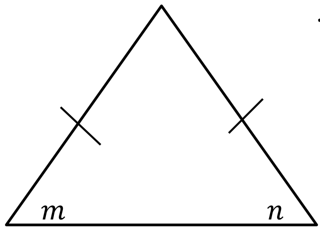
SPECIAL TRIANGLES

1. **Equilateral Triangle**



An equilateral triangle has;
(i) all interior angles equal (60° each)
(ii) all sides are of the same length.
(iii) three lines of symmetry.

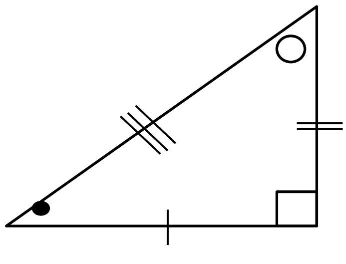
2. **Isosceles Triangle**



$< m = < n$

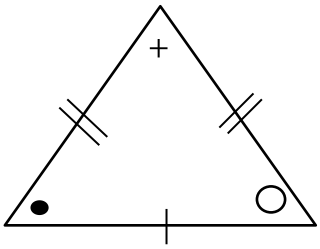
An isosceles triangle has
(i) two angles are equal
(ii) two sides are of the same length
(iii) two of its interior angles are equal

3. **Right – Angled Triangle**



Right – angled triangle has:
(i) one angle is 90°.
(ii) all sides may be different lengths.
(iii) all angles may be different.
(iv) may have 0 or 1 line of symmetry.

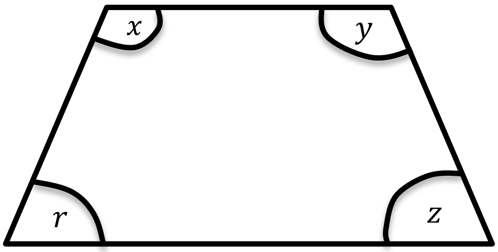
4. **Scalene Triangle**



Scalene triangle has:
(i) all angles different.
(ii) all sides (lengths) different.
(iii) no lines of symmetry.

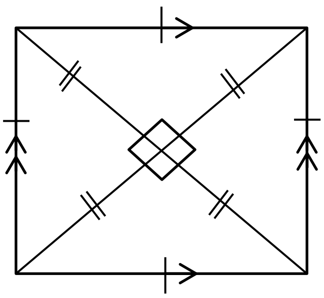
QUADRILATERALS
A quadrilateral is a polygon with four sides. It has four vertices and two diagonals.

For any quadrilateral, the sum of the interior angles is 360°.



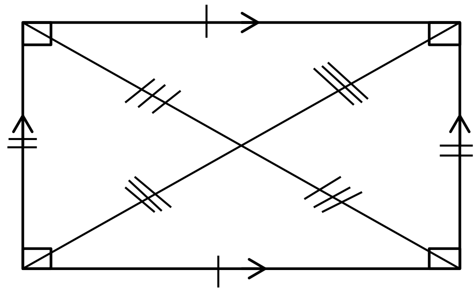
$$r + x + y + z = 360°$$

SPECIAL PROPERTIES OF QUADRILATERALS
1. **Square**



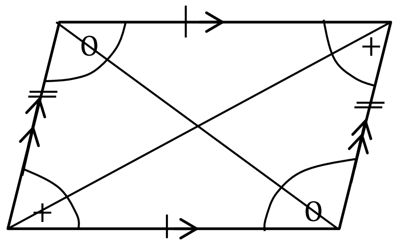
A square has
(i) all angles equal to 90°.
(ii) all sides equal.
(iii) opposite side parallel.
(iv) diagonals equal in length and bisect each other.
(v) diagonal cross at right angles.
(vi) diagonals bisect corner angles.

2. **Rectangle**



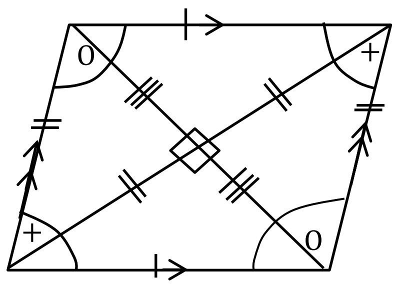
A rectangle has
(i) all angles equal to 90°.
(ii) opposite sides equal.
(iii) opposite sides parallel.
(iv) diagonals equal in length and bisect each other.
(v) two lines of symmetry.

3. **Parallelogram**



A parallelogram has:
(i) opposite angles equal.
(ii) opposite sides equal.
(iii) opposite sides parallel.
(iv) diagonals bisect each other.
(v) no lines of symmetry.

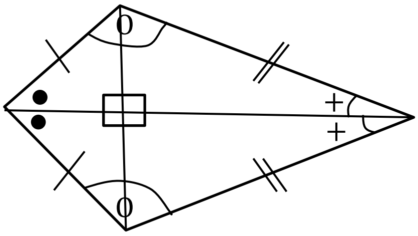
4. **Rhombus**



A rhombus has:
(i) all sides equal.
(ii) opposite sides parallel.

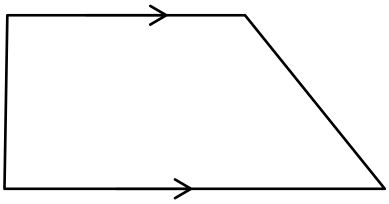
(iii) opposite angles equal.
(iv) diagonals bisect each other.
(v) diagonals crossing at right angles.
(vi) diagonals bisecting corner angles.
(vii) two lines of symmetry.

5. **Kite**



A kite has
(i) one pair of opposite angles equal.
(ii) two pairs of adjacent sides equal.
(iii) diagonals crossing at right angles.
(iv) only one diagonal bisected.
(v) only one pair of opposite angles bisected.
(vi) one of the diagonals as line of symmetry.

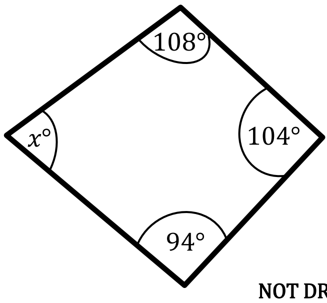
6. **Trapezium**



A trapezium has:
(i) one pair of opposite sides parallel.
(ii) all sides may be different lengths.
(iii) all angles may be different sizes.
(iv) no lines of symmetry.

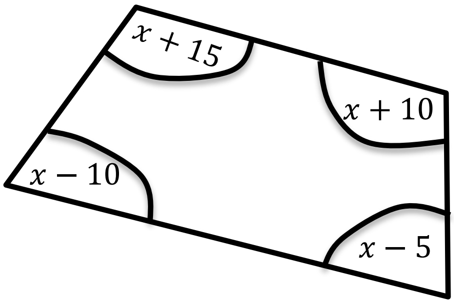
Exercise 28 **Date:……………………..**

1. Find the value of x.

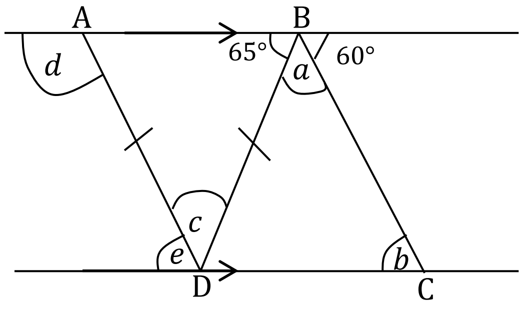


NOT DRAWN TO SCALE

2. Find x in the quadrilateral below.



3. Find the value of a, b, c, d and e in the diagram below.



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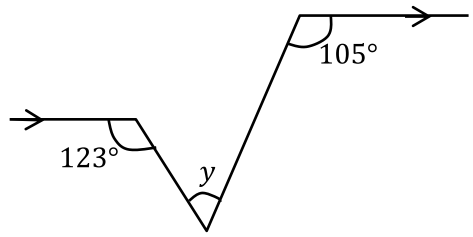
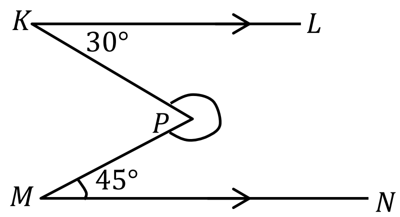
Exercise 29 **Date:……………………**

1. Calculate the value of y from the diagram.

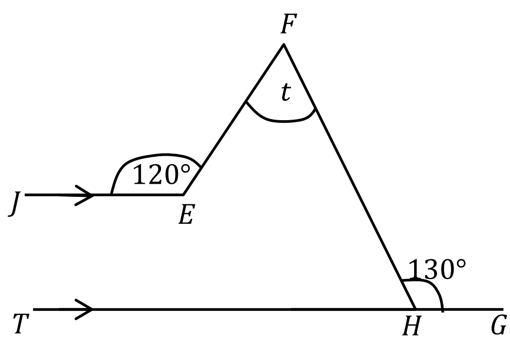


2.

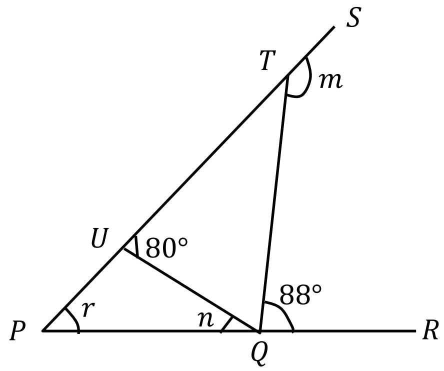


In the diagram, $KL//MN$, $< LKP = 30°$ and $< NMP = 45°$. Find the size of the reflex $< KPM$.

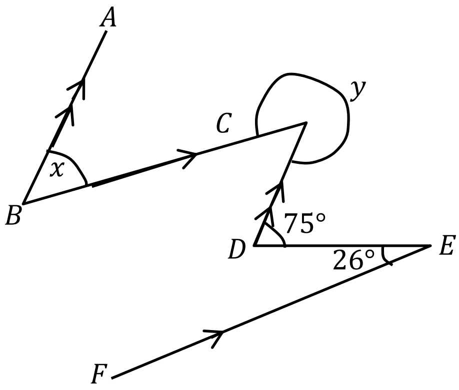
3. In the diagram, TG is parallel to JE, $< JEF = 120°$ and $< FHG = 130°$, find the angle marked t.



4. In the diagram, $< STQ = m$, $< TUQ = 80°$, $< UPQ = r$, $< PQU = n$ and $< RQT = 88°$. Find the value of $(m + n)$.



5. In the diagram, $AB//CD$ and $BC//FE$. $< CDE = 75°$ and $< DEF = 26°$. Find the angles marked x and y.



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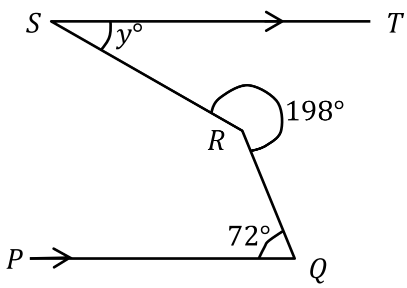
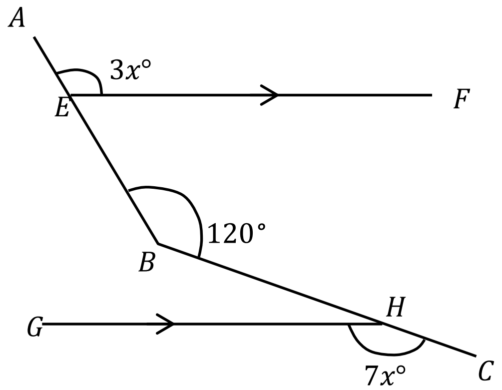
Exercise 30 **Date:....................**

1. In the diagram, $ST//PQ$, reflex angle $SRQ = 198°$ and $< RQP = 72°$.

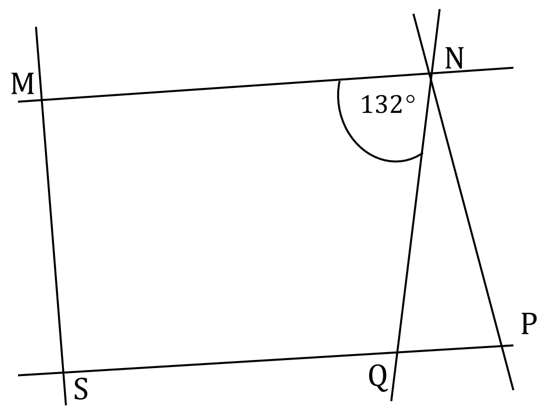


2. Two isosceles triangles PQR and PQS are drawn on opposite sides of a common base PQ. If $< PQR = 66°$ and $< PSQ = 109°$, calculate the value of $< RQS$.

3. In the diagram, $\overline{EF}$ is parallel to $\overline{GH}$. If $< AEF = 3x°$, $< ABC = 120°$ and $< CHG = 7x°$, find the value of $< GHB$.



4.



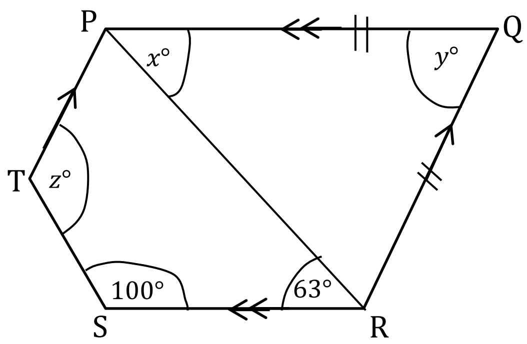
NOT DRAWN TO SCALE

In the diagram, $MNPS$ is a quadrilateral. A line is drawn through N to cut SP at Q. Angle $MNQ = 132°$, angle SMN is twice angle MSQ and angle NPQ is twice angle QNP. If NP bisects the acute angle at N, find

(i) angle SQN, (ii) angle MSQ

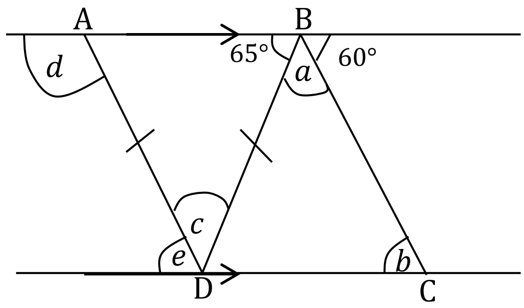
5. In the diagram $\overline{PQ}$ is parallel to $\overline{SR}$, and $\overline{QR}$ is parallel to $\overline{PT}$. $|PQ| = |QR|$, angle $PRS = 63°$ and angle $RST = 100°$.
Find the value of

(i) x (ii) y (iii) z



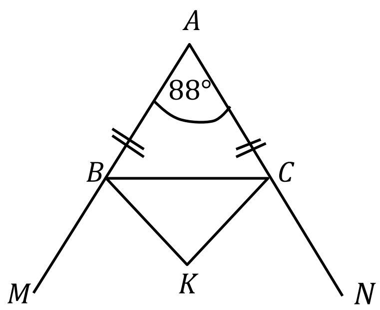
NOT DRAWN TO SCALE

6. Find the value of a, b, c, d and e in the diagram below.



NOT DRAWN TO SCALE

7.



In the diagram, $\overline{AM}$ and $\overline{AN}$ are straight lines. ABC is an isosceles triangle, $< BAC = 80°$, the bisectors of $< MBC$ and $< NCB$ meet at K. Calculate $< BKC$.

8. PQR is a triangle in which $|PQ| = |PR|$ and S is a point on PR such that $|QS| = |QR|$. If $< PQS = 30°$, calculate $< QPR$.

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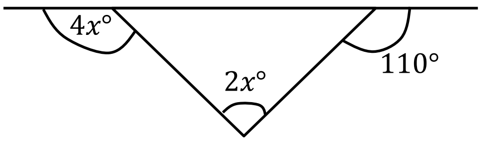
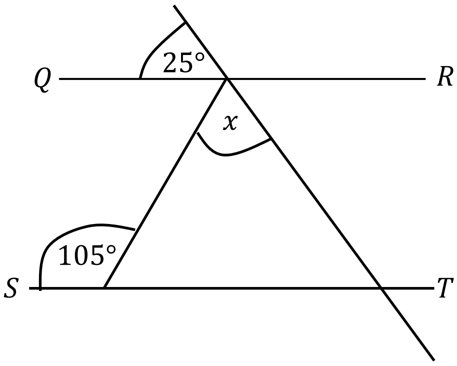
Exercise 31 **Date:……………………..**

1.



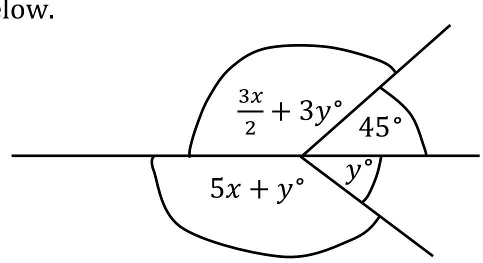
Find the value of x.

2.

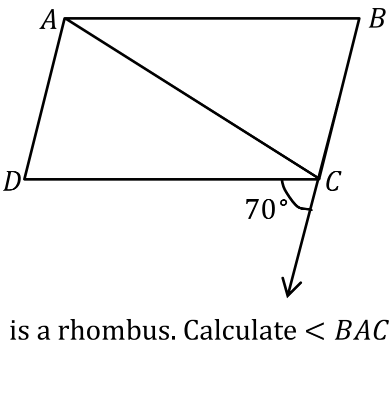


Find the value of x.

3. Find the values of x and y in the figure below.

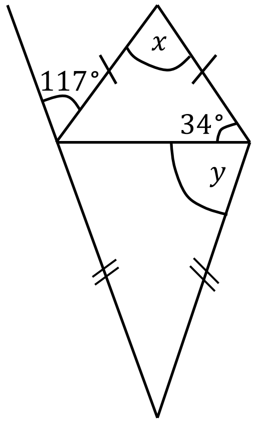


4.

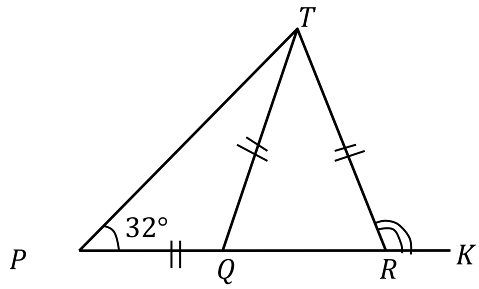


$ABCD$ is a rhombus. Calculate $< BAC$.

5. Find the angle marked x and y giving reasons for each step.

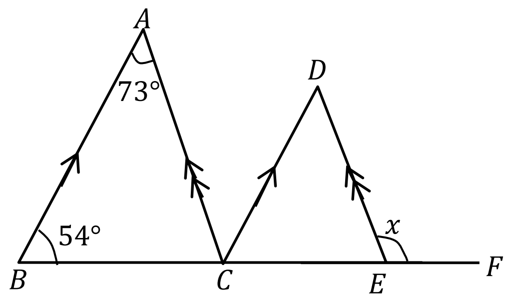


6.



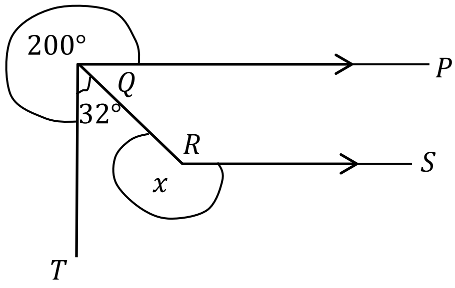
In the diagram, $< TPQ = 32°$ and $|PQ| = |QT| = |TR|$. Calculate $< TRK$.

7.

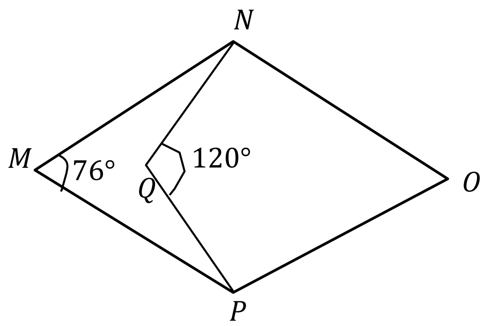


Find the value of x in the diagram above.

8. In the diagram, $< PQT = 200°$, $< TQR = 32°$ and $< QRS = x$. Find the value of x.

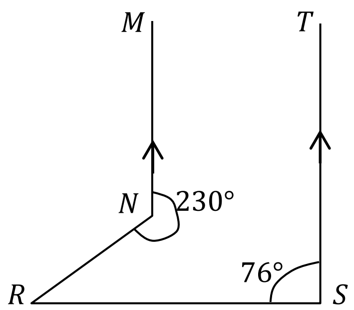


9.



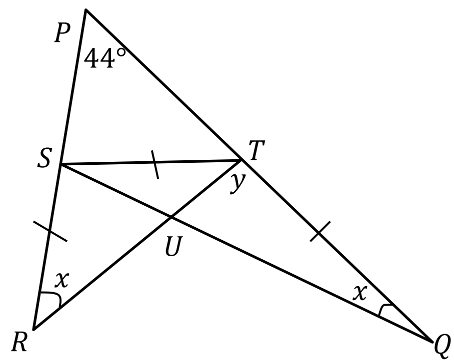
In the diagram, $MNOP$ is a rhombus and $PQNO$ is a kite, angle $PMN = 76°$ and angle $PQN = 120°$. Calculate the size of angle QPO.

10.



In the diagram, $\overline{MN} // \overline{ST}$, $< MNR = 230°$ and $< TSR = 76°$. Find the value of $< NRS$.

11.



In the diagram above, $\overline{ST} = \overline{SR} = \overline{TQ}$, angle $RPT = 44°$, angle $RTQ = y$, and angle $SRT =$ angle $SQT = x$. Calculate:

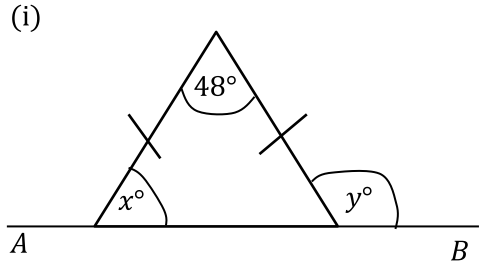
(i) the values of x and y

(ii) the value of angle RUQ

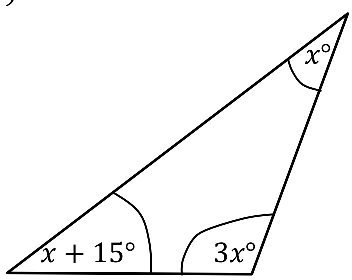
(iii) describe ΔPST

12. Find the size of the angles marked with a letter.

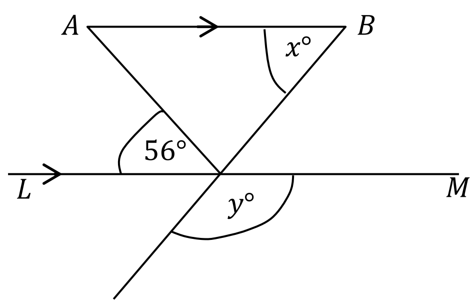
(i)



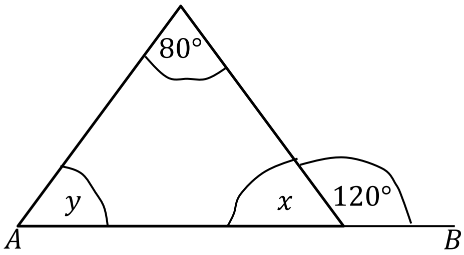
(ii)



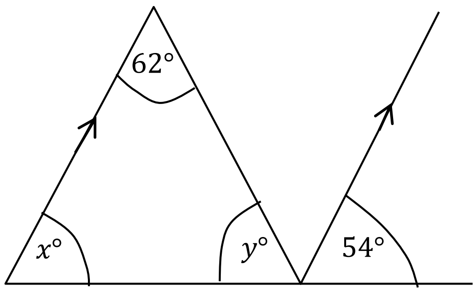
(iii)



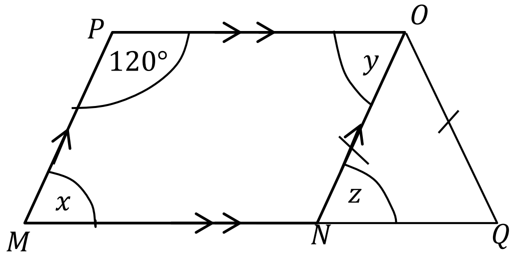
(iv)



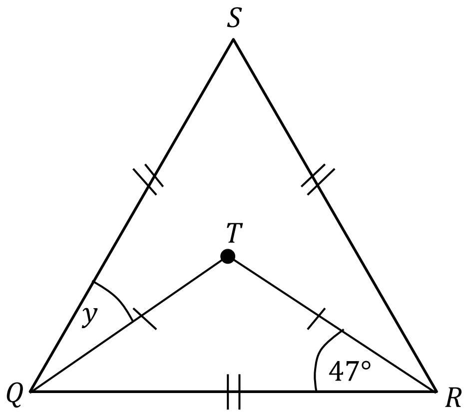
(v)



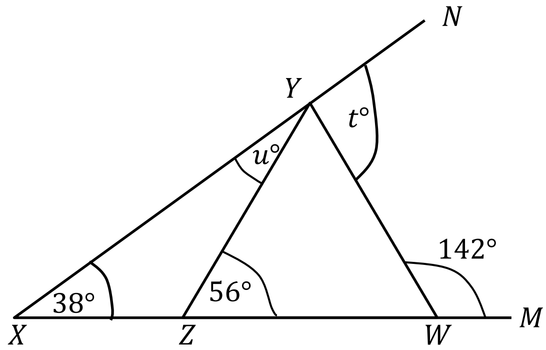
(vi)



(vii)



(viii)



Exercise 32 **Date:…………………..**

1. In the triangle PQR, M and N are points on the sides PQ and PR respectively such that MN is parallel to QR. If $< PRQ = 75°$, $|PN| = |QN|$ and $< PNQ = 125°$, determine
   (i) $< NOR$ (ii) $< NPM$

2. Prove that the sum of the angles of a triangle is two right angles. Hence or otherwise show that the exterior angle of a triangle equals the sum of the two interior opposite angles.

3. Prove that parallelogram on the same base and between that same parallel are equal in area.

4. PQRS is a parallelogram, T is any point on PQ between P and Q, prove that triangle QTS + triangle QTR = triangle STR.

5. If in the parallelogram $PQRS$, $|PQ| = 8$cm, $|PS| = 5$ cm and $< PQR = 30°$. Find the
   (i) distance between the parallel PQ and SR
   (ii) area of the parallelogram.

PROPERTIES OF POLYGONS

A polygon is any closed figure bounded by straight lines or edges. A polygon with all its sides equal is referred to as a regular polygon. All the interior angles of a regular polygon are of the same size.

Any polygon that does not have all congruent sides and angles is known as an irregular polygon.
Irregular polygons can still be pentagons, hexagons and nonagons, but they do not have congruent angles or equal sides.

Examples of Polygons
Quadrilateral – 4 sides
Pentagon – 5 sides
Hexagon – 6 sides
Heptagon – 7 sides
Octagon – 8 sides
Nonagon – 9 sides
Decagon – 10 sides

Exercise 1 **Date:......................**
Complete the table below.

Polygon	Number of		Sum of interior angles
	Sides	Triangles drawn from one vertex	
Triangle	3	1	$1 \times 180° = 180°$
Quadrilateral	4	2	$2 \times 180° = 360°$
Pentagon	5		
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			
Undecagon			
Dodecagon			
n sided polygon	n		

The sum of the interior angles of a regular polygon is given by:

$$S = (n - 2) \times 180°$$

Where n — number of sides of the polygon and $(n - 2)$ – number of triangles drawn from only one vertex.

Example 1
Find the sum of the interior angles of a regular polygon with 7 sides.

Solution...
Sum of interior angles:
$S = (n - 2) \times 180°$
Given, $n = 7$
$\therefore S = (7 - 2) \times 180°$
$= 5 \times 180° = 900°$

Exercise 1 **Date:......................**
Find the sum of the interior angles of a regular polygon with

1. 5 sides
2. 12 sides
3. 8 sides
4. 13 sides
5. 15 sides
6. 18 sides
7. 20 sides
8. 25 sides

Example 2
The sum of the interior angles of a regular polygon is 1080°. Find the number of sides of the regular polygon.

Solution…
$S = (n - 2) \times 180°$
Given, $S = 1080°, n = ?$
$\Longrightarrow 1080° = (n - 2) \times 180°$
$1080° = 180°n - 360°$
$1080° + 360° = 180°n$
$180°n = 1440°$
$n = \frac{1440°}{180°} = 8$
∴ The number of sides of a regular polygon is 8.

Exercise 2 **Date:……………………**

1. The sum of the interior angles of a regular polygon is 720°. Find the number of sides of the regular polygon.

2. The sum of the interior angles of a regular polygon is 900°. Find the number of sides of the regular polygon.

3. The sum of the interior angles of a convex polygon is 1260°. How many sides has the polygon?

4. The sum of the interior angles of an n −sided polygon is 3420°. Find the value of n.

Example 3
The interior angles of a pentagon are 126°, 114°, y, 92° and 83°. Find the value of y.

Solution…
$S = (n - 2) \times 180°$
Pentagon, so $n = 5$

$\Rightarrow 126° + 114° + y + 92° + 83° = (5 - 2) \times 180°$
$y + 415° = 3 \times 180°$
$y + 415° = 540°$
$y = 540° - 415°$
$\therefore y = 125°$

Exercise 3 **Date:……………………**

1. If the interior angles of a pentagon are $2x°$, $x°$, $2x°$, $3x°$ and $x°$, find the value of $x°$.

2. The interior angles of a hexagon are $2r$, $3r$, $2r$, $5r$, r and $2r$. Find the value of r.

3. Three angles of a hexagon are $2m°$ each. The others are $3m°$, $5m°$ and $m°$. Find $m°$.

Exercise 4 **Date:……………………**

1. Four angles of a hexagon are 130°, 160°, 112° and 80°. If the remaining angles are equal, find the size of each of them.'

2. Three interior angles of a pentagon are 100°, 120° and 108°. Find the size of each of the remaining two angles, if one of them is three times the other.

3. Two angles of a pentagon are 120° and 108°. The remaining three angles are congruent. Find the measure of each angle.

4. Two angles of a hexagon are 120° and 180°. The remaining four angles are equal. Find the measure of each equal angle.

5. Three interior angles of a polygon are 160° **each**. If the other interior angles are 120° **each**, find the number of sides of the polygon.

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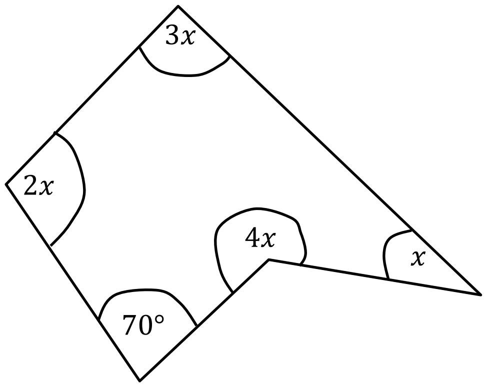
Example 4
Find the value of x in the diagram.

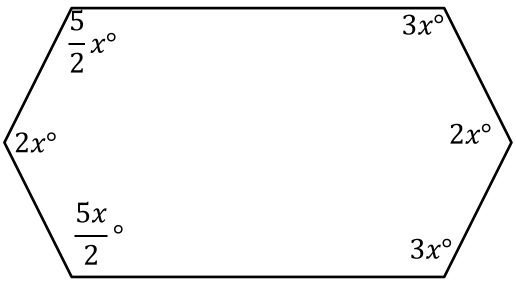


$S = (n - 2) \times 180°$
$2x + 3x + x + 4x + 70° = (5 - 2) \times 180°$

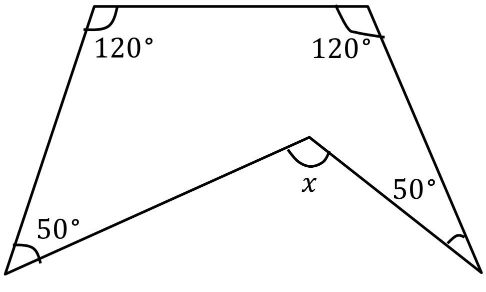
Since $n = 5$,
$\Rightarrow 10x + 70° = 540°$
$10x = 540° - 70°$
$10x = 470°$
$x = 47°$

Exercise 5 **Date:......................**
Find the value of x in the following diagrams.

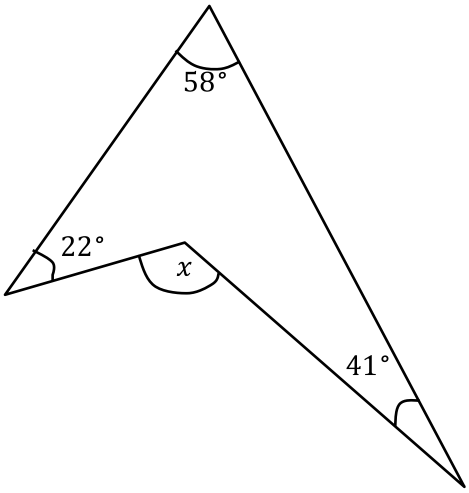
1.



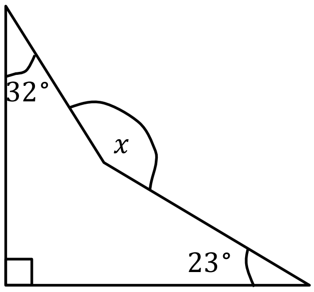
2.



3.



4.



> Each interior angle of a regular polygon:
> $\frac{(n-2)\times 180°}{n}$, where n is the number of sides.

Example 5

Calculate the number of sides of a regular polygon whose interior angles are each 150°.

Solution…

Each interior angle $= \frac{(n-2)\times 180°}{n}$

$150° = \frac{(n-2)\times 180°}{n}$

$150°n = (n-2) \times 180°$

$150°n = 180°n - 360°$

$360° = 180°n - 150°n$

$360° = 30°n$

$n = 12$

∴The number of sides of a regular polygon is 12.

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Exercise 6 **Date:......................**

1. Each of the interior angles of a regular polygon is 140°. How many sides have it?

2. The interior angle of a regular polygon is 108°. How many sides has the polygon?

3. The interior angle of a regular polygon is 120°. How many sides has the polygon?

Exercise 7 **Date:......................**

Determine the number of sides of these regular polygons, if each interior angle is of the following size.

1. 165°	5. 135°
2. 170°	6. 120°
3. 160°	7. 157.5°
4. 108°	8. 162°

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Exercise 8 **Date:……………………**

1. Find the interior angle of a 12 sided regular polygon.

2. The interior angle of a regular is 135°. How many sides has the polygon.

3. Calculate the number of sides of a regular polygon whose interior angles are each 156°.

Exercise 9 **Date:......................**

1. Find the sum of the interior angles of the following polygons
   (a) A hexagon
   (b) A nonagon
   (c) A heptagon

2. Find the value of each interior angle of the following polygons
   (i) A square
   (ii) A decagon
   (iii) An octagon
   (iv) A dodecagon

Exercise 10 **Date:......................**

1. The sum of the interior angles of a polygon of n sides is 2160°. Find n.

2. Each angle of a regular polygon is 168°. How many sides has it?

3. Two angles of a hexagon are 120° and 180°. The remaining four angles are equal. Find the measure of each equal angle.

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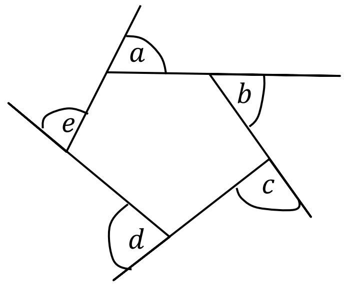
Exercise 11 **Date:……………………**

1. The angles of a pentagon are $x°$, $2x°$, $(x+60)°$, $(x+10)°$, $(x-10)°$. Find the value of x.

2. Three angles of a hexagon are each $x°$. The others are each $3x°$. Find x.

3. The interior angles of a pentagon are in the ratio 2: 3: 4: 4: 5. Find the value of the largest angle.

Sum of the Exterior Angles of a Polygon Add Up To 360°



$\hat{a} + \hat{b} + \hat{c} + \hat{d} + \hat{e} = 360°$

The exterior angle of a regular polygon is given by $\frac{360°}{n}$.

Example 6

A regular polygon has 6 sides. Find

(a) the exterior angle
(b) the interior angle
(c) the sum of interior angles

Solution…

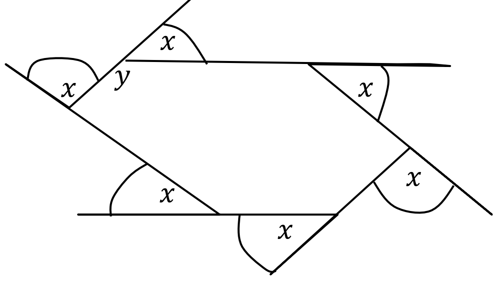
(a) the exterior angle of a regular polygon $= \frac{360°}{n}$, where n is the number of sides.
Given, $n = 6$
$\therefore$ the exterior angle $= \frac{360°}{6} = 60°$

(b) the exterior angle + the corresponding interior angle of a polygon $= 180°$
$\Rightarrow 60° +$ interior angle $= 180°$
interior angle $= 180° - 60°$
$= 120°$

(c) $S = (n - 2) \times 180°$
Given, $n = 6$
$\Rightarrow S = (6 - 2) \times 180°$
$= 4 \times 180°$
$= 720°$

Example 7

Find x and y.



Solution…

Sum of the exterior angles add up to 360°

$\Rightarrow x + x + x + x + x + x = 360°$

$6x = 360°$

$\frac{6x}{6} = \frac{360°}{6}$

$x = 60°.$

The exterior angle + corresponding interior angle of a polygon = 180°

i.e. from the diagram,

$\Rightarrow x + y = 180°$

$60° + y = 180°$

$y = 180° - 60°$

$y = 120°$

Exercise 12 **Date:……………………**

1. Calculate the size of an exterior of a regular pentagon.

2. The exterior angle of a regular polygon is 40°. Find the sum of its interior angle.
3. The exterior angle of a regular of a regular polygon is 70°. Find the sum of its interior angles.'

4. The exterior angles of a pentagon are $(3x + 5)°$, $(x + 2)°$, $5x°$, $(6x - 8)°$ and $(x + 9)°$. Find the value of x.

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Exercise 13 **Date:……………………**

1. The exterior angles of a pentagon are $x°$, $2x°, 3x°, 3x°$ and $2x°$. What is the size of the largest angle?

2. What is the size of the exterior angle of a regular polygon of 10 sides?

3. Each exterior angle of a polygon is 30°. Calculate the sum of the interior angles.

Exercise 14 **Date:……………………**

1. The ratio of the exterior angle to the interior angle of a regular polygon is 1: 11. How many sides has the polygon?

2. The ratio of the interior angle to the exterior angle of a regular polygon is 5: 2. Find the number of sides of the polygon.

3. In a regular polygon, the interior angle is 108° greater than the exterior angle. Calculate the number of sides of the polygon.

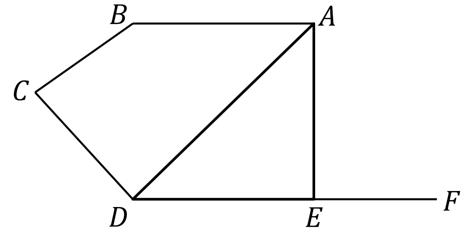
Exercise 15 **Date:……………………**

1. Find the number of sides of a regular polygon whose interior angle is three times the exterior angle.

2. The exterior of a polygon are $2x°$, $(x-20)°, x°, (3x+10)°, (x+15)°$ and $(2x+5)°$. Find the value of x.

3. The sum of all the interior and exterior angles of a regular polygon is 1080°. Find the number of sides.

4.



$ABCDEF$ is a regular pentagon
DEF is a straight line.
Calculate (i) $<AEF$ (ii) $<DAE$

5. A regular polygon of n sides is such that each interior angle is 120° greater than the exterior angle. Find
   (i) The value of n
   (ii) The sum of all the interior angles

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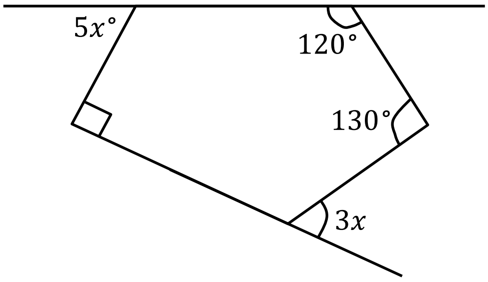
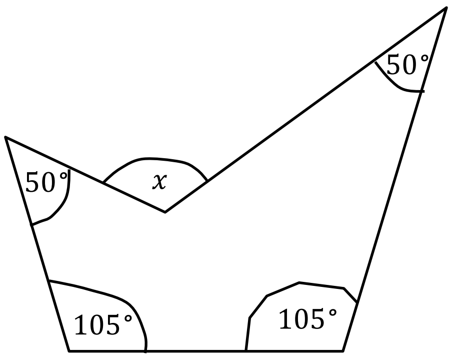
Exercise 16 **Date:.......................**

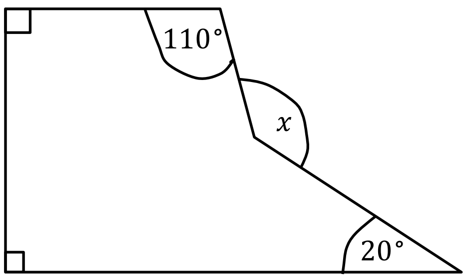
1. From the diagram, find the value of x.



2. Find the angle x in the diagram.

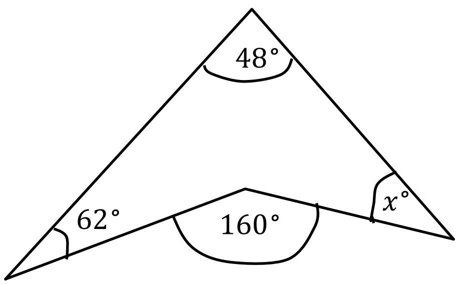


3.



Find the angle marked x in the diagram.

4.



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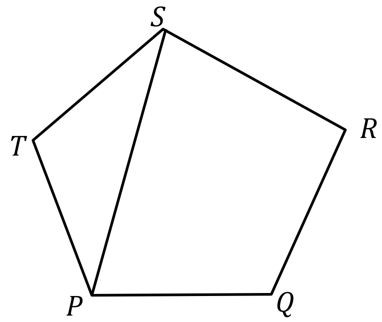
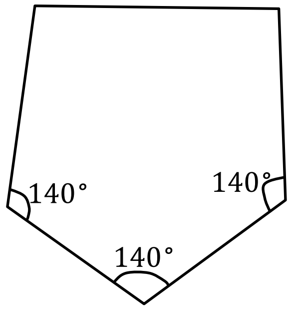


In the diagram, $PQRST$ is a regular pentagon. Calculate the size of $< SPQ$.

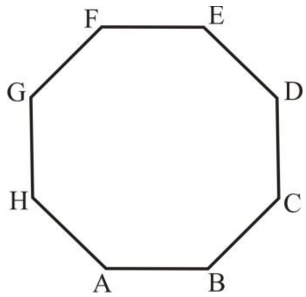
Exercise 17 **Date:……………………**

1.



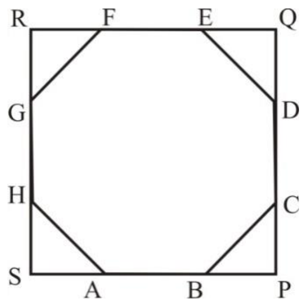
The pentagon has three angles which are 140°. The other two angles are equal. Calculate the size of one of these angles.

2.



$ABCDEFGH$ is a regular octagon.
(a) Show that angle $BCD = 135^{\circ}$
(b) Find
  (i) angle DEB
  (ii) angle FEB

3.



The sides of the octagon are extended to from the square $PQRS$. The length of each side of the octagon is $12cm$ and the length of BP is $8.485cm$. Calculate the area of
(i) triangle BPC
(ii) the octagon $ABCDEFGH$.

4. $ABCDE$ is a regular pentagon and a rectangular $AXYE$ is drawn on the side AE such that the vertices X and Y lie on the sides BC and CD respectively. Calculate the size of
   (i) an interior angle of the pentagon.
   (ii) $< BXA$.

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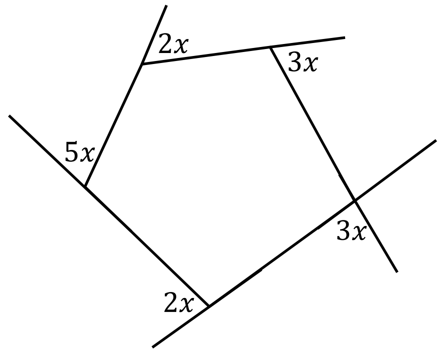
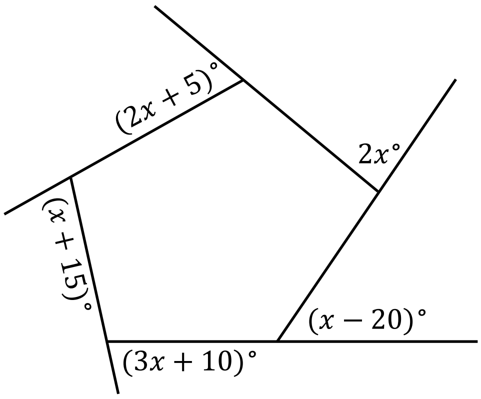
Exercise 18 **Date:……………………**

Find x in the following diagrams.

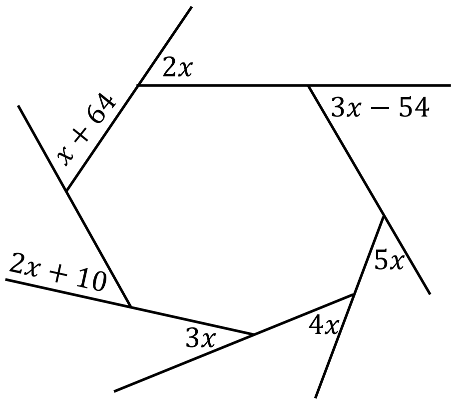
1.



2.



3.



4. If each interior angles of a regular polygon is five times the exterior angle, how many sides has the polygon?

5. The sum of the interior angles of a regular polygon is 1440°, calculate:
   (i) the number of sides.
   (ii) the size of one exterior angle of the polygon.

6. In a given regular polygon, the ratio of the exterior angle to the interior angle is 1 : 3. How many sides has the polygon?

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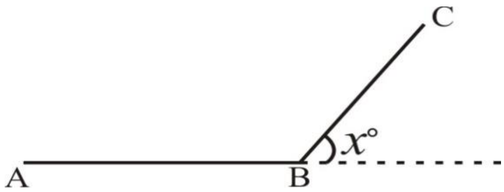
Exercise 19 **Date:......................**

1.
   (a) The table show how many sides different polygons has. Complete the table.

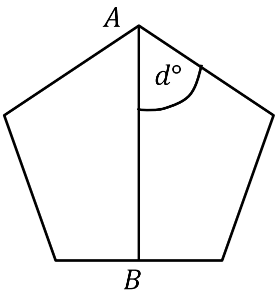
Name of polygon	Number of sides
	3
Quadrilateral	4
	5
	6
	7
	8
Nonagon	9

   (b) Two sides AB and BC of a regular nonagon are shown in the diagram below,

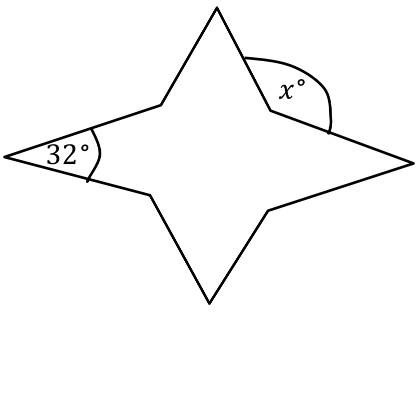


   (i) Work out the value of x, the exterior angle
   (ii) Find the value of angle ABC, the interior angle of a regular nonagon.

2. The diagram shows a regular pentagon. AB is a line of symmetry.
   (a)



   (b)



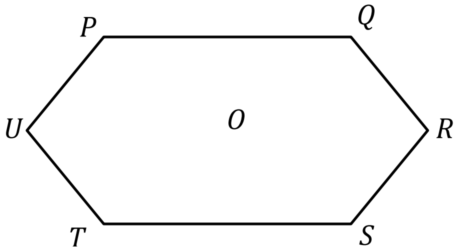
   The diagram shows an octagon. All of the sides are the same length. Four of the interior angles are each 32°. The other four interior angles are equal. Find the value of x.

3. The formula for finding the interior angle of a regular polygon with $n-$ sides is given below:

   interior angle $= \frac{180(n-2)}{n}$
   (i) Find the size of the interior angle of a regular polygon with 9 sides
   (ii) A regular polygon has and interior of 156°. How many sides does this polygon have?

4. The interior angle of a regular $n-$sided polygon is 48° more than the interior angle of a regular hexagon.
   (a) Find the size of the interior angle
   (b) Find the value of n

5. $PQRSTU$ is a polygon. O is any point inside the region of the polygon.



6. An irregular polygon consists of 16 triangles. The sum of 12 of its interior angles is 1236°, the next three interior angles sum up to 522°, and the remaining are equal and acute.
   (i) How many sides has the polygon?
   (ii) What is the size of the remaining equal angles?

CHANGE OF SUBJECT

Suppose $m = n + 3$, to make n the subject means rewriting this relation in an equivalent form, where n will be alone on one side of the equality sign. We can rewrite the relation as $m - 3 = n$. We normally write relations with the subject on the left – hand side (LHS), so the form we want is $n = m - 3$.

CASE 1: LINEAR FORM

Example 1

Make x the subject in the following:

1. $x + m = t$
2. $y = 3x - 2$
3. $a = b(x - 3)$

Solution…

1. $x + m = t$

   $x = t - m$

2. $y = 3x - 2$

   $y + 2 = 3x$

   $\frac{y+2}{3} = \frac{3x}{3}$

   $\therefore x = \frac{y+2}{3}$

3. $a = b(x - 3)$

   $a = bx - 3b$

   $a + 3b = bx$

   $x = \frac{a+3b}{b}$

Exercise 1 **Date:…………………..**

Make x the subject in the following

1. $x + c = 4$
2. $m + x = 7$
3. $3x = 2y + t$
4. $z = 8x - y$
5. $b(x - 1) = 7(x - 5)$

Exercise 2 **Date:…………………..**

Make the variable shown in brackets the subject if the formula in each case.

1. $e = m(x - c)$ (x)
2. $t(y + x) = v$ (x)
3. $J = mv - mu$ (m)
4. $ap = px + c$ (p)
5. $r(s - x) = 2x + r$ (x)
6. $n(m + n) = m(h + r)$ (h)
7. $a - b = x(c - nd)$ (n)
8. $A = 2\pi r + 2\pi rh$ (h)
9. $a(x + 3) = by - x$ (x)

Exercise 3 **Date:……………………**

1. (i) Find the value of $a^3 - 3ab - b^2$, when $a = -2$ and $b = 2$
   (ii) Find the value of $x^3 + xy^2 + 3y^3$ when $x = -2, y = 1$.
   (iii) Evaluate $m^2p - p(m - p)$ if $m = -3$ and $p = 5$.
   (iv) Find the value of $(2x - y)(x^2 + y^2)$ when $x = 4$ and $y = -2$.
   (v) Given that $x = -2$ and $y = 3$, evaluate $x^3 + 5x^2y - y^3$.

2. (i) Make l the subject of the relation $A = \pi rl + \pi r^2$.
   (ii) Find l when $r = 3, A = 176$ and $\pi = \frac{22}{7}$.
3. If $\frac{x}{x-z} = \frac{y}{z-y}$, find z when $x = 2$ and $y = 3$.
4. If $x = -1, y = -3, z = -4$ and $w = -7$, evaluate: $\frac{x^3-y^2}{2w-z}$.

CASE 2: FRACTIONAL FORM

Example 2

Make x the subject in the following

1. $m = \frac{1}{2}xt$
2. $\frac{x+b}{c} = d$
3. $r = \frac{x+1}{x-2}$
4. $t = \frac{1}{p} + \frac{1}{x}$

Solution…

1. $m = \frac{1}{2}xt$

$$2 \times m = 2 \times \frac{1}{2}xt$$
$$2m = xt$$
$$\frac{2m}{t} = \frac{xt}{t}$$
$$\therefore x = \frac{2m}{t}$$

2. $\frac{x+b}{c} = d$

$$\frac{x+b}{c} \times c = d \times c$$
$$x + b = dc$$
$$x = dc - b$$

3. $r = \frac{x+1}{x-2}$

$r(x-2) = x+1$

$rx - 2r = x+1$

$rx - x = 1 + 2r$

$x(r-1) = 1 + 2r$

$x = \frac{1+2r}{r-1}$

4. $t = \frac{1}{p} + \frac{1}{x}$

$xp \times t = xp \times \frac{1}{p} + xp \times \frac{1}{x}$

$xpt = x + p$

$xpt - x = p$

$x(pt-1) = p$

$x = \frac{p}{pt-1}$

Exercise 4 **Date:………………….**

Make the variable shown in brackets the subject of the formula in each case.

1. $E = \frac{1}{2}mc^2$ (m)
2. $p = \frac{3m+1}{m}$ (m)
3. $w = \frac{n-q}{q}$ (q)
4. $v = \frac{1}{3}\pi r^2 h$ (h)
5. $\frac{x-2y}{xy} = 3$ (y)

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Exercise 5 **Date:…………………..**

Make the variable shown in brackets the subject of the formula in each case.

1. $z = \frac{3y+2}{y-1}$ (y)
2. $\frac{m+n}{m-n} = \frac{3}{4}$ (n)
3. $\frac{3m-n}{5m-n} = \frac{p}{q}$ (m)
4. $t = \frac{a-m}{1+am}$ (a)
5. $\frac{m}{m-y+2} = \frac{r}{y+r-1}$ (y)

Exercise 6 **Date:…………………..**

Make the variable shown in brackets the subject of the formula in each case.

1. $\frac{1}{y} = \frac{1}{c} + \frac{1}{x}$ (y)
2. $f = \frac{t}{s} - m$ (s)
3. $\frac{1}{m} + \frac{1}{n} = \frac{1}{p}$ (m)
4. $\frac{1}{m} = t - \frac{1}{n}$ (n)
5. $\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$ (u)

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Exercise 7 **Date:.....................**

Make the variable shown in brackets the subject of the formula in each case

1. $y = \frac{a+p}{a-p}$ (p)
2. $\frac{a+bc}{wd+f} = y$ (w)
3. $S = \frac{n}{2}[2a + (n-1)d]$ (d)
4. $h = \frac{mt}{d(m+p)}$ (m)
5. $\frac{m}{n-y} = \frac{n}{m+y}$ (y)

Exercise 8 **Date:…………………..**

Make the variable shown in brackets the subject of the formula in each case.

1. $x + 5 = k + \frac{x}{3}$ (x)
2. $d(x - 7) = 3 + \frac{x-b}{5}$ (x)
3. $2x - 5 = \frac{3}{5}(x - f)$ (x)
4. $p + 5 = \frac{1-2r}{r}$ (r)
5. $q = \frac{3p}{r} + \frac{s}{2}$ (p)
6. $p = s + \frac{sm^2}{nr}$ (s)
7. $u = 1 - \frac{3v}{vt-w}$ (t)

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Exercise 9 **Date:.......................**

1. $S = \frac{n}{2}[2a + (n-1)d]$, where $n > 0$.
   Find the value of n, if $a = 3$, $d = 4$ and $S = 210$.

2. If $\frac{1}{m} + \frac{1}{n} = \frac{1}{p}$,
   (i) Express m in terms of n and p
   (ii) Calculate m correct to one decimal place when $n = 17.24$ and $p = 16.41$

3. Given that $S = \frac{m^2(tP-n)}{2Pn}$
   (i) Make P the subject of the relation
   (ii) Find the value of P if $m = 4$, $S = 60$, $n = \frac{1}{3}$ and $t = 4$.

CASE 3: EXPONENTIAL AND ROOTS FORM

Example 3

Make the variable shown in brackets the subject of the formula in each case.

1. $A = \pi r^2$ (r)
2. $a^2 + b^2 = c^2$ (b)
3. $\frac{xt}{7} = \frac{p+2}{3x}$ (x)

Solution…

1. $A = \pi r^2$

   $\frac{A}{\pi} = \frac{\pi r^2}{\pi}$

   $r^2 = \frac{A}{\pi}$

   $r = \pm\sqrt{\frac{A}{\pi}}$

2. $a^2 + b^2 = c^2$

   $b^2 = c^2 - a^2$

   $b = \pm\sqrt{c^2 - a^2}$

3. $\frac{xt}{7} = \frac{p+2}{3x}$

   $(xt) \times (3x) = 7(p + 2)$

   $3x^2 t = 7p + 14$

   $x^2 = \frac{7p+14}{3t}$

   $x = \pm\sqrt{\frac{7p+14}{3t}}$

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Exercise 10 **Date:…………………..**

Make the variables shown in brackets the subject of the formula in each case.

1. $v = \frac{1}{3}\pi r^2 h$ (r)
2. $J = \frac{1}{2}mv^2$ (v)
3. $v^2 = u^2 + 2ax$ (u)
4. $E = \frac{m}{2g}(v^2 - u^2)$ (v)
5. $P = s + \frac{sm^2}{nr}$ (m)
6. $R = \frac{h}{2} + \frac{d^2}{8h}$ (d)
7. $v = \frac{1}{6}\pi h(3r^2 + h^2)$ (r)
8. $l = g\left(\frac{T}{2\pi}\right)^2$ (T)

Exercise 11 **Date:…………………..**

1. (α) Given that $p = x + ym^3$, find m in terms of p, x and y.

   (β) If $V = \sqrt[3]{p + r}$, make r the subject

2. If $p = \frac{m}{2} - \frac{n^2}{5m}$
   (i) Make n the subject of the relation
   (ii) Find, correct to three significant figures, the value of n when $p = 14$ and $m = -8$.

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Example 4

Make x the subject in the following

1. $\sqrt{x} = b$
2. $r = m\sqrt{\left(\frac{x}{n}\right)}$
3. $p = \sqrt{q^2 - \frac{r^2}{x^2}}$

Solution…

1. $\sqrt{x} = b$

   Squaring both sides

   $\left(\sqrt{x}\right)^2 = b^2$

   $x = b^2$

2. $r = m\sqrt{\left(\frac{x}{n}\right)}$

$\frac{r}{m} = \sqrt{\left(\frac{x}{n}\right)}$

Squaring both sides

$\left(\frac{r}{m}\right)^2 = \left(\sqrt{\left(\frac{x}{n}\right)}\right)^2$

$\left(\frac{r}{m}\right)^2 = \frac{x}{n}$

$x = n\left(\frac{r}{m}\right)^2$

3. $p = \sqrt{q^2 - \frac{r^2}{x^2}}$

Squaring both sides

$p^2 = \left(\sqrt{q^2 - \frac{r^2}{x^2}}\right)^2$

$p^2 = q^2 - \frac{r^2}{x^2}$

$p^2x^2 = q^2x^2 - r^2$

$r^2 = q^2x^2 - p^2x^2$

$r^2 = x^2(q^2 - p^2)$

$x^2 = \frac{r^2}{q^2-p^2}$

$\therefore x = \pm\sqrt{\frac{r^2}{q^2-p^2}}$

Exercise 12 **Date:.......................**

Make the variable shown in brackets the subject of the formula in each case.

1. $d = \sqrt{\frac{4h}{5}}$ (h)
2. $x = \sqrt{\frac{a-y}{b}}$ (y)
3. $r = \sqrt{\frac{t-3}{9-t}}$ (t)
4. $d = \sqrt{\frac{6}{x} - \frac{y}{2}}$ (x)
5. $t = \sqrt{\frac{pq}{2} - r^2q}$ (q)
6. $v = \frac{\sqrt{25-u^2}}{4}$ (u)

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Exercise 13 **Date:……………………**

Make the variable shown in brackets the subject of the formula in each case.

1. $T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$ (l)
2. $2p = q + \sqrt{q^2 + r}$ (r)
3. $t = \sqrt{\frac{tk-h}{k-h}}$ (k)
4. $y = \frac{2(\sqrt{x^2+m})}{3N}$ (x)
5. $d = \sqrt[3]{\frac{P+k^2}{Q-P}}$ (P)
6. $x = q + \sqrt{(y^2 + z^2)}$ (y)
7. $w^2 = \sqrt{\frac{m^2-n}{r}} - v$ (n)

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Exercise 14 **Date:………………….**

1. If $T = \sqrt{\frac{p-r}{p+r}}$, find
   (a) r in terms of p and T
   (b) The value of r, when $T = 3$ and $p = 10$

2. Given that $S = k\sqrt{m^2 + n^2}$
   (i) Make m the subject of the relation
   (ii) If $S = 12.2$, $k = 0.02$ and $n = 1.1$, find correct to the nearest whole number, the positive value of m.

3. Make r the subject of the formulae $V = mg\sqrt{(1 - r^2)}$. Hence, find the value of r when $V = 15$, $m = 20$ and $g = 10$.

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Exercise 15 **Date:……………………**

1. (i) Make h the subject of the formula,

$$\frac{1}{n} = \sqrt{\frac{k^2 + a^2}{hg}}$$

(ii) If $n = \frac{8}{5}$, $a = 3$, $h = 2$, $g = 32$, find the value of k.

2. Given the relation

$$T = \sqrt{\frac{u}{\frac{1}{f} + \frac{1}{g}}}$$

(i) Make g the subject of the relation

(ii) Find g when $T = 3$, $f = 4$ and $u = 5$.

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Exercise 16 **Date:…………………..**

1. If $p = \frac{3}{5}\sqrt{\frac{q}{r}}$, express q in terms of p and r.

2. If $T = WP[M^2 - (M - S)^2]$, express M in terms of T, W, P and S.

3. (i) Make q the subject of the relation.

$$r = \frac{f}{2} + \left(\frac{f^2}{4} + q^2\right)^{\frac{1}{2}}$$

(ii) Find, correct to **3** significant figures, the positive value of q when $f = 4$ and $r = 5$

CASE 4: OTHER FORMS

Example 5
If $V = \pi r^2 h$ and $S = 2\pi rh$, express V in terms of r and S.

Solution…
$V = \pi r^2 h$ ………………………………….. (1)
$S = 2\pi rh$
$\Rightarrow h = \frac{S}{2\pi r}$………………………………….. (2)

Eliminating h

Put (2) into (1)

$$\Rightarrow V = \pi r^2 \left(\frac{S}{2\pi r}\right)$$
$$= \frac{rS}{2}$$

Example 6
If $p = kq$ and $r = \frac{mk}{eq}$, express r in terms of m, p, e and q.

Solution…
Note: The final answer must be without k.

$p = kq$
$\Rightarrow k = \frac{p}{q}$…………………………………….. (1)
$r = \frac{mk}{eq}$……………………………………….. (2)

Put (1) into (2)
$$\Rightarrow r = \frac{m\left(\frac{p}{q}\right)}{eq}$$

$$r = \frac{mp}{eq^2}$$

Exercise 17 **Date:**…………………..

1. If $a = \frac{3b+2}{2b+3}$ and $b = \frac{2d-1}{d-2}$, express d in terms of a.

2. If $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$, express V in terms of S and π.

3. Given that $p = x - \frac{1}{x}$ and $q = x^2 + \frac{1}{x^2}$, express q in terms of p.

4. Given that $a = bc$ and $n = \frac{mk}{ec}$,
   (i) express k in terms of a, b, e, m and n;
   (ii) find, correct to **three** significant figures, the value of k, when $a = \frac{1}{2}$, $b = -4, e = 3, m = 7$ and $n = -5$.

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Exercise 18 **Date:…………………..**

1. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and $d=u+v$.
   (i) Express d in terms of f and u
   (ii) Find the value of d, when $f=6$ and $u=6.5$

2. If $\left(\frac{x+y}{n-1}\right)^2=p$ and $2S=n(y+x)$, express S in terms of n and p only

3. If $\frac{l+a}{n-1}=d$ and $2S=n(a+l)$, express S in terms of n and d.

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Exercise 19 **Date:……………………..**

1. Make R the subject of $p = \frac{m}{R^2} + k$

2. Given the relation $A = \sqrt{\frac{x-a}{x+a}}$
   a. Express a in terms of A and x
   b. Find the value of a, if $x = 2$ and $A = 3$.

3. In the relation $t = m\sqrt{n^2 + 4r}$:
   (i) Make n the subject of the relation;
   (ii) Find the positive value of n when $t = 25, m = 5$ and $r = 4$.

4. Make t the subject in $\frac{q}{m} = \sqrt[3]{\frac{r}{a} - \frac{a}{t^2}}$

5.
   (a) Make the variable shown in brackets the subject of the formula in each case.
   (i) $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ (c)
   (ii) $m = \frac{q^{-1}}{\sqrt[3]{x-1}}$ (x)
   (iii) $c = a + \sqrt{\frac{b-y}{d}}$ (y)

   (b) The tangent t of the angle between two straight lines with gradient m_1 and m_2 is given by $t = \frac{m_1-m_2}{1+m_1m_2}$
   (i) Find t when $m_1 = 3$ and $m_2 = \frac{1}{3}$.
   (ii) Make m_2 the subject of the formula.
   (iii) If $t = 0.5$ and $m_1 = 4$, find m_2.

6. In a certain electrical circuit the voltage V, the total current I and the three resistances R_1, R_2 and R_3 are connected by the following relations $V = I\left(\frac{R_1R_2}{R_1+R_2} + R_3\right)$.
   (a) Calculate V, if $I = 0.5$ when $R_1 = 4$, $R_2 = 6$ and $R_3 = 10$
   (b) Calculate R_3, if $I = 0.5$ when $R_1 = 4$, $R_2 = 6$ and $V = 10$
   (c) Make R_1 the subject of the formula.

LINEAR EQUATIONS

Solving Linear Equations in One Variable
Example 1
Solve the following.
1. $x + 7 = 9$
2. $x - 2 = 4$
3. $8x - 5 = 22 - 4x$

Solution…
1. $x + 7 = 9$
   $x = 9 - 7$
   $x = 2$

2. $x - 2 = 4$
   $x = 4 + 2$
   $x = 6$

3. $8x - 5 = 22 - 4x$
   $8x + 4x = 22 + 5$
   $12x = 27$
   $x = \frac{27}{12}$
   $x = \frac{9}{4}$
   $x = 2\frac{1}{4}$

Exercise 1 **Date:……………………**
Solve:
1. $x + 7 = 15$
2. $x - 2 = 9$
3. $4x - 7 = 29$
4. $8x - 5 = 7$
5. $3x - 4 = 14$
6. $2x + 3 = 15 - x$
7. $8x - 5 = 22 - 4x$
8. $3x + 8 = 7x - 8$
9. $15x - 4 = 10 - 3x$
10. $22x - 8 = 10 - 5x$
11. $3r + 2 = 5r - 1$

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Exercise 2 **Date:……………………**

Solve:

1. $4x + 3 = 11$
2. $7 - 3n = 11n + 2$
3. $11x + 15 = 3x - 7$
4. $5x + 4 = 19 + 2x$
5. $2 - x = 5x + 11$
6. $5 - 2x = 3x - 19$
7. $4x + 3 = 2 + 6x$
8. $4x + 3 = 7 - x$
9. $3x + 5 = 8 - x$
10. $11x - 5 = 6x - 3$

Exercise 3 **Date:……………………**

Find the truthset of the following

1. $4x + 3 = 23$
2. $8x - 15 = 6x + 2$
3. $3x - 4 = 14$
4. $8 - 8x = -9x$
5. $13x - 12 = 5x + 60$
6. $3x + 1 = 2x + 6$
7. $9x + 13 = 6x + 10$
8. $8x + 9 = 6x + 8$
9. $6x + 6 = 3x + 5$
10. $7x + 5 = 5x + 1$

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Exercise 4 **Date:……………………..**

Find the truthset of the following

1. $2(2x + 3) = 2 + 1$
2. $5(p - 4) = 3(p + 2)$
3. $3(2x - 4) = 4(x + 7)$
4. $6(k - 8) = 78$
5. $2(3x - 7) = 13$
6. $3(x + 4) = 2(4x - 1)$
7. $5(x + 1) = 4(x + 2)$
8. $8(x + 5) = 10(x + 3)$
9. $7(x + 2) = 6(x + 3)$
10. $p = 2(p - 3)$

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Exercise 5 **Date:......................**

Find the solution set of the following

1. $p + 3 = 3(p - 5)$
2. $1 - 2(3x) = 2x$
3. $3(x + 4) = 2(4x - 1)$
4. $2(q - 3) - (q - 2) = 5$
5. $4 + 2(2 - x) = 3 - 2(5 - x)$
6. $5(3x + 8) - 10 = 0$
7. $3(2z - 7) - 2(z - 3) = -9$
8. $10(x + 4) - 7(x - 3) = 100$
9. $3(3x + 1) - 8(2x - 3) + 1 = 0$
10. $7(5x - 3) - 10 = 2(3x - 5) - 3(5 - 7x)$

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Example 2

Find the solution set for each of the following.

1. $\frac{1}{2}(x-1)-\frac{1}{3}(x-2)=4$
2. $3(y-4)+\frac{y}{2}=9$

Solution…

1. $\frac{1}{2}(x-1)-\frac{1}{3}(x-2)=4$

   Multiply through with the L.C.M

   $\Rightarrow 6\times\frac{1}{2}(x-1)-6\times\frac{1}{3}(x-2)=4\times 6$

   $3(x-1)-2(x-2)=24$

   $3x-3-2x+4=24$

   Group like terms

   $\Rightarrow 3x-2x=24+3-4$

   $\Rightarrow x=23$

   $\{x: x=23\}$

2. $3(y-4)+\frac{y}{2}=9$

   Multiply through with the L.C.M

   $2\times 3(y-4)+2\times\frac{y}{2}=9\times 2$

   $6(y-4)+y=18$

   $6y-24+y=18$

   Group like terms

   $6y+y=18+24$

   $7y=42$

   $\frac{7y}{7}=\frac{42}{7}$

   $y=6$

   $\{y: y=6\}$

Exercise 6 **Date:……………………**

Solve the following linear equations.

1. $\frac{x}{3}=7$
2. $9=\frac{x}{6}$
3. $\frac{1}{3x}=\frac{1}{9}$
4. $\frac{4}{9}=\frac{16}{x}$
5. $\frac{\frac{1}{2}}{3}=\frac{x}{9}$
6. $\frac{1}{x}=\frac{\frac{1}{6}}{\frac{1}{4}}$

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Exercise 7 **Date:……………………**

Find the truthset for each of the following

1. $\frac{x}{2} + 3 = 8$
2. $\frac{1-p}{3} = 4$
3. $\frac{y+1}{5} = 2$
4. $\frac{p-3}{5} = 3$
5. $\frac{x-2}{4} = \frac{2x+5}{4}$
6. $\frac{2y-1}{3} = 7$
7. $\frac{15-x}{2} = 3 - 2x$

Exercise 8 **Date:……………………..**

Find the solution set for each of the following

1. $4(y-1)+3(y+2)=5(y-4)$
2. $\frac{2x+5}{3}=8$
3. $\frac{x+5}{x}=\frac{7}{3}$
4. $\frac{n-8}{2}=11$
5. $2=\frac{1}{u-1}$
6. $\frac{9y+16}{6}=\frac{113}{12}$
7. $\frac{5}{w}=\frac{3}{w+1}$

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Exercise 9 **Date:……………………**

Find the solution set for each of the following

1. $\frac{1}{2+x} = \frac{1}{5-x}$
2. $\frac{2x+3}{4} = \frac{4x-2}{6}$
3. $\frac{5x-7}{6} + \frac{2x-3}{4} = -\frac{2}{3}$
4. $\frac{3}{x} - \frac{2}{x+1} = 0$
5. $\frac{3x}{5} + 2 = \frac{x-2}{3}$
6. $\frac{3x-1}{6} - \frac{2x-1}{3} = \frac{1}{4}$
7. $\frac{3}{4+p} = \frac{2}{1+p}$

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Exercise 10 **Date:……………………..**

Find the truthset for each of the following equations.

1. $8(x-2)-9(x-4)=13$
2. $2(x-1)-7(3x-2)=7(x-4)$
3. $3(6+7y)+2(1-5y)=42$
4. $3(x-2)-5(1-2x)=15$
5. $5(3x-1)-4(x+3)=9x+5$
6. $5(6x+2)-7(3x-5)-72=0$
7. $-3(x-2)-5(3x-2)+74=0$
8. $-2(x+3)-6(2x-4)+108=0$
9. $10(x+4)-9(x-3)-1=8(x+3)$
10. $(6-x)-(x-5)-(4-x)=-\frac{x}{2}$

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Exercise 11 **Date:……………………..**

Solve the following.

1. $5(a-5)-\frac{1}{2}(2a+6)=4$
2. $\frac{x}{7}-1=\frac{2x-3}{3}+4$
3. $\frac{4x-3}{2}=\frac{8x-10}{8}+2\frac{3}{4}$
4. $\frac{1}{3}(x+3)-2(x-5)=4\frac{1}{3}$
5. $2x-\frac{4}{5}(3-x)=2$
6. $\frac{x}{4}-\frac{x+3}{2}-\frac{2(x+2)}{3}=5$
7. $\frac{3}{4}(x-4)-\frac{2}{3}\left(\frac{1}{4}+x\right)=0$

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Exercise 12 **Date:……………………..**

Find the solution set of the following.

1. $0.2x - 3 = 0.5x$
2. $0.3x - 2(x + 2) = 3(2 - x)$
3. $\frac{x+1}{3} - \frac{x-2}{5} = 2$
4. $\frac{2}{3} + \frac{5}{6} = \frac{x}{2}$
5. $\frac{5}{3} \div \frac{3}{y} = \frac{40}{9}$
6. $\frac{m-3}{4} + \frac{m+4}{3} = -7$
7. $\frac{3x-2}{5} + \frac{x+2}{10} = 4$
8. $2\left(\frac{5-x}{3}\right) + 3 = -3\left(\frac{x-2}{2}\right)$

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Exercise 13 **Date:……………………..**

Find the solution set of the following

1. $\frac{2}{3}(3x-5)-\frac{3}{5}(2x-3)=3$
2. $\frac{1}{5x}+\frac{1}{x}=3$
3. $\frac{1\frac{1}{2}}{2\frac{2}{3}}=\frac{x}{96}$
4. $3(x-2)-5(1-2x)=15$
5. $\frac{3x-8}{2}=\frac{6x-4}{5}+1$
6. $3(2x-3)-2(1-2x)=29$
7. $\frac{3x-8}{2}=\frac{6x+1}{5}$
8. $6x+17=5(2x-3)$
9. $\frac{x}{3}+\frac{5}{2}x=3-2$
10. $2\left(x-3(x+4)\right)=8-5(x+1)-x$
11. $2\left(x-3(x+2)\right)-5=3x-1$
12. $\frac{1}{2}\left(\frac{x}{3}+1\right)=\frac{x}{4}-2$
13. $\frac{3}{2p-\frac{1}{2}}=\frac{\frac{1}{3}}{\frac{1}{4}p+1}$
14. $\frac{3}{2-x}-\frac{2}{2x+1}-\frac{3}{2-x}=0$

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Exercise 14 **Date:……………………..**

Solve the following equations.

1. $3(x-6) = 12-2x$
2. $2(x-4)+27 = 3(3-x)$
3. $4x = x-(x-2)$
4. $5x-3(x-1) = 39$
5. $7-(x+1) = 9-(2x-1)$
6. $5(2x-1)-2(x-2) = 7+4x$
7. $3(x-3)-7(2x-8)-(x-1) = 0$
8. $3x+2(x+1)+3(x+2) = 8$
9. $7(x+4)-5(x+3)+4-x = 0$
10. $0.05x+0.07(10{,}000-x) = 550$
11. $\frac{3x-1}{7} = 5$
12. $\frac{5}{2x-1} = \frac{1}{2}$
13. $\frac{2}{3x-1} = \frac{5}{3x+1}$
14. $\frac{2x+7}{4} - \frac{x+1}{3} = \frac{3}{4}$
15. $\frac{3x+2}{5} - \frac{2x+5}{3} = x+3$
16. $\frac{1}{x} + \frac{1}{2x} - \frac{1}{3x} = 2\frac{1}{3}$
17. $1\frac{2}{3}(x+1) = x+5\frac{2}{3}$
18. $\frac{2}{y+2} - \frac{y}{2-y} = \frac{y^2+4}{y^2-4}$
19. $\frac{4r-3}{6r+1} = \frac{2r-1}{3r+4}$
20. $(x+1)(x-3)+(x+1)^2 = 5x+10$
21. $(2x+1)^2-4(x-3)^2 = 5x+10$
22. $2(x+1)^2-(x-2)^2 = x(x-3)$

Exercise 15 **Date:……………………..**

Solve:

1. $5x-7-8x = 4x-17-6x$
2. $6(5+4x)-3(x-4) = 0$
3. $3-\frac{4}{x} = 6\left(-\frac{4}{3}\right)$
4. $\frac{4}{9-2x}+3 = 7$
5. $\frac{1}{2}(1-x)-\frac{1}{3}(2+x)+\frac{1}{4}(3-x) = 1$
6. $4-\frac{x-2}{2} = x+\frac{1-2x}{5}$
7. $\frac{1}{5}(x+1)-\frac{1}{15}(2x+3)-\frac{1}{3}(1-3x) = -1$
8. $\frac{1}{7}(2y-4)-\frac{1}{3}(2y-26) = 5-\frac{1}{2}(3y+5)$
9. $\left(\frac{x}{14}+3\frac{1}{2}\right)$ is twice $\left(\frac{x}{21}+1\frac{2}{3}\right)$
   Find the value of x
10. $\left(1-\frac{2x}{5}\right)$ is 3 less than three times $\left(\frac{x}{2}-5\right)$. Find the value of x.
11. $\frac{4}{2x+3}+\frac{5}{x-4} = \frac{1}{4x^2-10x-24}$
12. $\frac{1}{x}+\frac{3}{x-1} = \frac{4}{x+1}$
13. $\frac{3}{6x^2-2x+1}-\frac{1}{2x^2-4x+7} = 0$

14. $\frac{x-1}{x+2} - \frac{19-22x}{x^2-x-6} = \frac{x+1}{x-3}$
15. $\frac{5}{6(x-3)} + \frac{7-x}{2(x^2+3x+9)} = \frac{5-2x-x^2}{81-3x^3}$
16. $\frac{4}{x} + \frac{4+4x}{x^2+4x} = \frac{-5}{x+4}$

WORD PROBLEM

NOTE

1. **Words Denoting Addition**

sum	More than
Plus	Greater than
Gain	Larger than
Increase	Enlarge
Rise	Grow
Expand	Augment

2. **Words Denoting Subtraction**

Difference	Less than
Minus	Smaller than
Lose	Fewer than
Decrease	Shorten
Drop	Depreciate
Lower	Diminish

3. **Words Denoting Multiplication**

Multiplied	Double
Times	Triple or treble
Product	Quadruple
Twice	Quintuple

4. **Words Denoting Division**

Divided by	Ratio
Quotient	Half

Exercise 16 **Date:.......................**

Write the following as a mathematical sentence.

1. The sum of two equal numbers is 7.
2. The sum of y and 3 is 71.
3. The sum of m and n is equal to the product of m and n.
4. 5 less than the number and the result is 7 times the number.
5. 11 more than $\frac{1}{4}$ the number and the result is 8.
6. 2 more than 5 times the number is 3 less than 9 times the number.
7. When 13 is added to a certain number and the sum is multiplied by 5, the result is 90.
8. Two consecutive odd numbers are such that 3 times the smaller, subtracted from six times the bigger, give 102.

Example 3

The sum of three consecutive odd numbers is 57. Find the numbers.

Solution...

Let x be the first odd number.

i.e. $x, x+2, x+4$.

$$x + x + 2 + x + 4 = 57$$
$$3x + 6 = 57$$
$$3x = 51$$
$$x = 17$$

$\therefore$ The numbers are 17, 19, 21.

Example 4
A man is twice as sold as his son. Five years ago, the ratio of their ages was 9 : 4. Find the son's present age.

Solution…

Let the son's age $= x$
$\therefore$ the man's age $= 2x$

Five years ago,
The son's age was $= x - 5$
The man's age was $= 2x - 5$
The ratio of their ages in 5 years is given as 9 : 4.

$(2x - 5): (x - 5) = 9 : 4$

$$\frac{2x-5}{x-5} = \frac{9}{4}$$
$$4(2x - 5) = 9(x - 5)$$
$$8x - 20 = 9x - 45$$
$$8x - 9x = -45 + 20$$
$$-x = -25$$
$$x = 25$$

$\therefore$ The son's age is 25 years.

Example 5
A T – shirt cost 5 times as much as a singlet. For GH¢800.00, a trader can buy 32 more singlets than T – shirts. How much does a T – shirt cost?

Solution…
Let the cost of a singlet = GH¢ x
$\Longrightarrow$ The cost of a T – shirt = GH¢$5x$

Number of singlets that can be bought $= \frac{800}{5x}$

If the number of singlets that can be bought is 32 more than the number of T – shirts,
$\frac{800}{x} = \frac{800}{5x} + 32$

Multiply through by x
$$x \times \frac{800}{x} = x \times \frac{160}{x} + x \times 32$$
$$800 = 1600 + 32x$$
$$32x = 640$$
$$x = 20$$

$\therefore$ Cost of a T – shirt $= 5 \times 20 =$ GH¢100.00

Exercise 17 **Date:……………………**

1. The sum of two consecutive even numbers is 54. Find the numbers.
2. The sum of two consecutive odd numbers is 208. Find the numbers.
3. The sum of four consecutive odd numbers is 1112. Find the least of the four numbers.
4. Two consecutive integers are such that the greater added to twice the smaller gives 52. Find the numbers.
5. The cost of a cup of tea is t cents. The cost of a cup of coffee is $(t + 5)$ cents. The total cost of 7 cup of tea and 11 cups of coffee is 2215 cents. Find the cost of one cup of tea.
6. Find the two consecutive integers such that three times the smaller integer added to two times the greater integer equals 42.
7. At a football match, the price of a child's ticket is \$2.50 less than the price of an adult ticket. There are 18500 adults and 2400 children attending the football match. The total amount paid for the tickets is \$320040. Find the price of an adult ticket.

Exercise 18 **Date:……………………..**

1. Three quarters of a number added to two and a half of that number gives 13. Find the number.

2. The sum of 6 and one – third of x is one more than twice x. Find x.

3. Two positive numbers are in the ratio of $3 : 4$. The sum of 3 times the first and 2 times the second is 64. Find the smaller number.
4. If integer x is divided by 7, the quotient is 11 with remainder 1. Find the value of x.

5. Awa is m years old now and Fatou is y years older than Awa. If $(x - 5)$ years ago, Fatou was twice as old as Awa, express x in terms of y and m.

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Exercise 19 **Date:……………………**

1. Find three consecutive odd integers such that the sum of the last two is 15 less than 5 times the first.

2. Kofi is now five times as old as Kweku. In ten years' time Kofi will be three times as old as Kweku. How old are they now?

3. If two angles of a triangle are equal and the third angle is three times either of the other two angles, find the size of each angle.

4. The sum of three numbers is 81. The second number is twice the first, and the third number is six more than the second. Find the numbers.

5. A man walked 5 kilometers, then travelled a certain distance by Nissan Urvan bus, and twice as far by train. If the whole journey was 104 kilometers, how far did he travel by the bus?

Exercise 20 **Date:……………………..**

1. The sum of a number, $\frac{3}{4}$ of the number and $\frac{5}{8}$ of the number is 19. Find the number.

2. Mrs. Augustina Jackson bought 6 plates and 10 drinking cups from a shop. A plate cost her GH¢20.00 more than a drinking cup. If she spent GH¢1,080 altogether, how much did a plate and a drinking cup cost?

3. A boy is three years younger than his sister. If his age three years ago was two – thirds of her age at that time. What are their present ages?

4. Ama's age is two – thirds that of her elder sister. Four years ago Ama's age was half the sister's. How old is the sister?

5. A bag contains some balls of which $\frac{1}{4}$ are red. Forty more balls of which 5 are red. If $\frac{1}{5}$ of all the balls are red, how many balls were there originally?

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Exercise 21 **Date:……………………**

1. Sony is twice as Wale. Four years ago, he was four times as old as Wale. When will the sum of their ages be 66?

2. If $\frac{3}{4}$ of a number added to $\frac{5}{6}$ gives the same result as subtracting $\frac{7}{8}$ of the number from $20\frac{1}{3}$, find the number.

3. A man bought some shirts for GH¢720.00. If each shirt was GH¢2.00 cheaper, he would have received 4 more shirts. Calculate the number of shirts bought.

4. A student plans to spend ₦200 on p notebooks. But the price of the notebooks had increased by ₦10.00. As a result, the number of notebooks the students could buy was reduced by 1. Find the price of each notebook before the increase.

5. Two minibuses start from the same station and travel in opposite directions, along the same straight road. The first bus travels at a speed of 72km/h, the second at 48km/h. in how many hours will they be 360km apart?

Exercise 22 **Date:……………………**

1.

(a) Bottles of water cost 25 cents each.
  (i) Find the cost of 7 bottles in cents.
  (ii) Write down an expression in b for the cost of b bottles in cents.
  (iii) Change your answer to part (i) into dollars.
  (iv) Write down an expression in b for the cost of b bottles in dollars.

(b) The total cost, T, of n bars of chocolate is given by $T = nc$.
  (i) Write c in terms of T and n.
  (ii) What does c represent?

(c) The average cost of a book is $\$A$.
  (i) The total cost of 8 books is \$36, find the value of A.
  (ii) One of the 8 books is removed. The cost of this book is \$6.60. Find the new value of A.
  (iii) The total cost of x books is $\$y$. Write an expression for A in terms of x and y.
  (iv) One of the x books is removed. The cost of this book is \$7. Write a new expression for A in terms of x and y.

2. In a school, there are 1000 boys and number of girls. The 48% of the total number of students that were successful in an examination was made up of 50% of the boys and 40% of the girls. Find the number of girls in the school.

3. Two tanks X and Y are filled to capacity with petrol. Tank X hold 600 litres more than tank Y. If 100 litres of petrol were pumped out of each tank, tank X would then contain 3 times as much as petrol as tank Y. Find the capacity of each tank.

4. With GH¢184, I can buy x packet of biscuits. If I can but two more packets of the same biscuit with GH¢230.00, find the value of x.

5. Two cars travelled along the same road. The first car travelled at 45km/h for a certain time. The second car travelled at 80km/h for six minutes less than this time, but covered 55 more kilometers. For how long did the car travel?

6. Three times the age of Felicia is four more than the age of Asare. In three years, the sum of their ages will be 30 years. Find their present ages.

7.

(a) At a football match, the price of an adult ticket is \$ x and the price of a child ticket is \$ 2.50 less than the price of an adult. There are 18,500 adults and 2,400 children attending the football match. The total amount paid for the tickets is \$ 320,040. Find the price of an adult ticket.

(b) In a shop, the price of a monthly magazine is \$ m and the price of a weekly magazine is \$ 0.75 less than the price of a monthly magazine. One day, the shop receives.
- \$ 168 from selling monthly magazines.
- \$ 207 from selling weekly magazines.

The total number of these magazines sold during this day is 100. Find the price of a monthly magazine. Show all your working.

LINEAR INEQUALITIES

An inequality is a statement that shows that two algebraic expressions are not equal in a specific way, one expression being greater than or less than the other.

INEQUALITY SYMBOLS

SYMBOL	MEANING
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to

1. $a < b \Longrightarrow a$ is less than b
2. $a \leq b \Longrightarrow a$ is less than or equal to b.
3. $a > b \Longrightarrow a$ is greater than b.
4. $a \geq b \Longrightarrow a$ is greater than or equal to b.

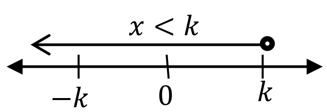
NOTE:
When both sides of an inequality are multiplied or divided by a negative number, the sign of the inequality changes.
i.e. if $a > b$ then multiply through by $-k$
$\Longrightarrow -ka < -kb$
Also if $-ka \leq b$

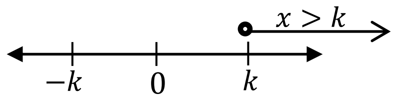
$\frac{-ka}{-k} \geq \frac{b}{-k}$

$a \geq -\frac{b}{k}$

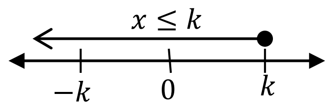
REPRESENTATION OF SOLUTION SET ON THE NUMBER LINE

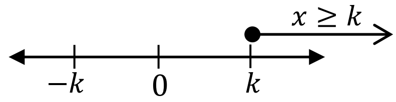
(a) If $x < k$ or $x > k$, where x is a variable and k is a constant, we use 'O' an open circle to indicate that k is not included in the solution set. The number line for $x < k$ and $x > k$ are as follows:





(b) If $x \leq k$ or $x \geq k$, where x is a variable and k is a constant, we use '●' a dot to indicate that k is included in the solution set. The number line for $x \leq k$ or $x \geq k$ are as follows:



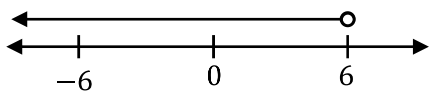


Example 1
Determine the solution set of the following and illustrate the answer on the number line.

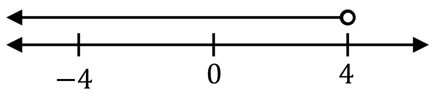
1. $2p + 4 < 16$
2. $\frac{21+x}{5} > x + 1$
3. $\frac{2x+1}{3} \leq \frac{5x-8}{4}$

Solution…

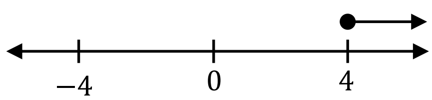
1. $2p + 4 < 16$
   $2p < 16 - 4$
   $2p < 12$
   $p < 6$
   $\{p: p < 6\}$



2. $\frac{21+x}{5} > x + 1$
   $5 \times \frac{21+x}{5} > 5 \times x + 5 \times 1$
   $21 + x > 4x + 5$
   $x - 5x > 5 - 21$
   $-4x > -16$
   $\frac{-4x}{-4} < \frac{-16}{-4}$
   $x < 4$
   $\{x: x < 4\}$



3. $\frac{2x+1}{3} \leq \frac{5x-8}{4}$
   $12 \times \frac{2x+1}{3} \leq 12 \times \frac{5x-8}{4}$
   $4(2x + 1) \leq 3(5x - 8)$
   $8x + 4 \leq 15x - 24$
   $-7x \leq -28$
   $x \geq 4$



THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 1 **Date:.......................**

Determine the solution set of the following:

1. $3p + 7 < 16$
2. $3x - 13 > 26$
3. $-2 - 4x > -6$
4. $3x - 1 \leq 11x + 2$
5. $5t + 23 < 17 - 2t$
6. $3x - 4 - 2(x + 1) < 3$

Exercise 2 **Date:.......................**

Find the truthset of the following

1. $x + 3 > 19 - 3x$
2. $5 - 2x > x + 2$
3. $7x - 5 > 3(2 - 5x)$
4. $3n + 23 < n + 41$
5. $2x - 6 \leq 5(3 - x)$

Exercise 3 **Date:......................**

Find the solution set of each of the following inequalities and represent it on the number line.

1. $3x - 9 \geq 12(x - 3)$
2. $5x - 3(x - 1) \geq 39$
3. $7x - 5 > 3(2 - 5x)$
4. $5(x - 4) < 3(12 - x)$
5. $(p - 1) - 3(1 - 2p) \geq 0$

Exercise 4 **Date:......................**

Find the truthset of each of the following inequalities and represent it on the number line.

1. $3p > (7 + p) - 5$
2. $-2 + 5 - 3x < 4x + 7 - 2x$
3. $11 - 3(2z + 1) - 2(1 - 6z) < 0$
4. $3 - 4(1 - z) + 3(z - 3) > 0$

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Exercise 5 **Date:……………………**

Determine the solution set of each of the following.

1. $\frac{2x+1}{3} \leq \frac{5x-8}{4}$
2. $\frac{21+x}{5} > x+1$
3. $\frac{1}{2}x + \frac{3}{4} < 2$
4. $\frac{4}{5}x - \frac{3}{4}x < \frac{7}{15}$
5. $\frac{1}{2}(5x-4) < x + \frac{11}{24}$

Exercise 6 **Date:……………………**

Find the solution set of each of the following and illustrate your answer on the number line.

1. $\frac{2x-1}{4} - \frac{x-2}{3} > 1$
2. $\frac{2x-2}{4} - \frac{2x-1}{3} \leq 1$
3. $\frac{1}{3}x + 1 \geq \frac{1}{2}x + \frac{1}{4}(2 - x)$
4. $\frac{1}{3}(x - 1) - \frac{1}{2}(x - 3) \leq 1\frac{1}{4}$

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THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 7 **Date:......................**

Find the truthset of the following and illustrate the answer on the number line.

1. $\frac{1}{3}x + 1\frac{2}{3} < \frac{3}{4}x - \frac{1}{2}$
2. $\frac{3}{4}(x+1) + 1 \leq \frac{1}{2}(x-2) + 5$
3. $\frac{1}{2}x - \frac{5}{6}(x+2) \leq 1 + x$
4. $\frac{1}{3}x - \frac{1}{4}(x+2) \geq 3x - 1\frac{1}{3}$

Exercise 8 **Date:……………………**

Solve

1. $\frac{1}{2}(4x-6)-\frac{1}{3}(5-4x)\geq 8$
2. $6(x-4)+2(x+1)<3(x+7)+1$
3. $5(2x-3)-2(4x-5)\geq 8(x+1)$
4. $\frac{3}{8}\left(2x-\frac{1}{3}\right)-\frac{2}{9}(7-3x)\geq\frac{1}{4}x+\frac{2(x-8)}{7}$

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Exercise 9 **Date:……………………**

Solve:

1. $\frac{1}{4}(x+1)-\frac{1}{2}(x+2) \geq -3\frac{1}{4}$
2. $\frac{1}{2}(4x-6)-\frac{1}{3}(5-4x) \geq 8$
3. $\frac{x}{3}-\frac{1}{4}(x+2) > 3x-2\frac{1}{5}$
4. $7(x+4)-\frac{2}{3}(x-6) \leq 2[x-3(x+5)]$

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Example 2

Find the truthset and illustrate the answer on the number line.

$$0 \leq 3x - 1 \leq 2$$

Solution…

$0 \leq 3x - 1 \leq 2$

Add 1 to each part

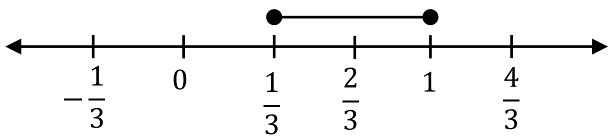
$\Rightarrow 1 + 0 \leq 3x - 1 + 1 \leq 2 + 1$

$\Rightarrow 1 \leq 3x \leq 3$

Divide both sides by 3

$\Rightarrow \frac{1}{3} \leq \frac{3x}{3} \leq \frac{3}{3}$

$\left\{x: \frac{1}{3} \leq x \leq 1\right\}$



Exercise 10 **Date:……………………**

Solve the following inequalities.

1. $4 < 3x + 1 \leq 12$
2. $4 < 2x + 1 \leq 7$
3. $x < 2x + 1 \leq 7$
4. $2x < 3x + 1 \leq 13$
5. $3 < 2x - 5 < 7$
6. $16 < 2x - 5 < 48$
7. $9 < 3n + 6 \leq 21$
8. $-3 < 2x - 1 \leq 6$

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Exercise 11 **Date:.......................**

Solve the following inequalities.

1. $-35 < 6x + 7 < 1$
2. $-9 \leq 6x + 5 \leq 0$
3. $1 < 4x - 9 < 5$
4. $8 < 3x - 2 < 11$
5. $6x + 3 < x < 3x + 9$

Exercise 12

Solve and represent the solution on the number line.

1. $-5 < 2x - 3 < 6$
2. $0 < \frac{1}{3}t + \frac{1}{2} < 1$
3. $0 < \frac{1}{2}(2 - 3x) < 2$
4. $-2 \leq 2p - 3 < 1$
5. $-1 < (3t - 5 + 7 - 4t) \leq 2$
6. $-2 - 3x < 10 + x < 2 - 3x$
7. $8 - x < 7 - 2x < 4 - x$
8. $-\frac{1}{4} < \frac{3}{4}(3x - 2) \leq -\frac{1}{2}$
9. $-3 \leq 3x + \frac{1}{3} - \left(x - \frac{2}{3}\right) \leq 6$

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

BEARINGS AND VECTORS

BEARINGS

Exercise 1

1. A ship sails due North from a point P to a point Q, 4km away. It then sails on a bearing of 090° to a point R, 3km from Q. Find the distance between P and R.
2. A ship sails from port R on a bearing 065° to port S a distance of 54km. It then sails on a bearing of 155° from port S to port Q, a distance of 80km. Find, correct to one decimal place,
   (i) The distance between R and Q
   (ii) The bearing of Q from R

Exercise 2

1. A, B, X and Y are four points in a horizontal plane. B is on a bearing of 090° from A. X is 7.5m due North of B and on a bearing of 052° from A. Y is due North of A and on a bearing of 340° from B. Calculate correct to **three** significant figures:
   (a) $|AB|$
   (b) $|AY|$
   (c) The components of $\overrightarrow{XY}$
   (d) The distance and bearing of Y from X

2. A ship sails from a point A in a direction 065° to a point B, 24km away. From B, the ship sails 18km due East to a point C. From C, the ship then sails 30km due North to a point D. Calculate the bearing of D from A.

3. The locations of four towns, P, Q, R and T are such that Q is on a bearing of 270° from P. T is 12km due North of P and on a bearing of 047° from Q. R is due North of Q and 16km from P. Calculate, correct to **three** significant figures;
   (a) The distance between P and Q
   (b) The distance between Q and R
   (c) The bearing of R and P
   (d) The components of $\overrightarrow{RT}$
   (e) $|\overrightarrow{RT}|$

Exercise 3

1. P, Q nd R are three villages on a level ground. Q is 4km on a bearing of 040° from P, while R is 3km on the bearing 130° from Q. Calculate the distance and bearing of P from R. State $\overrightarrow{PR}$ in distance and bearing form.

2. Salifu walks 500 metres due North, then 250 metres due East and finally 500 metres on a bearing of 055°.
   (i) Sketch a diagram to illustrate the movement.
   (ii) Calculate, correct to the nearest whole number, how far North Salifu has moved from the starting point.
   (iii) Calculate, correct to the nearest whole number, how far East Salifu has moved from the starting point.
   (iv) Calculate the bearing of Salifu's final position from the starting point.

3. Two cyclists left a point P at the same time. The first cyclist covered 300m on a bearing of 296° and the second cyclist covered 250m on a bearing of 206°. Calculate, correct to **three** significant figures,
   (a) The distance between the two cyclists
   (b) The bearing of the second cyclist from the first.

Exercise 4

1. Three towns, P, Q and R are such that the distance between P and Q is 50km and the distance between P and R is 90km. If the bearing of Q from P is 075° and the bearing of R from P is 310°, find the:
   (a) Distance between Q and R;
   (b) Bearing of R from Q

2. A man travels from a village X on a bearing of 060° to a village Y which is 20km away. From Y he travels to a village Z, on a bearing of 195°. If Z is directly East of X, calculate correct to three significant figures, the distance of
   (i) Y from X (ii) Z from X

3. Two men P and Q set off from a base camp R prospecting for oil, moves 20km on a bearing of 205° and Q moves 15km on a bearing of 060°. Calculate the:
   (a) Distance of Q from P,
   (b) Bearing of Q from P

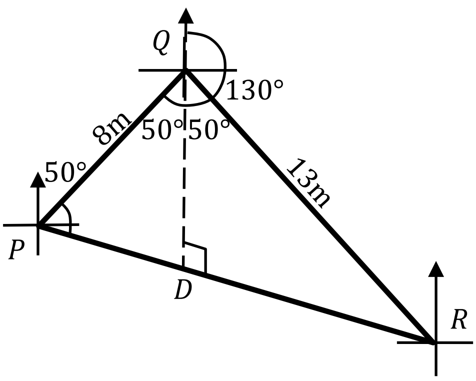
(Give your answer in each case correct to the nearest whole number)

Exercise 5

1. A surveyor standing at a point X sights a pole Y due East of him and a tower Z of a building on a bearing of 046°. After walking to a point W, a distance of 180m in the South – East direction, he observes the bearing of Z and Y to be 337° respectively.
   (a) Calculate, correct to the nearest metre
   (i) $|XY|$ (ii) $|ZW|$
   (b) If N is on XY such that $XN = ZN$, find the bearing of Z from N.

2. The bearing of Q from P is 150° and the bearing of P from R is 015°. If Q and R are 24km and 32km respectively from P:
   (i) Represent this information in a diagram;
   (ii) Calculate the distance between Q and R, correct to two decimal places;
   (iii) Find the bearing of R and Q, correct to the nearest degree.

3.

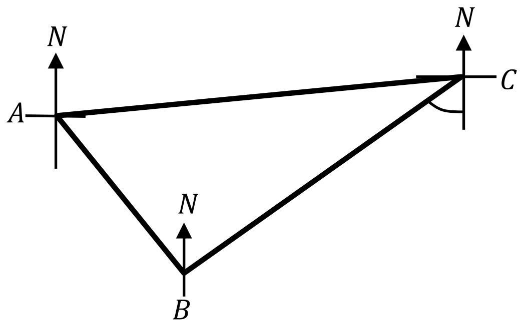


In the diagram, $|PQ| = 8$m, $QR = 13$m, the bearing of Q from P is 050° and the bearing of R from Q is 130°.
   (a) Calculate correct to 3 significant figures
   (i) PR
   (ii) The bearing of R from P
   (b) Calculate the shortest distance between point Q and PR, hence the area of triangle PQR

Exercise 6

1. A boy walks 6km from a point P to a point Q on a bearing of 065°. He then walks to a point R, a distance of 13km, on a bearing of 146°.
   (i) Sketch the diagram of his movement
   (ii) Calculate, correct to the nearest kilometre, the distance PR.

2.



In the diagram, $|AB| = 8$km, $|BC| = 13$km, The bearing of A from B is 310° and the bearing of B from C is 230°. Calculate, correct to 3 significant figures,
   (a) The distance AC
   (b) The bearing of C from A
   (c) How far East of B, C is

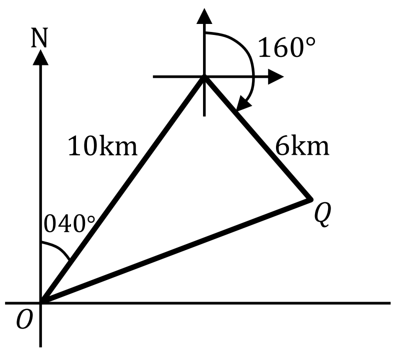
3. A cyclist starts from a point X and rides 3km due West to a point Y. At Y, he changes direction and rides 5km North – West to a point Z.
   (i) How far is he from the starting point, correct to the nearest km?
   (ii) Find the bearing of Z from X, to the nearest degree.

Exercise 7

1. A ship leaves port P and sails on a bearing of N45°E to a port Q, 15km away. It then sails on a bearing of S45°E to port R, 20km away.
   (i) Represent the information in a diagram.

(ii) Calculate, correct to the **nearest** whole number, the
(a) Distance from P to R;
(b) Bearing of P from R.

2.



In the diagram, $|OP|10$km, $|PQ| = 6$km, the bearing of P from O is 040° while the bearing of Q from P is 160°.
(a) Calculate, correct to **three** significant figures,
  (i) $|OQ|$;
  (ii) The bearing of Q from O;
(b) How far South of P is Q? Correct to **three** significant figures.

Exercise 8

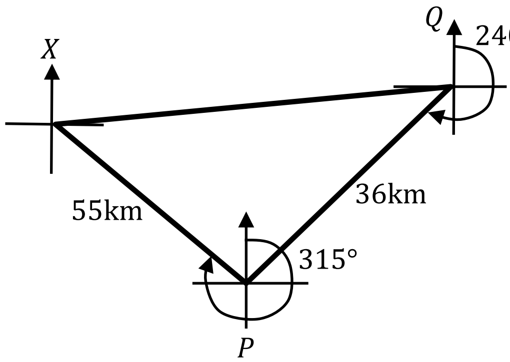
1. The bearing of points X and Y from Z are 040° and 300°, respectively. If $|XY| = 19.5$km and $|YZ| = 11.5$km,
   (a) Illustrate the information in a diagram,
   (b) Calculate, correct to the **nearest** whole number,
     (i) $< ZXY$;
     (ii) $|XZ|$;
     (iii) Bearing of X from Y.

2. The bearing of Q from P is 150° and the bearing of P from R is 015°. If Q and R are 24km and 32km respectively from P.
   (i) Represent this information in a diagram;
   (ii) Calculate the distance between Q and R, correct to **two** decimal places;
   (iii) Find the bearing of R from Q, correct to the **nearest** degree.

Exercise 9

1. Town Q is 20km due North of P. The bearing of town R from Q is 140°. If R is 8km from Q, calculate
   (a) The bearing of R from P, to the **nearest** degree.
   (b) How far North of P, R is, correct to **two** significant figures.

2.



The diagram shows the position of three points P, Q and X on a horizontal plane. The bearing of P from Q is 240° and that of X from P is 315°. If $|PQ| = 36$km, and $|PX| = 55$km, evaluate, correct to one decimal place,
(i) $|QX|$;
(ii) The bearing of Q from X

3. A boat sails 6km on a bearing of 037° and then 7km on a bearing of 068°. Calculate:
   (a) The distance of the boat from the starting point, correct to **two** decimal places:
   (b) The bearing of the boat from its starting point, correct to the **nearest** degree.

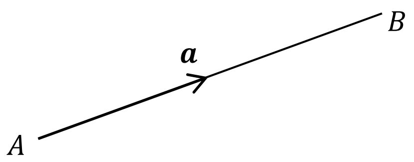
4. The bearing of P from X, 10km away is 025°. Another point Q is 6km from X and on a bearing of 162°. Calculate the:
   (i) Distance PQ;
   (ii) Bearing of P from Q.

5. Two boats A and B leave a port at the same time. A travels 15km on a bearing of 020° while B travels 14km on a bearing of 290°. Calculate, correct to **two** decimal places, the
   (a) Distance between A and B;
   (b) Bearing of A from B

BEARINGS AND VECTORS

Vectors are physical quantities that have both magnitude and direction.

In Mathematics, vectors are represented by line segments and arrow head showed direction.



$\overrightarrow{AB}$: point

A is the (initial) origin and B the destination (terminal or end point).

REPRESENTATION OF VECTORS IN STANDARD BASIC FORM

1. **The column or Component Form**

   i.e. $\begin{pmatrix} a \\ b \end{pmatrix}$.

   For example, $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

2. **The Magnitude and Bearing Form**

   $\overrightarrow{OB} = (3\text{km}, 060°)$

MODULUS OR MAGNITUDE OR LENGTH OF A VECTOR

If $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$, then $|\mathbf{a}| = \sqrt{x^2 + y^2}$

Example 1

Find the length of the vector $a = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Solution…

$|\boldsymbol{a}| = \sqrt{3^2 + 4^2}$

$|\boldsymbol{a}| = \sqrt{25} = 5$ units

Example 2

If $\overrightarrow{CD} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$. Find

(i) $|\overrightarrow{CD}|$ (ii) $|\overrightarrow{AB}|$

Solution…

(i) $|\overrightarrow{CD}| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$ units

(ii) $|\overrightarrow{AB}| = \sqrt{5^2 + 12^2} = \sqrt{169}$

$= 13$ units

Exercise 1 **Date:……………………..**

Find the magnitude of the following vectors.

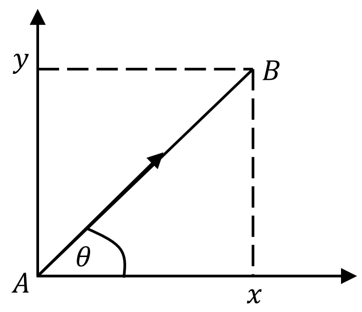
(i) $\boldsymbol{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ (iii) $\boldsymbol{c} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

(ii) $\boldsymbol{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ (iv) $\boldsymbol{d} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

DIRECTION OF A VECTOR

IF $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, then the direction is the angle measured from the geographical north to the vector through a clockwise direction.

i.e.



$\tan\theta = \frac{x}{y}$

$\theta = \tan^{-1}\left(\frac{x}{y}\right)$

In finding the bearing (direction) of $\overrightarrow{AB}$, we subtract the angle that the line $\overrightarrow{AB}$ makes with the positive $x-$axis from the north, i.e. 90°.

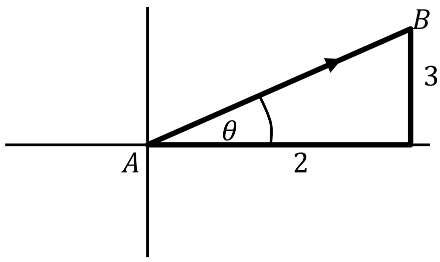
$\therefore$ Direction of $\overrightarrow{AB} = 90 - \tan^{-1}\left(\frac{x}{y}\right)$

Example 2
Find the magnitude and direction of the following.

1. $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$
2. $\overrightarrow{MN} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$
3. $\overrightarrow{PQ} = \begin{pmatrix} -7 \\ -5 \end{pmatrix}$

Solution…

1. $|\overrightarrow{AB}| = \sqrt{2^2 + 3^2} = \sqrt{13} = 3.61\text{units}$



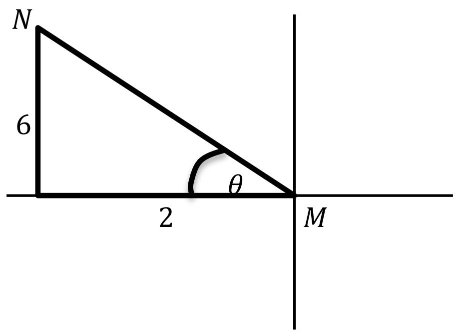
$\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$

$\therefore$ Direction of $\overrightarrow{AB}$
$= 90° - 56.3° = 33.7°$

$\therefore \overrightarrow{AB} = (3.61 \text{ units}, 33.7°)$

2. $|\overrightarrow{MN}| = \sqrt{(-2)^2 + (6)^2} = 6.32 \text{ units}$



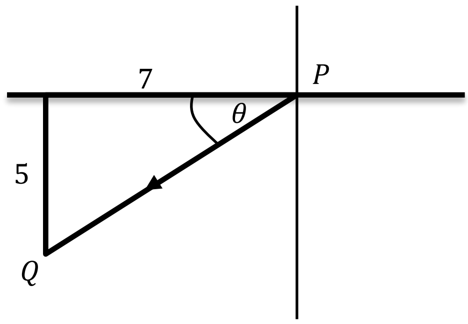
$\tan\theta = \left(\frac{6}{2}\right)$

$\theta = \tan^{-1}(3) = 71.6°$

$\therefore$ Direction of $\overrightarrow{MN}$
$= 270° + 71.6° \cong 342°$

$\therefore \overrightarrow{MN} = (6.32 \text{ units}, 342°)$

3. $|\overrightarrow{PQ}| = \sqrt{(-7)^2 + (-5)^2} = 8.6 \text{ units}$



$\tan\theta = \frac{5}{7}$

$\theta = \tan^{-1}\left(\frac{5}{7}\right) = 35.5°$

$\therefore$ Direction of $\overrightarrow{PQ}$
$= 270° - 35.5° \cong 234°$

$\therefore \overrightarrow{PQ} = (8.6 \text{ units}, 234°)$

Exercise 2 **Date:……………………..**
Find the magnitude and bearing of the following.

1. $\overrightarrow{QR} = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$
2. $\overrightarrow{AB} = \begin{pmatrix} -6 \\ -9 \end{pmatrix}$
3. $\overrightarrow{MN} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

EQUAL VECTORS

Two vectors $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ are said to be equal (or equivalent) if they have the same magnitude and direction. i.e. $\overrightarrow{AB} = \overrightarrow{PQ} \Leftrightarrow |AB| = |PQ|$

If $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, then $x_1 = x_2$ and $y_1 = y_2$

Example 3

If $\boldsymbol{m} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix} 3x \\ 3+y \end{pmatrix}$ are equal vectors, find the values of x and y.

Solution…

Since the two vectors are equal, then $\boldsymbol{m} = \boldsymbol{n}$.

$\Rightarrow \begin{pmatrix} 6 \\ -3 \end{pmatrix} = \begin{pmatrix} 3x \\ 3+y \end{pmatrix}$

$\Rightarrow 3x = 6$ $\therefore x = 2$

Also $= -3 = 3 + y$ $\therefore y = -6$

Exercise 3 **Date:……………………..**

1. If $\boldsymbol{m} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix} 5x \\ y-4 \end{pmatrix}$ are equal vectors, find the values of x and y.

2. If $\boldsymbol{a} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 2x \\ 3+y \end{pmatrix}$ are equal vectors, find the values of x and y.

3. If $\boldsymbol{m} = \begin{pmatrix} 2-x \\ 2y+1 \end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix} 3 \\ -2-y \end{pmatrix}$; find the values of x and y if $\boldsymbol{m} = \boldsymbol{n}$.

4. Given that $\boldsymbol{p} = \begin{pmatrix} a-2 \\ 5b+1 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 7 \\ -9 \end{pmatrix}$; find a and b if $\boldsymbol{p} = \boldsymbol{q}$.

5. Given that the vectors $\begin{pmatrix} 2x-3 \\ y-x \end{pmatrix}$ and $\begin{pmatrix} x-y \\ 3x-5 \end{pmatrix}$ are equal; find the values of x and y.

MULTIPLICATION OF A VECTOR BY A SCALAR

If $\boldsymbol{a} = \begin{pmatrix} m \\ n \end{pmatrix}$, then $\lambda\boldsymbol{a} = \begin{pmatrix} \lambda m \\ \lambda n \end{pmatrix}$, $\forall \lambda \in \mathbb{R}$

Example 4

Given that $\boldsymbol{u} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$. Find $\frac{1}{3}\left(\boldsymbol{u} + \frac{1}{2}\boldsymbol{v}\right)$.

Solution…

$\boldsymbol{u} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} -2+1 \\ 3+3 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

$\therefore \frac{1}{3}\left(\boldsymbol{u} + \frac{1}{2}\boldsymbol{v}\right) = \frac{1}{3}\begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ 2 \end{pmatrix}$.

Exercise 4 **Date:……………………..**

1. If $\boldsymbol{p} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, evaluate $\boldsymbol{p} + \boldsymbol{q} - 2\boldsymbol{r}$.

2. If $\boldsymbol{u} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $\boldsymbol{s} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ and $\boldsymbol{t} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, find $2\boldsymbol{u} - 4\boldsymbol{s} + \boldsymbol{t}$.

3. If $\boldsymbol{p} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, evaluate $2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}$.

4. Given that $\boldsymbol{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, find $2\boldsymbol{p} + 3\boldsymbol{q} - 4\boldsymbol{r}$.

Exercise 5 **Date:..........................**

1. If $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, find $\boldsymbol{p} - 3\boldsymbol{q}$.

2. Given that $\boldsymbol{x} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{y} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, calculate $3\boldsymbol{x} - \boldsymbol{y}$.

3. If $\boldsymbol{q} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$, find $\boldsymbol{p}$ such that $\boldsymbol{q} = \frac{1}{2}\boldsymbol{p}$.

4. If $\boldsymbol{p} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $2\boldsymbol{r} = \boldsymbol{p} + \boldsymbol{q}$, find $\boldsymbol{r}$.

5. If $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\boldsymbol{r} = \boldsymbol{a} + \frac{1}{2}(\boldsymbol{a} - \boldsymbol{b})$, find:
   (i) $\boldsymbol{r}$ (ii) $|\boldsymbol{r}|$

Exercise 6 **Date:..........................**

1. If $\boldsymbol{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, evaluate $|\boldsymbol{u} + \boldsymbol{v}|$.

2. Find the values of x and y in the equation $\begin{pmatrix} x+3 \\ 2 \end{pmatrix} - \begin{pmatrix} y \\ x+y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

3. Given that $\boldsymbol{m} = \begin{pmatrix} 3x+1 \\ -2y+3 \end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix} -8 \\ 5 \end{pmatrix}$, find (x, y) if $\boldsymbol{m} = \boldsymbol{n}$.

4. If $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$. Find $|\boldsymbol{a} - 2\boldsymbol{b}|$.

5. If $\boldsymbol{x} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{y} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, calculate $|3\boldsymbol{x} - \boldsymbol{y}|$.

6. If $\boldsymbol{a} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, find
   (a) $|\boldsymbol{b}|$ (b) $|\boldsymbol{a} + \boldsymbol{b}|$ (c) $|2\boldsymbol{a} - \boldsymbol{b}|$

Exercise 7 **Date:……………………..**

1. If $4\begin{pmatrix}1\\3\end{pmatrix}+2\begin{pmatrix}1\\m\end{pmatrix}=3\begin{pmatrix}n\\-6\end{pmatrix}$, find the values of m and n.

2. Find the values of λ and μ such that $\begin{pmatrix}\lambda+4\\2\end{pmatrix}-\begin{pmatrix}\mu\\\lambda+\mu\end{pmatrix}=\begin{pmatrix}6\\-3\end{pmatrix}$.

3. If $3\begin{pmatrix}2\\x\end{pmatrix}-\begin{pmatrix}1-y\\4\end{pmatrix}=\begin{pmatrix}3-2y\\1-2x\end{pmatrix}$. Find the values of x and y.

4. If $\boldsymbol{m}=\begin{pmatrix}2x+1\\2-3y\end{pmatrix}$, $\boldsymbol{n}=\begin{pmatrix}6\\-8\end{pmatrix}$ and $\boldsymbol{m}+\boldsymbol{n}=\begin{pmatrix}9\\-12\end{pmatrix}$, find
   (i) Values of x and y
   (ii) Components of $\boldsymbol{m}$

5. Two vectors are represented by $\boldsymbol{r}=2\boldsymbol{a}$ and $\boldsymbol{s}=3\boldsymbol{a}$ where $\boldsymbol{a}=\begin{pmatrix}2\\-1\end{pmatrix}$. Find $|\boldsymbol{r}-\boldsymbol{s}|$.

6. If $\boldsymbol{p}=\begin{pmatrix}2\\-3\end{pmatrix}$, $\boldsymbol{q}=\begin{pmatrix}-3\\4\end{pmatrix}$ and $\boldsymbol{s}=\begin{pmatrix}4\\-11\end{pmatrix}$. Find $|\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{s}|$.

7. Given that $s=\begin{pmatrix}3\\-4\end{pmatrix}$, $\boldsymbol{m}=\begin{pmatrix}5\\0\end{pmatrix}$, if $\boldsymbol{n}=\frac{1}{4}(\boldsymbol{s}+\boldsymbol{m})$ and $\boldsymbol{t}=\frac{1}{2}(\boldsymbol{s}-\boldsymbol{m})$. Show that $|\boldsymbol{n}+\boldsymbol{t}|=|\boldsymbol{n}-\boldsymbol{t}|$.

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Example 5

The vectors $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\boldsymbol{r} = \frac{1}{2}(\boldsymbol{q} - \boldsymbol{p})$.

(i) Find the vector r

(ii) If $m\boldsymbol{p} + n\boldsymbol{q} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, find m and n, where m and n are scalars.

Solution…

(i) $\boldsymbol{r} = \frac{1}{2}(\boldsymbol{q} - \boldsymbol{p}) = \frac{1}{2}\left(\begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right)$

$= \frac{1}{2}\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(ii) $m\boldsymbol{p} + n\boldsymbol{q} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$m\begin{pmatrix} 2 \\ 3 \end{pmatrix} + n\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$2m + 2n = 4$

$\therefore m + n = 2$………………. (1)

$3m + 5n = 3$…………… (2)

$(2) - 3(1) \Longrightarrow 2n = -3 \quad \therefore n = -\frac{3}{2}$

Put $n = -\frac{3}{2}$ into (1)

$\Longrightarrow m - \frac{3}{2} = 2 \quad \therefore m = \frac{7}{2}$

Hence, $m = \frac{7}{2}, n = -\frac{3}{2}$

Exercise 8 **Date:……………………..**

1. The vectors $\boldsymbol{a} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ are in the same plane. If $3\boldsymbol{a} - 2\boldsymbol{b} = \boldsymbol{c}$, find
   (i) the vector $\boldsymbol{b}$;
   (ii) $|\boldsymbol{d}|$ and express your answer in the form $p\sqrt{q}$ where p and q are integers and $\boldsymbol{d} = \boldsymbol{b} - \boldsymbol{c}$.

2. If $\boldsymbol{a} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$, find
   (i) m and n such that $\boldsymbol{c} = m\boldsymbol{a} + n\boldsymbol{b}$ where m and n are scalars.
   (ii) $|\boldsymbol{d}|$ if $\boldsymbol{d} = \boldsymbol{c} - 2\boldsymbol{a}$.

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Exercise 9 **Date:.........................**

1. If $\boldsymbol{p} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$, find:
   (a) m and n, such that $\boldsymbol{r} = m\boldsymbol{p} + n\boldsymbol{q}$, where m and n are scalars.
   (b)
      (i) $|\boldsymbol{g}|$, if $\boldsymbol{g} = 3\boldsymbol{q} + \boldsymbol{r}$
      (ii) the bearing of $\boldsymbol{g}$

2. If $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 6 \\ 23 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$ and $\boldsymbol{s} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$. Find,
   (i) $|3\boldsymbol{p} + 2\boldsymbol{q} - 5\boldsymbol{r}|$
   (ii) the scalars m and n such that $\boldsymbol{s} = m\boldsymbol{p} + n\boldsymbol{q} + \boldsymbol{r}$.

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Exercise 10 **Date:..........................**

1. The vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ are given by $\mathbf{a} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} -1 \\ 17 \end{pmatrix}$. Find the numbers m and n so that $m\mathbf{a} + n\mathbf{b} = \mathbf{c}$

2. If $\boldsymbol{a} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$. Find the values of p and q such that $\boldsymbol{c} = p\boldsymbol{a} + q\boldsymbol{b}$.

3. Given that $\boldsymbol{p} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$, find the values of λ and μ from the expression $\boldsymbol{r} = \lambda\boldsymbol{p} + \mu\boldsymbol{q}$.
4. If $\boldsymbol{p} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$, find the values of the constants x and y such that $2p = 5x\boldsymbol{q} - 3y\boldsymbol{r}$.

5. If $\boldsymbol{p} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ respectively. Find
   (a) $|3\boldsymbol{p} - 2\boldsymbol{r}|$
   (b) the values of the scalars m and n such that $\begin{pmatrix} 8 \\ 8 \end{pmatrix} = m\boldsymbol{p} + n\boldsymbol{r}$.

6. Find the values of k for which the vector $\begin{pmatrix} k+4 \\ 3k-3 \end{pmatrix}$ has a magnitude of 15 units.

7. If $\boldsymbol{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{s} = \begin{pmatrix} \lambda \\ -3 \end{pmatrix}$. Find
   (i) the values of μ and of λ if $\mu\boldsymbol{r} + 2\boldsymbol{s} = \begin{pmatrix} 2 \\ -18 \end{pmatrix}$
   (ii) the values of λ such that $|4\boldsymbol{r} + \boldsymbol{s}| = 3|\boldsymbol{s}|$.

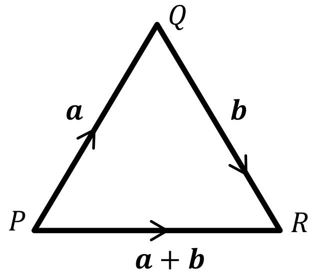
8. Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a} = \begin{pmatrix} 3+m \\ 5-2n \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 4-2n \\ 10+3m \end{pmatrix}$
   (i) Given that $3\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} 1+n \\ -5 \end{pmatrix}$, find the values of m and n
   (ii) Show that the magnitude of b is $k\sqrt{5}$, where k is an integer to be found.

9. Given that $\boldsymbol{p} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 5 \\ 17 \end{pmatrix}$, express r in terms of p and q.

RESULTANT OF VECTORS
(ADDITION OF VECTORS)
TRIANGULAR LAW OF VECTOR ADDITION

Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be any two vectors (represented by $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ in the diagram below) such that the end point a is the initial point of b.

i.e. $\boldsymbol{a} = \overrightarrow{PQ}, \boldsymbol{b} = \overrightarrow{QR}$



$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$
$\boldsymbol{a} + \boldsymbol{b} = \overrightarrow{PR}$

Example 6

Find the resultant of the vectors $\boldsymbol{a}$ and $2\boldsymbol{b}$ if $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$.

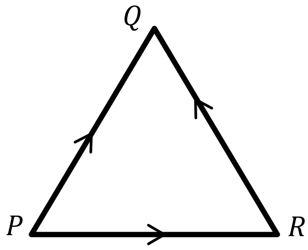
Solution…

$\boldsymbol{a} + 2\boldsymbol{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -6 \\ -4 \end{pmatrix}$
$= \begin{pmatrix} 2-12 \\ 3-8 \end{pmatrix} = \begin{pmatrix} -10 \\ -15 \end{pmatrix}$

Example 7

If $\overrightarrow{PQ} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\overrightarrow{RQ} = \begin{pmatrix} c \\ d \end{pmatrix}$, express $\overrightarrow{PR}$ as a column vector.

Solution…



From the diagram,
$\overrightarrow{PR} + \overrightarrow{RQ} = \overrightarrow{PQ}$
$\overrightarrow{PR} = \overrightarrow{PQ} - \overrightarrow{RQ}$
$\overrightarrow{PR} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a-c \\ b-d \end{pmatrix}$

Exercise 11 **Date:..........................**

1. PQR is a triangle such that $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$. Find $\overrightarrow{RQ}$.

2. Given that $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$, find $\overrightarrow{QR}$.

3. In the triangle PQR, $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$. Find $\overrightarrow{QR}$.

4. If $\overrightarrow{BA} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$, calculate the magnitude of $\overrightarrow{BC}$.

5. In the triangle ABC, $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$. Find $\overrightarrow{BC}$.

6. In triangle PQR, $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{RQ} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$, find $|\overrightarrow{PR}|$.

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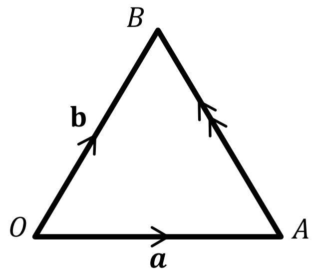
POSITION VECTORS

Let O be a point fixed in space.
Then O is called the origin or the reference point of a given coordinate system. Let A be any other point in space. The displacement vector $\overrightarrow{OA}$ is called the position vector of A relative to the origin O.
Thus the position vector of B relative to A is the directed line segment $\overrightarrow{AB}$.



In $\Delta OAB, \overrightarrow{OA} = \boldsymbol{a}, \overrightarrow{OB} = \boldsymbol{b}$

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$$

In particular,

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \boldsymbol{a} - \boldsymbol{b}$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = \boldsymbol{d} - \boldsymbol{c}$

$\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \boldsymbol{n} - \boldsymbol{m}$

Example 7

$A(-1, 5), B(-1, 2)$ and $C(3, 0)$ are points in the $x - y$ plane. Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in the form $\binom{x}{y}$.

Solution…

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$\overrightarrow{BA} = \binom{-1}{5} - \binom{-1}{2} = \binom{0}{3}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{BC} = \binom{3}{0} - \binom{-1}{2} = \binom{4}{-2}$

Exercise 12 **Date:……………………..**

1. The vertices of a triangle are $P(1, -3)$, $Q(7, 5)$ and $R(-3, 5)$.
   (i) Express $\overrightarrow{PQ}, \overrightarrow{QR}$ and $\overrightarrow{PR}$ as column vectors.
   (ii) Show that triangle PQR is isosceles.

2. Given $A(1, 3), B(-2, -1)$ and $C(2, 3m)$ where m is a constant, find
   (i) $|AB|$
   (ii) The value of m if $\overrightarrow{BC} = \binom{4}{-5}$

Exercise 13 **Date:……………………..**

1. If $A(3,-5), B(1,3)$ are two points in a plane and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$. Find
   (i) the co – ordinates of C
   (ii) the value of $|\overrightarrow{AC}|$, correct to two significant figures

2. $PQRS$ is a quadrilateral with $P(2,2)$, $S(4,4)$ and $R(6,4)$. If $\overrightarrow{PQ} = 4\overrightarrow{SR}$, find the co – ordinates of Q.

3.
   (a) The points $M(2,3), N(5,-2)$ and $T(3,-5)$ are in the $x - y$ rectangular plane.
   If $k\overrightarrow{OM} + l\overrightarrow{ON} = \overrightarrow{MT}$, where k and l are real numbers, calculate the value of
   (i) k (ii) l
   (b) Given that $\overrightarrow{AC} = \begin{pmatrix} -7 \\ 2 \end{pmatrix}$, calculate the
   (i) correct to three significant figures, the length of $\overrightarrow{AC}$
   (ii) bearing of C form A, correct to the nearest degree.

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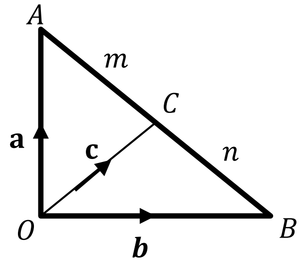
THE POSITION VECTOR OF A POINT THAT DIVIDES A GIVEN STRAIGHT LINE IN A GIVEN RATIO

1. **Internal Division**
   Suppose the point C divides the line AB internally in the ratio $m : n$.
   Let O be a reference point.



$|\overrightarrow{AC}| : |\overrightarrow{CB}| = m : n$

$\frac{|\overrightarrow{AC}|}{|\overrightarrow{CB}|} = \frac{m}{n}$

$n|\overrightarrow{AC}| = m|\overrightarrow{CB}|$

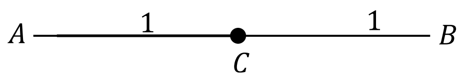
Since $\overrightarrow{AC}$ and $\overrightarrow{CB}$ are in the same direction.

$$n\overrightarrow{AC} = m\overrightarrow{CB}$$
$$n(\overrightarrow{OC} - \overrightarrow{OA}) = m(\overrightarrow{OB} - \overrightarrow{OC})$$
$$n\overrightarrow{OC} - n\overrightarrow{OA} = m\overrightarrow{OB} - m\overrightarrow{OC}$$
$$n\overrightarrow{OC} + m\overrightarrow{OC} = m\overrightarrow{OB} + n\overrightarrow{OA}$$
$$\overrightarrow{OC}(n + m) = m\overrightarrow{OB} + n\overrightarrow{OA}$$
$$\overrightarrow{OC} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$$
$$\overrightarrow{OC} = \frac{n\overrightarrow{OA} + m\overrightarrow{OB}}{m+n}$$
$$\boldsymbol{c} = \frac{n\boldsymbol{a} + m\boldsymbol{b}}{m+n}$$

Where a, b and c are the position vectors of A, B and C respectively.

DEDUCTION

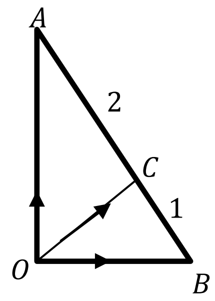
1. If C is the midpoint of AB, then:



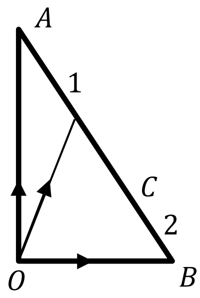
If $\lambda = \mu$, then $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$

2. **Point of Trisection**

(i) If



(ii)



If C is a point of trisection of AB near B, then,

$\boldsymbol{c} = \frac{2\boldsymbol{b}+\boldsymbol{a}}{2+1} = \frac{1}{3}(2\boldsymbol{b} + \boldsymbol{a})$

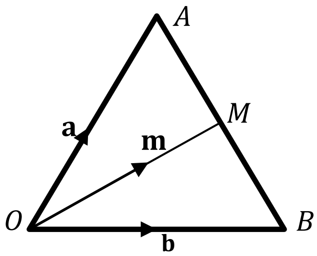
(ii) If C is a point of trisection of B, then A

$\boldsymbol{c} = \frac{2\boldsymbol{a}+\boldsymbol{b}}{2+1} = \frac{1}{3}(2\boldsymbol{a} + \boldsymbol{b})$

Example 8

A point A has position vector $\boldsymbol{a}$ and B has position vector $\boldsymbol{b}$. Prove that M, the midpoint of AB has position vector $\frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})$

Solution…



$|\overrightarrow{AM}| : |\overrightarrow{MB}| = 1:1$ since M is the midpoint.

$\frac{|\overrightarrow{AM}|}{|\overrightarrow{MB}|} = \frac{1}{1}$

$\overrightarrow{AM} = \overrightarrow{MB}$

$\overrightarrow{OM} - \overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{OA}$

$\boldsymbol{m} - \boldsymbol{a} = \boldsymbol{b} - \boldsymbol{m}$

$\boldsymbol{m} + \boldsymbol{m} = \boldsymbol{a} + \boldsymbol{b}$

$2\boldsymbol{m} = \boldsymbol{a} + \boldsymbol{b}$

$\boldsymbol{m} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})$ as required.

Exercise 14 **Date:……………………..**

1. The points O, A and B have coordinates $(0, 0)$, $(5, 0)$ and $(-1, 4)$ respectively. Write as column vectors.
   (a) $\overrightarrow{OB}$
   (b) $\overrightarrow{OA} + \overrightarrow{OB}$
   (c) $\overrightarrow{OA} - \overrightarrow{OB}$
   (d) $\overrightarrow{OM}$ where M is the mid – point of AB.

2. Given that $\overrightarrow{OP} = \begin{pmatrix}3\\2\end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix}0\\4\end{pmatrix}$ and that M is the mid – point of PQ, express as column vectors
   (a) $\overrightarrow{PQ}$
   (b) $\overrightarrow{PM}$
   (c) $\overrightarrow{OM}$

3. $A(4, 7)$ is the vertex of triangle ABC. $\overrightarrow{BA} = \begin{pmatrix}5\\3\end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix}4\\-3\end{pmatrix}$.
   (a) Find the co – ordinates of B and C
   (b) If M is the midpoint of the line $\overrightarrow{BC}$, find $\overrightarrow{AM}$.

4. $P(7, 4)$ is a vertex of triangle PQR. If $\overrightarrow{QP} = \begin{pmatrix}3\\5\end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix}-3\\4\end{pmatrix}$, find
   (i) the co – ordinates of Q and R
   (ii) $\overrightarrow{PN}$, where N is the midpoint of $\overrightarrow{QR}$

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Exercise 15 **Date:……………………..**

1. Given that $P(2, -3)$ is a vertex of a triangle PQR, $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{RP} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$,
   (i) Find
   α) the co – ordinates of Q and R
   β) $|\overrightarrow{QR}|$
   (ii) If M is the midpoint of $\overrightarrow{PR}$, find $\overrightarrow{MQ}$.

2. In triangle ABC, $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$. If P is the midpoint of $\overrightarrow{AB}$, express $\overrightarrow{CP}$ as a column vector.

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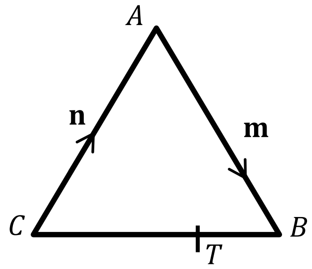
Exercise 16 Date:.........................

1. If triangle ABC is an isosceles triangle such that $\overrightarrow{AB} = \overrightarrow{BC}$ and M is a point on $\overrightarrow{AC}$ such that $|AM|:|MC| = 1 : 1$. Show that $\overrightarrow{BA} + \overrightarrow{BC} = 2\overrightarrow{BM}$.

2. In the diagram, $\overrightarrow{AC} = \boldsymbol{n}$, $\overrightarrow{AB} = \boldsymbol{m}$ and T is such that $|CT|:|TB| = 2:1$.



(i) Find, in terms of $\boldsymbol{n}$ and $\boldsymbol{m}$, $\overrightarrow{BC}$.
(ii) Show that $\overrightarrow{TA} = -\frac{1}{3}(2\boldsymbol{m} + \boldsymbol{n})$

3. $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ are vectors in the same plane. A is the point $(1, 2)$.
   (i) Find the co – ordinates of B and C
   (ii) If D is the midpoint of BC, show that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$.

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Exercise 17 **Date:……………………..**

1. Points A and B are position vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Find the position vector of the point M on AB such that $|AM : |MB| = 3 : 2$.

2. Given that $\boldsymbol{p} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \boldsymbol{q} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, find
   (a) $4p - 2q + 5r$
   (b) The position vector which divides $\boldsymbol{p}$ and $\boldsymbol{q}$ in the ratio $2 : 3$.
3.
   (a) Given that $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \boldsymbol{q} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$, find, $|6\boldsymbol{p} - 3\boldsymbol{q} - 4\boldsymbol{r}|$.
   (b) If $\boldsymbol{m} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \boldsymbol{n} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 5 \\ 17 \end{pmatrix}$, find $|3\boldsymbol{m} - 2\boldsymbol{n} - \boldsymbol{r}|$.
   (c) S is the midpoint of the side QR of a triangle PQR. If $\overrightarrow{PQR}$. If $\overrightarrow{PR} = \boldsymbol{u}$ and $\overrightarrow{PQ} = \boldsymbol{v}$, express $\overrightarrow{PS}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$.

4. $A(3, 1), B(-1, 9)$ and $C(1, -2)$ are the vertices of the triangle ABC. E is the point on $\overrightarrow{AB}$ such that $|\overrightarrow{AE}| = \frac{1}{3}|\overrightarrow{EB}|$. Find
   (a) $\overrightarrow{AB}$ (b) $\overrightarrow{AC}$ (c) $\overrightarrow{CE}$

5.
   α) OAB is a triangle, where O is the origin. Let $\boldsymbol{a}, \boldsymbol{b}$ be the position vectors of A and B respectively. If A is a point on AB such that $\overrightarrow{AX} = 2\overrightarrow{XB}$ and Y is the midpoint of $\overrightarrow{OX}$, show that $\overrightarrow{BY} = \frac{1}{6}\boldsymbol{a} - \frac{2}{3}\boldsymbol{b}$.
   β) What is the sum of the following five vectors?
   $3\overrightarrow{OA}, 6\overrightarrow{BZ}, 2\overrightarrow{AO}, \overrightarrow{AB}$ and $5\overrightarrow{OB}$.

PARALLEL VECTORS

If $\boldsymbol{a} = k\boldsymbol{b}$, where k is a scalar (a number) $\neq 0$, then the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel and in the same direction if $k > 0$, but in opposite direction if $k < 0$.

i.e. $|\boldsymbol{a}| = |k| \times |\boldsymbol{b}|$

In particular, the position vectors, $\boldsymbol{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$ are parallel since $\boldsymbol{b}$ can be express in the form $\boldsymbol{b} = k\boldsymbol{a}$, where k is a scalar, i.e. $\begin{pmatrix} 6 \\ 10 \end{pmatrix} = 2\begin{pmatrix} 3 \\ 5 \end{pmatrix} \Leftrightarrow \boldsymbol{b} = 2\boldsymbol{a}$. Since $\boldsymbol{b}$ can be expressed as a scalar multiple of $\boldsymbol{a}$ it follows that $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

Exercise 18 **Date:..........................**

1. Given that the vectors $\boldsymbol{a} = \begin{pmatrix} k \\ 2 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. Find the value of k for which $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$, where O is the origin.

2. If $\boldsymbol{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Find the value of λ if the vector $\boldsymbol{p} + \lambda\boldsymbol{q}$ is parallel to $\boldsymbol{r}$.

3. The position vectors of A, B and C are $\boldsymbol{a} + 2\mu\boldsymbol{b}$, $\mu\boldsymbol{a} - \boldsymbol{b}$ and $2\boldsymbol{a} - 3\boldsymbol{b}$ respectively. If $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$, where O is the origin, find the value of μ.

Exercise 19 **Date:..........................**

1. A, B, C and D are four points such that $A(-3, 2), C(6, 3), \overrightarrow{AB} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$
   (a) Calculate
      (i) The coordinates of B and D
      (ii) The vectors $\overrightarrow{BC}$ and $\overrightarrow{AD}$
   (b) What is the relationship between $\overrightarrow{BC}$ and $\overrightarrow{AD}$?

2. $P(6, 4), Q(-2, -2)$ and $R(4, -6)$ are the vertices of triangle PQR.
   (i) Determine the co – ordinates of M and S, the midpoints $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ respectively.
   (ii) Find $\overrightarrow{QR}$ and $\overrightarrow{MS}$
   (iii) State the relationship between $\overrightarrow{QR}$ and $\overrightarrow{MS}$
   (iv) Find the equation of $\overrightarrow{MS}$

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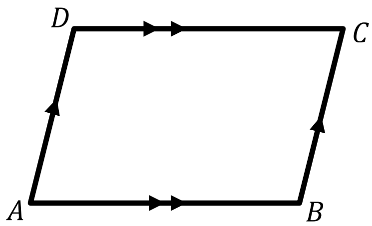
Exercise 20 **Date:……………………..**

1. Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and equal to half of it.

2. In a triangle ABC, X lies on $\overrightarrow{AB}$ such that $|\overrightarrow{AX}| : |\overrightarrow{XB}| = 1 : 2$ and Y lies on $\overrightarrow{AC}$ such that $|\overrightarrow{AY}| : |\overrightarrow{YC}| = 1 : 2$. Show that $\overrightarrow{XY}$ is parallel to $\overrightarrow{BC}$ and one – third its length.

QUADRILATERALS

(i) Parallelogram

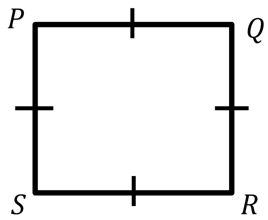


If $ABCD$ is a parallelogram then in terms of vectors:

$$\overrightarrow{AB} = \overrightarrow{DC} \text{ and } \overrightarrow{AD} = \overrightarrow{BC}$$

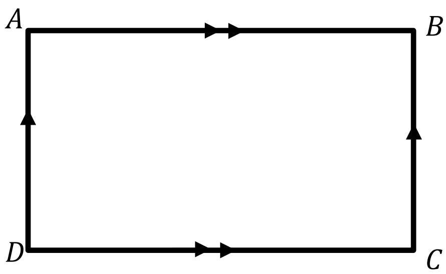
(ii) Square

It is a parallelogram with all the sides and angles equal. Each is 90°.



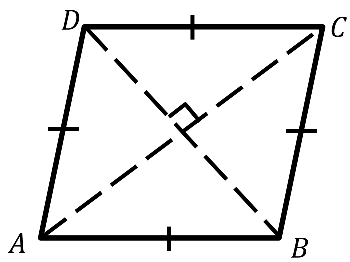
(iii) Rectangle

A rectangle is a parallelogram with each angle being 90°.



(iv) Rhombus

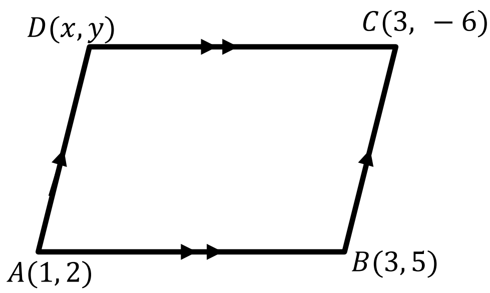
A rhombus is a parallelogram with all the sides equal. The diagonals bisect and meet at right angles. The opposite angles are equal.



Example 9

$A(1, 2), B(3, 5), C(3, -6)$ and $D(x, y)$ are the vertices of the parallelogram $ABCD$. Find the co – ordinates of D.

Solution…



Since $ABCD$ is a parallelogram

$\Longrightarrow \overrightarrow{AB} = \overrightarrow{DC}$

$\Longrightarrow \overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OD}$

$\begin{pmatrix}3\\5\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\-6\end{pmatrix} - \begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}3-x\\-6-y\end{pmatrix}$

$\Longrightarrow 2 = 3 - x \qquad \therefore x = 1$

$\Longrightarrow 3 = -6 - y \qquad \therefore y = -9$

$\therefore D(x, y) = D(1, -9)$

Exercise 21 **Date:..........................**

1. $P(-1,1), Q(1,3), R(m,n)$ and $S(3,-3)$ are the vertices of the parallelogram $PQRS$. Calculate the values of m and n.

2. Show that $A(-2,1), B(1,2), C(0,-1)$ and $D(-4,-2)$ are the vertices of a parallelogram.

3. $ABCD$ is a parallelogram such that $\overrightarrow{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$. Find
   (i) $\overrightarrow{BC}$ (ii) $\overrightarrow{BD}$

Exercise 22 **Date:..........................**

1. A rhombus $ABCD$ has A at $(0, 2)$ and B at $(5, 3)$. If the diagonals intersect at $(0, 3)$, find the co – ordinates of C and D.

2. The co – ordinates of the vertices of a parallelogram $WXYZ$ are $W(1, 6)$, $X(2, 2)$, $Y(5, 4)$ and $Z(a, b)$. Find
   (i) $\overrightarrow{WX}$
   (ii) $\overrightarrow{ZY}$
   (iii) The co – ordinates of Z

3. $A(-2, 3), B(2, -1), C(5, 0)$ and $D(x, y)$ are the vertices of the parallelogram $ABCD$.
   (a) Find $\overrightarrow{AB}$ and $\overrightarrow{DC}$. Hence find the coordinates of D
   (b) Calculate, correct to one decimal place, $|\overrightarrow{DB}|$.

4. In the parallelogram OABC, M is the mid – point of AB and N is the mid – point of BC.
   If $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$, express in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.
   (a) $\overrightarrow{CA}$ (b) $\overrightarrow{ON}$ (c) $\overrightarrow{NM}$
   Describe the relationship between CA and NM.

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Exercise 23 **Date:..........................**

1. The points P, Q, R and S are vertices of a parallelogram in the Cartesian plane. The co – ordinates of P and R are $(-8, 2)$ and $(5, -2)$ respectively and $\overrightarrow{QR} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$. Find
   (i) the co – ordinates of Q and S
   (ii) the magnitude of $\overrightarrow{PR}$

2. The coordinates of the vertices of a parallelogram $QRST$ are $Q(1, 6), R(2, 2), S(5, 4)$ and $T(x, y)$.
   (i) Find the vectors $\overrightarrow{QR}$ and $\overrightarrow{TS}$ and hence determine the values of x and y
   (ii) Calculate the magnitude of $\overrightarrow{RS}$
   (iii) Express $\overrightarrow{RS}$ in the form $(k, \theta°)$ where k is the magnitude and θ, the bearing.

3. The points $A(4, 7), B(x, y), C(-5, -8)$ and $D(1, 4)$ are the vertices of a parallelogram. Find
   (i) $\overrightarrow{AB}, \overrightarrow{DC}, \overrightarrow{DA}$
   (ii) The values of x and y

4. If $A92, 6), B(2, 2), C(7, 3)$ and $D(x, y)$ are the vertices of a parallelogram $ABCD$. Calculate the coordinates of D.

5. Two vectors p and q are defined by $\boldsymbol{p} = \begin{pmatrix} 5\cos x \\ 5\sin x \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 2\cos x \\ 2\sin x \end{pmatrix}$.
   (a) If $\boldsymbol{p} + \boldsymbol{q} = \begin{pmatrix} 4.690 \\ 5.208 \end{pmatrix}$, find to the nearest whole number, the value of x, where x is acute.
   (b) Find the value of $\boldsymbol{c}$ if $\boldsymbol{c} = 2\boldsymbol{p} + \boldsymbol{q}$.

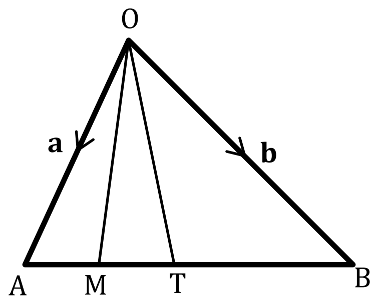
6. The position vectors of points P, Q and R with respect to the origin are $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ respectively. If $PQRM$ is a parallelogram, find:
   (a) The position vector of M;
   (b) $|\overrightarrow{PM}|$, and $|\overrightarrow{PQ}|$.

7. Given that $\boldsymbol{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix} 1 \\ q \end{pmatrix}$, where q is a scalar and $2|\boldsymbol{m} - \boldsymbol{n}| = |\boldsymbol{m} + \boldsymbol{n}|$,

8.
   α) The position vectors of A and B are $\boldsymbol{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ respectively. Find, correct to two decimal places, $|4\boldsymbol{a} - 2\boldsymbol{b}|$.

   β) The position vectors of A and B are $\boldsymbol{a} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.
   Find:
   (a) The scalars m and n such that $m\boldsymbol{a} + n\boldsymbol{b} = \begin{pmatrix} 11 \\ 19 \end{pmatrix}$
   (b) $|2\boldsymbol{a} + 7\boldsymbol{b}|$

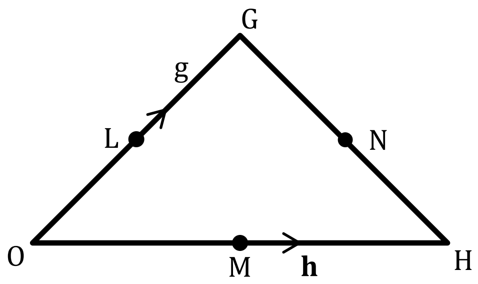
9.



   (a) In the diagram, T is the mid – point of AB and M is the mid – point of AT. Given that $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$, express as simply as possible in terms of **a** and **b**,
   (i) **AB** (iii) **OM**
   (ii) **AM**
   (b) Two points P and Q have position vectors **p** and **q** respectively, relative to the origin O. Given that $\mathbf{p} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{PQ} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, find
   (i) **q**
   (ii) **|PQ|**
   (iii) The coordinates of the point R, which is such that $\mathbf{OR} = \mathbf{QP}$.

   (c) Given also that $\mathbf{s} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{t} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ and $l\mathbf{p} + m\mathbf{s} = \mathbf{t}$, write down two simultaneous equations in l and m, and solve them.

10.



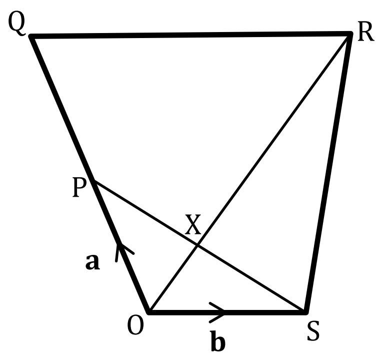
In the triangle OGH the midpoints of OG, OH and GH are L, M and N respectively. $\mathbf{OG} = \mathbf{g}$, $\mathbf{OH} = \mathbf{h}$

(i) Write down expression, in terms of **g** and **h**, for
 (a) **OL**
 (b) **GN**
 (c) **ON**
 (d) **OL + OM + ON**

(ii)
 (a) Use the vector equation $\mathbf{OG} + \mathbf{GM} = \mathbf{OM}$ to express **GM** in terms of **g** and **h**.
 (b) By a similar method, express **HL** in terms of g and h.
 (c) Hence obtain an expression from **GM + HL + ON** and simplify it.

(iii)
 (a) Use your results from (ii) to express **GM – HL** in terms of **g** and **h**.
 (b) K is a point, not shown on the diagram, such that $\mathbf{OK} = \mathbf{GM} - \mathbf{HL}$. What can you say about **OK** and **GH**?

11.



In the diagram $\mathbf{OP} = \mathbf{a}$ and $\mathbf{OS} = \mathbf{b}$.

(i) Express **SP** in terms of **a** and **b**.

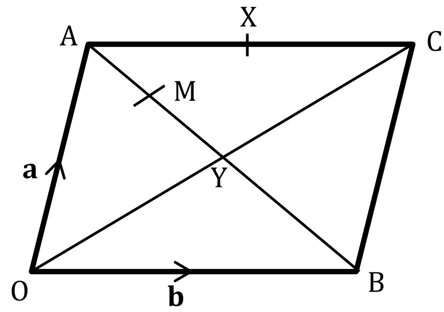
(ii) Given that $\mathbf{SX} = h\mathbf{SP}$, show that $\mathbf{OX} = h\mathbf{a} + (\mathbf{1} - h)\mathbf{b}$.

(iii) Given that $\mathbf{OQ} = 3\mathbf{a}$ and $\mathbf{QR} = 2\mathbf{b}$, write down an expression for **OR** in terms of **a** and **b**.

(iv) Given that $\mathbf{OX} = k\mathbf{OR}$ use the results of parts (ii) and (iii) to find the values of h and k.

(v) Find the numerical value of the ratio $\frac{PX}{XS}$.

12.



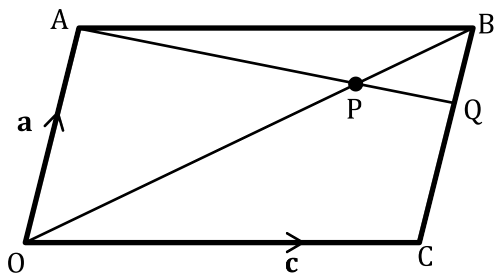
In the diagram $\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$. OACB is a parallelogram, X is the mid – point of **AC**, and M is the point on **AB** such that $\mathbf{AM} = \frac{1}{3}\mathbf{AB}$.

(i) Express the following vectors in terms of **a** and **b** as simply as possible: **OX**, **AB**, **AM**, **OM**.

(ii) L is the point on **OX** such that $\mathbf{OL} = \frac{2}{3}\mathbf{OX}$. Express the vector **OL** in terms of **a** and **b**. what can you now say about L and M?

(iii) **AB** meets **OC** at Y. Express the vectors **OY** and **AY** in terms of **a** and **b**, and hence, or otherwise show that $\mathbf{AM} : \mathbf{AY} = \mathbf{OM} : \mathbf{OX}$.

13.



In the parallelogram OABC, $\mathbf{OP} = \frac{3}{4}\mathbf{OB}$ and **APQ** is a straight line.
$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.

(i) Find **OB, OP** and **AP** in terms of **a** and **c**.

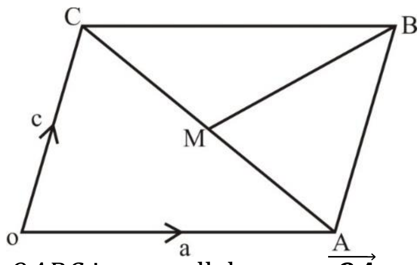
(ii) By writing **OQ** as $\mathbf{OA} + x\mathbf{AP}$ express **OQ** in terms of **a**, **c** and x.

(iii) By writing **OQ** as $\mathbf{OC} + y\mathbf{CB}$ express **OQ** in terms of **a, c** and y.

(iv) Find the value of x which makes the terms in **c** equal in the two expressions for **OQ.** Hence find the value of y.

(v) Use the value of y to find $\frac{\mathrm{CQ}}{\mathrm{QB}}$.

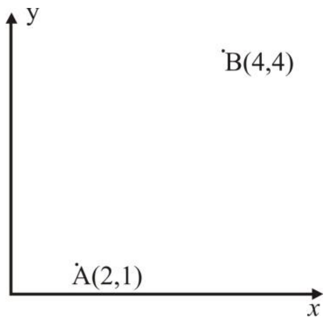
14.



$OABC$ is a parallelogram. $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OC} = \boldsymbol{c}$ and M is the midpoint of $\boldsymbol{CA}$. Find m terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

(a) $\overrightarrow{OB}$ (b) $\overrightarrow{CA}$ (c) $\overrightarrow{BM}$

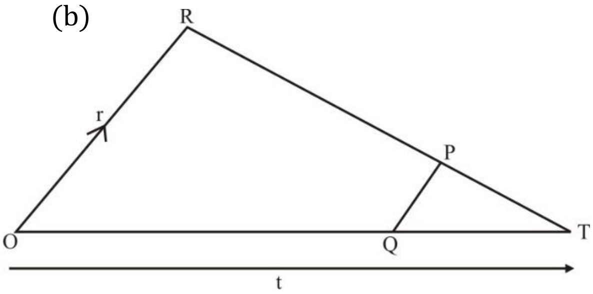
15.



(a) (i) Write down $\overrightarrow{AB}$ as a column vector.

(ii) $\overrightarrow{AC} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$. Work out $\overrightarrow{BC}$ as a column vector

(b)



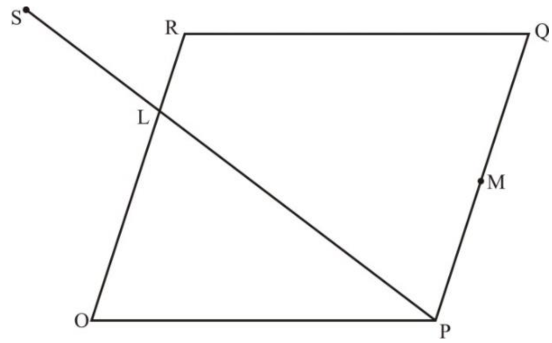
$\overrightarrow{OR} = \boldsymbol{r}$ and $\overrightarrow{OT} = \boldsymbol{t}$. P is on RT such that $RP : PT = 2 : 1$. Q is on OT such that $\boldsymbol{RP} : \boldsymbol{PT} = \mathbf{2} : \mathbf{1}$, Q is on $\boldsymbol{OT}$ such that $OQ = \frac{2}{3}OT$.

Write the following in terms of $\boldsymbol{r}$ and / or $\boldsymbol{t}$. Simplify your answers where possible .

(i) $\overrightarrow{QT}$ (ii) $\overrightarrow{TP}$ (iii) $\overrightarrow{QP}$

(iv) Write down two conclusions you can make about the line segment $\boldsymbol{QP}$.

16.



$OPQR$ is a parallelogram . O is the origin. $\overrightarrow{OP} = \boldsymbol{p}$ and $\overrightarrow{OR} = \boldsymbol{r}$. M is the mid-point of PQ and PQ and L is on OR such that $\boldsymbol{OL} : \boldsymbol{LR} = \mathbf{2} : \mathbf{1}$. The line PL is extended to the point $\boldsymbol{S}$.

(a) Find, in terms of $\boldsymbol{p}$ and $\boldsymbol{r}$, in their simplest terms,

(i) $\overrightarrow{OQ}$

(ii) $\overrightarrow{PR}$

(iii) $\overrightarrow{PL}$

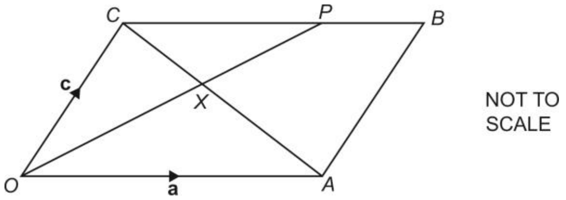
(iv) The position vector of $\boldsymbol{M}$

(b) PLS is a straight line and $PS = \frac{3}{2}PL$.
Find, in terms of $\boldsymbol{p}$ and / or $\boldsymbol{r}$, in their simplest forms,

(i) $\overrightarrow{PS}$ (ii) $\overrightarrow{QS}$

(c) What can you say about the points Q, R and S?

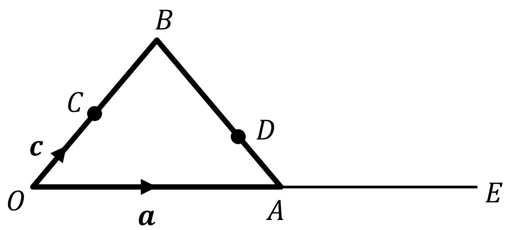
17.



In the diagram $OABC$ is a parallelogram. OP and CA intersect at X and $CP : PB = 2 : 1$. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$.

(a) Find $\overrightarrow{OP}$, in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$, in its simplest form.
(b) $CX : XA = 2 : 3$.
  (i) Find $\overrightarrow{OC}$, in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$, in its simplest form.
  (ii) Find $OX : XP$.

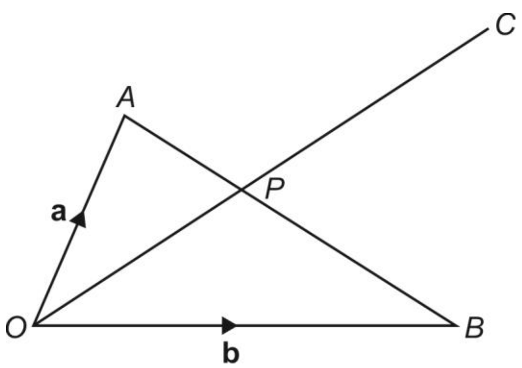
18.
(a) $\boldsymbol{a} = \begin{pmatrix}-3\\2\end{pmatrix}, \boldsymbol{b} = \begin{pmatrix}5\\4\end{pmatrix}, \boldsymbol{c} = \begin{pmatrix}14\\9\end{pmatrix}$
  (i) Find $3\boldsymbol{a} - 2\boldsymbol{b}$
  (ii) Find $|\boldsymbol{a}|$.
  (iii) $m\boldsymbol{a} + n\boldsymbol{b} = \boldsymbol{c}$, find the value of m and n
(b)



OAB is a triangle and C is the midpoint of OB. D is on AB such that $AD : DB = 3 : 5$. OAE is a straight line such that $OA : AE = 2 : 3$. $\overrightarrow{OA} = \boldsymbol{a}$ and $OC = \boldsymbol{c}$.
(i) Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$, in its simplest form,
  (a) $\overrightarrow{AB}$ (c) $\overrightarrow{CE}$
  (b) $\overrightarrow{AD}$ (d) $\overrightarrow{CD}$

(ii) $\overrightarrow{CE} = k\overrightarrow{CD}$. Find the value of k.

19.



In the diagram, O is the origin and P lies on $\overrightarrow{AB}$ such that $\overrightarrow{AP} : \overrightarrow{PB} = 3 : 4$, $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

(i) Find $\overrightarrow{OP}$, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, in its simplest form.
(ii) The line OP is extended to C such that $\overrightarrow{OC} = m\overrightarrow{OP}$ and $\overrightarrow{BC} = k\boldsymbol{a}$. Find the value of m and the value of k.

Exercise 24 **Date:……………………..**

1. The vertices of a quadrilateral, $OABC$, are $(0,0), (4,2), (6,10)$ and $(2,8)$ respectively.
Use a vector method to answer the questions which follow
  (a) Write as a column vector, in the form $\begin{pmatrix}x\\y\end{pmatrix}$, the vector
    (i) $\overrightarrow{OA}$ (ii) $\overrightarrow{CB}$

  (b) Calculate $|\overrightarrow{OA}|$, the magnitude of $\overrightarrow{OA}$
  (c)
    (i) State two geometrical relationships between the line segments OA and CB.
    (ii) Explain why $OABC$ is a parallelogram.

  (d) If M is the midpoint of the diagonal OB, and N is the midpoint of the diagonal AC, determine the position vector,
    (i) $\overrightarrow{OM}$ (ii) $\overrightarrow{ON}$

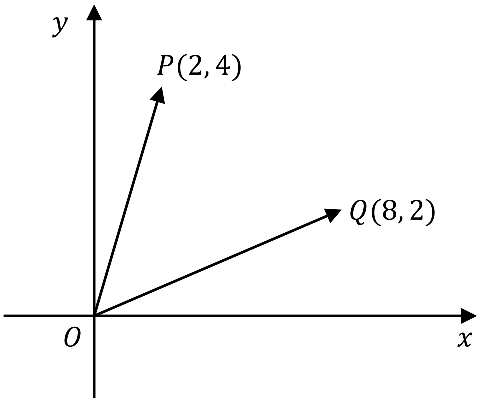
    Hence, state one conclusion which can be made about the diagonals of the parallelogram $OABC$.

2. $\overrightarrow{OM}$ and $\overrightarrow{ON}$ are position vectors with respect to the origin O, such that $\overrightarrow{OM} = \boldsymbol{m}$ and $\overrightarrow{ON} = \boldsymbol{n}$. L is a point on MN such that $ML : LN = 2 : 1$.
  (i) Draw a sketch of the triangle OMN and label the points O, M, N and L.
  (ii) Write in terms of $\boldsymbol{m}$ and $\boldsymbol{n}$, an expression for:
    (a) $\overrightarrow{MN}$ (b) $\overrightarrow{ML}$

  (iii) If $\boldsymbol{m} = \begin{pmatrix}3\\6\end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix}9\\0\end{pmatrix}$, determine the position vector of L.

3. In the diagram below, the coordinates of P and Q are $(2, 4)$ and $(8, 2)$ respectively. The line segment joining

the $(0, 0)$ to the point P may be written as $\overrightarrow{OP}$.



(i) What form is used to describe $\overrightarrow{OP}$?

(ii) Write each of the following in column form

(a) $\overrightarrow{OP}$ (c) $\overrightarrow{PQ}$

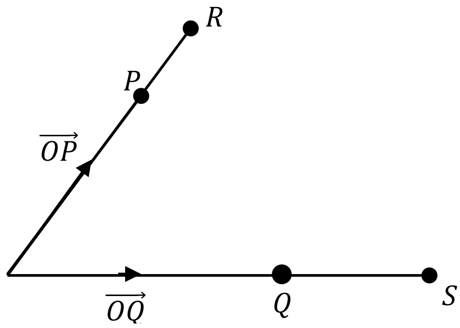
(b) $\overrightarrow{OQ}$

(iii) Given that $\overrightarrow{OP} = \overrightarrow{RQ}$, determine the co – ordinates of the point, R.

(iv) State the type of quadrilateral formed by $PQRO$. Justify your answer.

4.

(a) The position vectors of the points P and Q relative to an origin, O, are $\overrightarrow{OP} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ respectively. The diagram shows that $\overrightarrow{PR} = 30P$ and $\overrightarrow{QS} = 30Q$.



(i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, vector

α) $\overrightarrow{OS}$ β) $\overrightarrow{PQ}$ γ) $\overrightarrow{RS}$

(ii) State two geometrical relationship between PQ and RS.

(b) Given $\overrightarrow{OA} = \boldsymbol{a}, \overrightarrow{OB} = \boldsymbol{b}, \overrightarrow{AP} = \frac{1}{2}\overrightarrow{OA}$ where $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(i) Write $\overrightarrow{BP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Find $|\overrightarrow{BP}|$

(c) The position vectors of points A, B and C, relative to the origin O, are $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$; $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$ respectively.

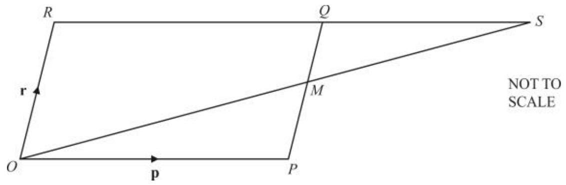
(i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, the vector

α) $\overrightarrow{AB}$ β) $\overrightarrow{AC}$

(ii) Hence, determine whether A, B and C are collinear, giving the reasons for your answer.

Exercise 25 **Date:..........................**

1.

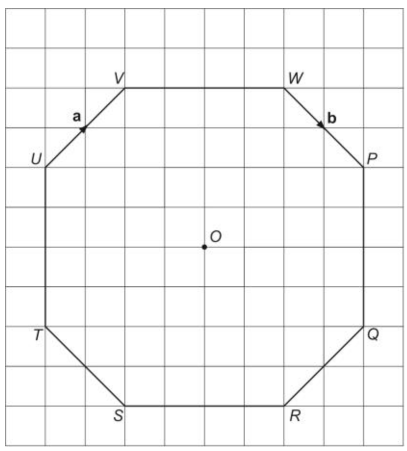


$OPQR$ is a parallelogram, with O the origin. M is the midpoint of PQ. OM and RQ are extended to meet at S.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{r}$.

(a) Find, in terms of $\boldsymbol{p}$ and $\boldsymbol{r}$, in its simplest form,
  (i) $\overrightarrow{OM}$
  (ii) The position vector of S.

(b) When $\overrightarrow{PT} = -\frac{1}{2}\boldsymbol{p} + \boldsymbol{r}$, what can you write down about the position of T?

2.



The origin O is the centre of the octagon $PQRSTUVW$. $\overrightarrow{UV} = \boldsymbol{a}$ and $\overrightarrow{WP} = \boldsymbol{b}$.

(a) Write down in terms of a and b.
  (i) $\overrightarrow{VW}$
  (ii) $\overrightarrow{TU}$
  (iii) $\overrightarrow{TP}$
  (iv) The position vector of the point P.

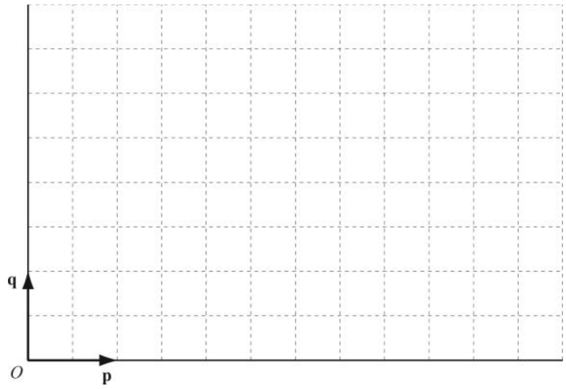
(b) In the diagram, 1 centimetre represents 1 unit. Write down the value of $|\boldsymbol{a} - \boldsymbol{b}|$.

3. The position vector r is given by $\boldsymbol{r} = 2\boldsymbol{p} + \boldsymbol{t}(\boldsymbol{p} + \boldsymbol{q})$.

(a) Complete the table below for the given values of t. Write each vector in its simplest form. One result has been done for you.

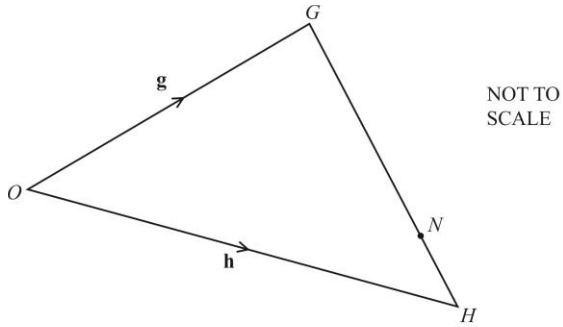
t	0	1	2	3
r			$4\boldsymbol{p} + 2\boldsymbol{q}$	

(b) O is the origin and p and q are shown on the diagram.
  (i) Plot the 4 points given by the position vectors in the table.



  (ii) What can you say about these four points?

4.

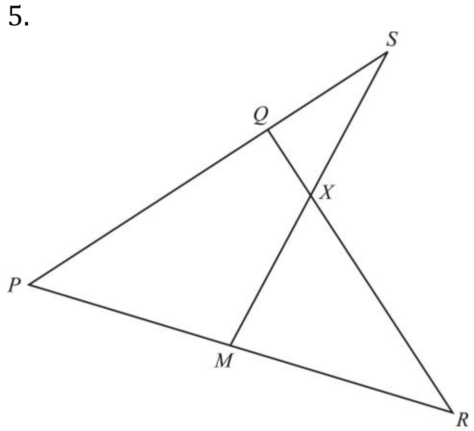


In triangle OGH, the ratio $GN : NH = 3 : 1$
$\overrightarrow{OG} = \boldsymbol{g}$ and $\overrightarrow{OH} = \boldsymbol{h}$.

Find the following in terms of $\boldsymbol{g}$ and $\boldsymbol{h}$, giving your answers in their simplest form.

(a) $\overrightarrow{HG}$ (b) $\overrightarrow{ON}$

5.



NOT TO SCALE

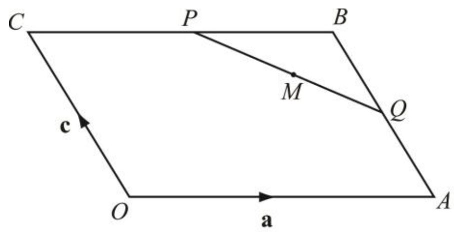
In the diagram PQS, PMR, MXS and QXR are straight lines.
$PQ = 2QS$.

M is the midpoint of PR.
$QX : XR = 1 : 3$

$\overrightarrow{PQ} = \boldsymbol{q}$ and $\overrightarrow{PR} = \boldsymbol{r}$.

(a) Find, in terms of q and r,
 (i) $\overrightarrow{RQ}$ (ii) $\overrightarrow{MS}$
(b) By finding $\overrightarrow{MX}$, show that X is the midpoint of MS.

6.

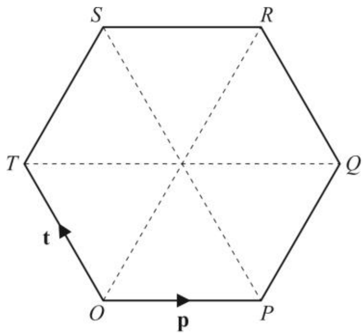


NOT TO SCALE

O is the origin and $OABC$ is a parallelogram. $CP = PB$ and $AQ = QB$.

$\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$. Find in terms of a and c, in their simplest form,
(a) $\overrightarrow{PQ}$
(b) The position vector of M, where M is the midpoint of PQ.

7.



O is the origin and $OPQRST$ is a regular hexagon.
$\overrightarrow{OP} = \boldsymbol{p}$ and $\overrightarrow{OT} = \boldsymbol{t}$.

Find, in terms of p and t, in their simplest forms.
(a) $\overrightarrow{PT}$
(b) $\overrightarrow{PR}$
(c) The position vector of R.