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RATIO AND RATES

A ratio is a comparison which expresses one as a fraction of the other.

A ratio has no units, since it is a comparison of similar quantities.

NOTE
To express two or more quantities as ratio, their units must be the same.

Comparing Two Quantities in the Form $x: y$
Suppose x and y be two quantities. The fraction telling us how many times bigger or smaller x is than y is called **the ratio of x to y** written as $x: y$ or $\frac{x}{y}$.

Example 1
Express 12km and 18km as a ratio.

Solution…
$\frac{12}{18} = \frac{2}{3}$
$\therefore 12: 18 = 2: 3$

Exercise 1 Date:……………………

1. A mother is 40 years old and her son is 12 years old. Find the ratio of their ages.

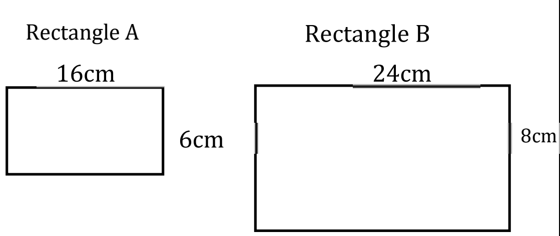
2. The distance from Accra to Kumasi is 270km, and the distance from Accra to Cape Coast is 144km. find the ratio of the distances.

3. Kofi and Abena share 300 oranges in such a way that Kofi receives 250 oranges. Find the ratio of their shares.

4. Two items cost GH¢50.00 and GH¢35.00 respectively. Find the ratio of their prices.

5. In a class, the tallest person is 1.8m and the shortest 1.5m. Find the ratio of their heights.

Exercise 2 Date:……………………
Look at these two rectangles.



Express the following as ratios.
i) length of rectangle A to length of rectangle B.
ii) width of rectangle A to width of rectangle B.
iii) perimeter of rectangle A to perimeter of rectangle B.
iv) area of rectangle A to area of rectangle B.

Exercise 3 Date:……………………
The table gives the mean life expectancy (in years) of some African animals.

Animal	Life Expectancy (years)
Tortoise	120
Parrot	50
Elephant	35
Gorilla	30
Lion	15
Giraffe	10

Find the ratio of the life expectancies of
a) Parrot and tortoise.
b) Lion and tortoise.
c) Lion and gorilla.
d) Elephant and gorilla.
e) Parrot and lion.

Exercise 4 Date:……………………
Simplify the following ratios.
1. 24m : 6m
2. 1.4m : 70cm
3. 400kg : 1 tonne
4. 45km/h : 60km/h
5. 2days : 1 week

Exercise 5 Date:……………………
What is the ratio of:
(a) A millimetre to a centimetre?
(b) A metre to a centimetre?
(c) A litre to a millilitre?
(d) A gram to a kilogram?
(e) A centimetre to a metre?
(f) A minute to an hour?

EQUIVALENT RATIOS
Two ratios $a: b$ and $m: n$ are said to be equivalent if the fraction they represent are equivalent.

i.e. $\boxed{\frac{a}{b} = \frac{m}{n} \Rightarrow a \times n = b \times m}$

Example 2
Find x if $3 : 4 = x : 24$

Solution…
$\frac{3}{4} = \frac{x}{24}$
$x = 24 \times \frac{3}{4} = 18$

Example 3
If $A : B = 2 : 3$ and $B : C = 2 : 3$, find $A : C$.

Solution…
$A : B = 2 : 3$
$\Longrightarrow \frac{A}{B} = \frac{2}{3}$

And $B : C = 2 : 3$
$\Longrightarrow \frac{B}{C} = \frac{2}{3}$

Now, $\frac{A}{C} = \frac{A}{B} \times \frac{B}{C}$
$= \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Exercise 6 **Date:……………………**
Find the values of the letters in the following.
1. $6 : 9 = m : 15$
2. $p : 12 = 3 : 4$
3. $6x : 64 = 18 : 32$
4. $2x : (x + 1) = 3 : 2$
5. $1\frac{1}{2} : x = 24 : 36$
6. $1 : x = \frac{1}{6} : \frac{1}{4}$
7. $\frac{1}{6} : \frac{1}{3} = 2 : y$
8. $n : (n + 3) = 6 : 8$
9. $3x^2 : 8 = 24 : 16$

Exercise 7 **Date:……………………**
1. If $X : Y = 3 : 5$ and $Y : Z = 4 : 7$, find the ratio $X : Z$.
2. If $m : n = \frac{3}{5}$ and $n : p = 2 : 5$, find $m : p$.
3. If $x : y = \frac{1}{5} : \frac{1}{4}$ and $y : z = \frac{3}{7} : \frac{5}{8}$, find $x : z$.
4. If $a : b = 3 : 2$, find ab when $a = 9$
5. If $a : b = 4 : 5$, find $a + b : 2a - b$.
6. $p : q = 2 : 3$, find $\frac{p^2 + 2q^2}{q^2 - p^2}$.
7. If $x : y = 3 : 2$ and $y : z = 5 : 4$, find the value of x in $x : y : z$.
8. If four numbers a, b, c, d are in the ratio $5 : 4 : 3 : 2$, find the value of $\frac{6a - 3d}{5b + 2c}$.
9. If $\frac{3p + 4q}{3p - 4q} = 2$, find $p : q$.
10. If $m : n = 2 : 1$, evaluate $\frac{3m^2 - 2n^2}{m^2 + mn}$.

Example 4
A man shares GH¢4,800.00 to his three children in the ratio 3: 4: 5. How much does each receive?

Solution…
Sum of ratio $= 3 + 4 + 5 = 12$

Let the three children be A, B, C

$\therefore$ A : B : C = 3 : 4 : 5

A's share $= \frac{3}{12} \times 4{,}800 =$ GH¢1,200.00

B's share $= \frac{4}{12} \times 4{,}800 =$ GH¢1,600.00

C's share $= \frac{5}{12} \times 4{,}800 =$ GH¢2,000.00

Exercise 8 **Date:……………………**
Divide
1. 200 in the ratio 1 : 4
2. 1500 in the ratio 4 : 1
3. 60 in the ratio 3: 12
4. 45 minutes in the ratio 2 : 3
5. 1 hour in the ratio 1 : 5
6. GH¢2,300 in the ratio 1 : 2 : 7

Exercise 9 **Date:……………………**
1. GH¢200 is to be shared amongst Patience, Elizabeth and Abigail in the ratio 3: 4 : 5. How much will each child receive?
2. The angles of a triangle are in the ratio 3: 5: 4. Calculate the size of each angle.
3. The size of the angles of a quadrilateral are in the ratio 1 : 2 : 3 : 3. Calculate the size of each angle.

Exercise 10 **Date:……………………**
1. 45% of student in a school are boys.
   (a) What is ratio of boys to girls?
   (b) How many boys and how many girls are there if the school has 800 students?

2. An amount of GH¢30,000.00 was shared among Ama, Kojo and Esi. Ama received

GH¢6,000.00, Kojo received $\frac{5}{12}$ of the remainder, while the rest went to Esi. In what ratio was the money shared?

Exercise 11 Date:……………………

1. The development budget of a district council includes expenditure on feeder roads, schools and water supply. The expenditure on roads, schools and water supply are in the ratio $7 : 15 : 2$. If the expenditure on roads is GH¢2,800.00, find the expenditure on
   (a) schools
   (b) water supply
   (c) What is the total budget for these three projects?
   (d) The cost of maintaining libraries is GH¢900.00 and it is met from the expenditure on schools. What percentage, correct to 3 significant figures, of the expenditure on schools is spent on maintaining libraries?

Exercise 12 Date:……………………

1. The ratio of the number of boys to the number of girls in a school of 432 pupils is $5 : 4$. If the number boys increases by 12, the new ratio of boys to girls is $7 : 6$. Find the increase in the number of girls.

2. There are 5 more girls than boys in a class. If 2 boys join the class, the ratio of girls to boys will be 5: 4. Find the:
   (i) number of girls in the class;
   (ii) total number of pupils in the class;
   (iii) probability of selecting a boy as the class prefect.

Exercise 13 Date:……………………

1. A man gave out GH¢24,000.00 to his three brothers, X, Y and Z, to be shared among them. If X takes twice as much as Y and Y is given one – third of what Z takes, how much did each of them receive?

2. A man spent $\frac{2}{5}$ of a certain amount on food and shared the remainder between two brothers in the ratio $2 : 3$. If the brother with the smaller share has GH¢6,000.00, what is the value of the amount initially?

3. Kuffour and Yaw contributed GH¢15,000 and GH¢25,000 respectively as capital in a business partnership. Kuffour as Managing Director, is paid 15% of the profit as salary while the remaining profit is shared between them in the ratio of their contributions. If Kuffour received GH¢4,800.00 from the total profit made, how much did Yaw receive?

Example 5

Three partners Ali, Baba and Musa shared their profit in the ratio $7 : 13 : 5$ respectively. At the end of a certain year Baba had GH¢8,400 more than Ali. What was the total profit shared by the three partners?

Solution…

Sum of ratio $= 7 + 13 + 5 = 25$

Let, x be Ali's share
y be the total amount shared

Baba's share $= x + 8{,}400$

Ali's share: $\frac{7}{15} \times y = x$
$25x - 7y = 0$……………………… (1)

Baba's share: $\frac{13}{25} \times y = x + 8{,}400$
$13 = 25(x + 8{,}400)$
$13y - 25x = 210{,}000$…………….. (2)

$(1) + (2)$: $6y = 210{,}000$
$y = 35{,}000$

Put $y = 35{,}000$ into (1)
$25x - 7(35{,}000) = 0$
$25x = 7(35{,}000)$
$x = 9{,}800$

$\therefore$ Total profit shared = GH¢35,000.00

ALTERNATIVELY

A : B : M = 7 : 13 : 5

Sum of ratio $= 7 + 13 + 5 = 25$

Suppose the total profit $= x$

Ali's share $= \frac{7}{25} \times x$

Baba's share $= \frac{13}{25}x$

Baba had GH¢8,400.00 more than Ali.

$$\Rightarrow \frac{13}{25}x = \frac{7}{25}x + 8{,}400$$
$$\Rightarrow 13x = 7x + 210{,}000$$
$$\Rightarrow 13x - 7x = 210{,}000$$
$$6x = 210{,}000$$
$$x = 35{,}000$$

∴ Total profit shared = GH¢35,000.00

PROPORTION

Proportion is an expression which shows that ratios are equal. There are two types of proportion.

i) Direct proportion

ii) Indirect (or inverse) proportion.

DIRECT PROPORTION

Two quantities are said to be direct proportion if they increase or decrease at the same rate.

Example 6

3 pens cost GH¢15.00. What is the cost of 7 such pens?

Solution

Let the cost of 7 pens be x.

$3 \longrightarrow 15$

$7 \longrightarrow x$

If more, less divide.

$$x = \frac{15}{3} \times 7$$
$$= 35$$

∴ 7 such pens cost GH¢35.00.

INDIRECT PROPROTION

Two quantities are said to be inversely proportional if an increase in one quantity causes a decrease in the other and vice versa.

Example 7

It took 21 days for 14 students to complete weeding a school compound. How long do you expect to take 7 students to weed the same compound?

Solution

Let x be the number of days it will take the 7 students to weed the same compound.

$21 \longrightarrow 14$

$7 \longrightarrow x$

If more, less divide.

$x = 21 \times \frac{14}{7}$

$= 42$

It will take the 7 students 42 days to weed the same compound.

Exercise 14 Date:……………………

1. In a fishing village two boxes of matches are exchanged for 7 herrings.
   (i) How many boxes of matches can be exchanged for 98 herrings?
   (ii) How many herring can be obtained for 10 boxes of matches?

2. If the cost of 6 items is GH¢1800.00, find the cost of 10 items.

Exercise 15 Date:……………………

1. A man is paid GH¢500.00 for 10 days work. Find his pay for:
   (i) 3 days (ii) 24 days (iii) x days

2. Six copies of a book cost GH¢12.00. Find the cost of
   (i) 3 copies (iii) 98 copies
   (ii) 5 copies (iv) m copies

Exercise 16 Date:……………………

1. A car uses 20 litres of petrol in travelling 160km along a good road.
   (i) How far will it go on 5 litres?
   (ii) What quantity will be required for a journey of 100km on a good road?

2. A floor is covered by 1600 square tiles each of side length 10cm. How many

squares tiles of side 8cm would be needed to cover the same floor?

Exercise 17 Date:......................

1. It takes 6 students one hour to sweep their school compound. How long will it take 15 students to sweep the same compound?

2. A tin of rice is consumed by n boys in 8 days. When three more boys joined them, the rice lasted only 6 days. Find n, if the rate of consumption is uniformed.

3. Esther, a typist charged 28Gp for the first five sheets and 8Gp for additional sheets she types. How much will she earn, if she types 36 sheets?

Exercise 18 Date:......................

1. A paint manufacturing company has a machine which fills 24 tins with paints in 5 minutes.
   (i) How many tins will the machine fill in
      (α) 1 minute, correct to the nearest whole number?
      (β) 1 hour?

   (ii) How many hours will it take to fill 1440 tins?
2. The ratio of boys to girls in a school is 12 : 25. If there are 120 boys,
   (i) How many girls are in the school?
   (ii) What is the total number of boys and girls in the school?

APPLICATION OF PROPORTION

Example 8

Kofi, Abena and Ayongo shared GH¢540.00 in the ratio 2: 3: 4. Find the amount each received.

Solution

Ratio ≡ Kofi : Abena : Ayongo

≡ 2 : 3 : 4

Sum of ratio $= 2 + 3 + 4 = 9$

Amount shared = GH¢540.00

Kofi's share $= \frac{2}{9} \times 540 =$ GH¢120.00

Abena's share $= \frac{3}{9} \times 540 =$ GH¢180.00

Ayongo's share $= \frac{4}{9} \times 540 =$ GH¢240.00

Example 9

The ratio of sheep to goats on a farm is 7 : 5. If there are 910 sheep, how many goats are there on the farm?

Solution...

Let g be the number of goats on the farm.

$$\text{Sheep} : \text{Goats} = 7 : 5$$
$$910 : g = 7 : 5$$
$$\frac{910}{g} = \frac{7}{5}$$
$$g = \frac{5 \times 910}{7} = 650$$

∴ There are 650 goats on the farm.

Example 10

Three men share GH¢48,000.00 in the ratio 7 : 8 : 9. Find

(a) Greatest share
(b) Least share
(c) The difference between the least and greatest share.

Solution...

Total ratio $= 7 + 8 + 9 = 24$

(a) Greatest share $= \frac{9}{24} \times 48{,}000$
$=$ GH¢18,000.00

(b) Least share $= \frac{7}{24} \times 48{,}000$
$=$ GH¢14,000

(c) Difference between the least share and the greatest share
= GH¢18,000.00 − GH¢14,000.00
= GH¢4,000.00

Example 11

The total enrolment in a school is 432. The ratio of the number of boys to the number of girls is 5 : 4. How many more boys than girls in the school?

Solution...

Total ratio $= 5 + 4 = 9$

Number of boys in the school
$= \frac{5}{9} \times 432 = 240$

Number of girls in the school
$= \frac{4}{9} \times 432 = 192$

The number of boys more than girls in the school $= 240 - 192 = 48$.

Exercise 19 **Date:……………………**

1. A man shares GH¢51,170.00 among his children Dede, Kofi and Abena. Abena has $1\frac{1}{2}$ times as much as Kofi and Kofi has 3 times as much as Dede.
   (i) Find the ratio of their shares
   (ii) Find the amount each received

2. The lengths of the sides of a triangles are in the ratio $4 : 5 : 6$. Find the length of each side if the perimeter is 45.

3. Acquaye, Addo and Atinga shared GH¢240,000.00. Acquaye received twice as much as Addo, and Addo three times as much as Atinga. How much does Addo receive?

4. A brother and sister share 2400 kola nuts in the ratio of $5 : 3$. The brother then shares his kola nuts with two friends in the ratio $3 : 2 : 1$. How many nuts does each friend receive?

5. Three businessmen agreed to enter into partnership to import some spare parts for sale. Daniel invested GH¢45,000.00, Elvis invested GH¢15,000 and Stephen invested GH¢60,000.00. Stephen received a profit of GH¢24,720.00. If the total profit is shared in the ratio of their investments, find
   (a) the total profit
   (b) the profit made by Daniel and Elvis together

6.
   (i) Eight cakes of a brand of soap cost GH54.00. What is the cost of 12 cakes?
   (ii) Pineapples are priced at 3 for GH¢18.00. Find the cost of 78 pineapples.
   (iii) A piece of uniform wire of length 2 metres cost GH¢60.00. Find the cost of a piece of the same wire of length
   (a) 35cm (b) 5 metres

7.
   (i) A petrol tank will take a factory 30 weeks when it uses 150 litres per day. How many weeks will it take the factory if it decides to use 500 litres per day?
   (ii) A car covers 180m in $(t - 1)$seconds and 324m in $(t + 3)$seconds. If it is travelling at a constant speed, calculate the value of t.
   (iii) A car travels 245km at a constant speed in $3\frac{1}{2}$ hours. How far does it travel in 90 minutes?

8. Kofi and Yaw entered into a business partnership with a total capital of ¢81 million. They agreed to contribute the capital in the ratio $2 : 1$ respectively. The profit was shared as follows:
   Kofi was paid 5% of the total profit for his services as manager. Each partner was paid 3% of the capital invested. The remainder of the profit was then shared between them in the ratio of their contributions to the capital. If Kofi's share of the total profit was ¢7.5 million, calculate
   (a) The total profit for the year to the nearest thousand cedis.
   (b) Yaw's share of the profit as a percentage of his contribution to the capital.

9. Three friends Ato, Oko and Edem entered into a business partnership. They contributed 4.0 million, 2.4 million and 3.6 million cedis respectively. It was agreed that profits will be shared in proportion to their contributions. After one year of operation the profit made was 2.7 million cedis.
   (a) Find the amount received by each partner as his share of the profit.

(b) Express Edem's share of the profit as a percentage of his investment.

10. Esi and Mansah entered into a business partnership. Esi contributed 35% of the capital, while Mansah contributed the rest. At the end of the year, they made a profit of ¢5,600,000.00. 15% of the profit was paid into a reserve fund while 25% of the remaining profit was paid as income tax. They then shared the remaining profit in the ratio of their contributions. If Mansah contributed ¢10,400,000.00, find
    (a) The total contribution of Esi and Mansah;
    (b) The amount paid as income tax;
    (c) Correct to one decimal place, Esi's profit as a percentage of her contribution.

11. Three candidates K, L and M were voted into office as school prefects. K secured 45% of the votes, L had 35% of the votes and M had the rest of the votes, if M secured 1,430 votes, calculate,
    (a) The total number of votes cast;
    (b) How many more votes K received than L

12. In his will, a father left an estate worth ¢76,000,000.00. Out of this ¢16,000,000.00 was reserved for various purposes. The rest of the amount was shared among his three children. The eldest son received 20% of the amount. The remaining amount was shared between the other son and the daughter in the ratio $3 : 2$ respectively. Calculate
    (a) The amount that the eldest son received;
    (b) The amount that the daughter received;
    (c) The difference between the amounts the two sons received

13. A man left the sum of ₦1,260,000 to be shared equally among all his surviving children. Three of the children died before their father and so each of the survivors received ₦70,000 more than they would have received if all had lived. How many children survived their father?

14. Six people can dig a trench in 8 hours.
    (a) How long would it take:
        (i) 4 people
        (ii) 12 people
        (iii) 1 person
    (b) How many people would it take to dig the trench in:
        (i) 3 hours?
        (ii) 16 hours?
        (iii) 1 hour?

15. A train travelling at 100km/h takes 4 hours for a journey. How long would it take a train travelling at 60km/h?

16. A swimming pool is filled in 30 hours by two identical pumps. How much quicker would it be filled if five similar pumps were used instead?

17. A tap issuing water at a rate of 1.2 litres per minute fills container in 4 minutes.
    (a) How long would it take to fill the same container if the rate was decreased to 1 litre per minute? Give your answer in minutes and seconds.
    (b) If the container is to be filled in 3 minutes, calculate the rate at which the water should flow.

18. Three people share GH¢54,000.00 in the ratio $2 : 3 : 4$. Find how much each received.

19.
    (a) The sides of a triangle are in the ratio $4 : 5 : 7$ and its perimeter is 64cm. find the sides.
    (b) Thirty five coloured balls were shared among four teams such that one team takes all the red balls. If the reminder is shared to the other teams in the ratio $4 : 3 : 2$ and the smallest share was 6 balls, how many red balls were there?

SCALE DRAWING

Scale drawings are used when an accurate diagram, drawn in proportion, is needed. Common uses of scale drawings include maps and plans.

A scale of a map shows the ratio between the measurement on the map and its corresponding actual distance on the surface of the Earth. Such a scale is called **Representative Fraction** (R.F).

The numerator of the representative fraction is always 1 and it indicates the length on the map while the denominator represents the distance on the ground.in using the R.F scale, the larger the denominator of the fraction, the smaller the map and vice versa. In particular, a map with a scale $1 : 20{,}000$ is larger than the one with a scale $1 : 50{,}000$.

Note the following metric equivalents:
1km = 1000m
1m = 100cm
1cm =10mm

$\therefore$ 1km = 1,000m = 100,000cm

If the R.F of a map is $1 : n$ then,
$1 : n$ = Distance on map : Distance on ground
$\Rightarrow \frac{\text{Distance on map}}{1} = \frac{\text{Distance on ground}}{n}$
$\Rightarrow \text{Distance on map} = \frac{\text{Distance on ground}}{n}$
$\Rightarrow \text{Distance on ground} = n \times \text{Distance on map}$

$$\frac{\text{Area on the map}}{\text{Corresponding area on the ground}} = \left(\frac{1}{n}\right)^2$$

Example 12

A map is drawn to a scale of 1 :; 50,000. Convert them to distance in kilometers on the ground.
(a) Kumasi to Bosuso 3cm
(b) Accra to Nsuta 4.5cm
(c) The length on the map of a lake which is 5km long.

Solution…

(a) Distance on the ground
Distance from Kumasi to Bosuso
$= n \times$ Distance on the map
$= 50{,}000 \times 3$
$= 150{,}000$
$= 15 \times 10^4$
$= 1.5 \times 10^5$
$= 1.5$km

(b) Distance from Accra to Nsuta
$= 50{,}000 \times 4.5$
$= 225{,}000$
$= 2.25 \times 10^5$
2.25km

(c) $\text{Distance on map} = \frac{\text{Distance on ground}}{n}$
$= \frac{10\text{km}}{50{,}000\text{cm}}$
$= \frac{10\times10^5}{50{,}000}$
$= 20$cm

Example 13

Find the R.F for each of the following scales.
(i) 100cm to 30km
(ii) 15cm to 500m
(iii) 3cm to 40km
(iv) 5cm to 100m

Solution…

(i) 1km = 1,000m = 100,000cm
$\therefore$ 100cm : 30km = 100 : 3,000,000
= 1 : 30,000

$\therefore$ The R.F is 1 : 30,000

(ii) 20cm to 500m = 20cm : 50,000cm
$= \frac{20}{20} : \frac{50{,}000}{20}$
= 1 : 2,500

$\therefore$ The R.F is 1 : 2,500

(iii) 4cm to 40km = 4cm : 40×10^5
= 4 : 4,000,000
= 1 : 1,000,000

$\therefore$ The R.F is 1 : 1,000,000

(iv) 5cm to 100m = 5 : 10,000
= 1 : 200

$\therefore$ The R.F is 1 : 200

Example 14

An area of 169cm^2 on a map represents an area of 1km^2 on the ground. Find the R.F of the map.

Solution…

Let the R.F of the map be $1 : n$.

$$\frac{\text{Area on the map}}{\text{Corresponding area on the ground}} = \left(\frac{1}{n}\right)^2$$
$$\frac{100\text{cm}^2}{1\text{km}^2} = \left(\frac{1}{n}\right)^2$$
$$\frac{100}{(10^5)^2} = \left(\frac{1}{n}\right)^2$$
$$\frac{10^2}{(10^5)^2} = \left(\frac{1}{n}\right)^2$$
$$\left(\frac{10}{10^5}\right)^2 = \left(\frac{1}{n}\right)^2$$
$$\frac{1}{10^4} = \frac{1}{n}$$
$$n = 100{,}000$$

Hence the R.F of the map is $1 : 100{,}000$

Exercise 20 Date:……………………

The following measurements are taken from $1 : 1{,}000{,}000$ map of Ghana. Convert them to distance in kilometers on the ground.

(i) Accra to Koforidua, 5.7cm
(ii) Takoradi to Kumasi, 2.8cm
(iii) Ho to Kyereponi, 6.4cm
(iv) Amrahia to Bolgatanga, 9.9cm

Exercise 21 Date:……………………

1. If the scale is $1 : 10{,}000$, what length will 45cm on the map represent:
   (a) In cm (b) in m (c) in km
2. On a map of scale $1 : 100{,}000$, the distance between two towns is 12.3cm. What is the actual distance in km?

Exercise 22 Date:……………………

1. A map with a scale of 1cm to 5km has an area of 405cm^2. What is the actual area of this map on the ground?

2. A map has a scale of 1cm to 5km. a country on the map measures 27cm by 18cm. Calculate the area of the country on the ground.

Exercise 23 Date:……………………

1. A distance of 36km on the ground is represented by a line on 1.8cm on a map. What is the scale of the map?

2. On a map, a square field has an area of 2.25cm^2. If the actual perimeter of the field is 900m, what does 1cm on the map represent?

Exercise 24 Date:……………………

1. On a map, 1cm represents 5km. Find the area on the map that represents 100km^2.

2. The scale of a map is $1 : 20{,}000$. Calculate the area, in square centimetres, on the map of a forest reserve which covers 85km^2.

RATES

A rate is a comparison of two different quantities.

SPEED, DISTANCE AND TIME

(i) Average speed $= \frac{\text{Total distance covered}}{\text{Time taken}}$
The unit of speed is km/h or m/s (ms^{-1}).

(ii) Distance Travelled
$=$ Time taken $\times$ Average speed
The unit is km or m.

(iii) Time taken $= \frac{\text{Total distance travelled}}{\text{Average speed}}$
The unit is hours or minutes or seconds.

Example 15

A train travels at a speed of 160km per hour. How long will it take to travel a distance of 1120km?

Solution…

Given,
Speed = 160km/h, Distance = 1120km

But time $= \frac{\text{distance}}{\text{speed}} = \frac{1120}{160} = 7$ hours

Example 16

A car covered 180m in $(t-1)$seconds and 324m in $(t+3)$seconds. If it is travelling at a constant speed, calculate the value of t.

Solution…

Speed $= \frac{\text{distance}}{\text{time}}$
$$\Rightarrow \frac{180}{t-1} = \frac{324}{t+3}$$

$\Rightarrow 180(t+3) = 324(t-1)$
$\Rightarrow 180t + 540 = 324t - 324$
$180t - 324t = -324 - 540$
$-144t = -864$
$t = 6$

Example 17
A cyclist travels from town X to town Y at an average speed of 10km/h and then travels back to town X by the same route at an average speed of 10.5km/h. If the total time for the journey is $1\frac{1}{4}$ hrs, find, correct to 1 decimal place, the distance between X and Y.
Solution…
Given,
Total time for the journey $= 1\frac{1}{4}\text{hrs} = \frac{5}{4}\text{hrs}$

Speed $= \frac{\text{distance}}{\text{time}}$
$\Rightarrow$ Time $= \frac{\text{distance}}{\text{speed}}$

Suppose the distance between X and Y is d.

$\Rightarrow$ Time used from X to Y $= \frac{\text{distance}}{\text{speed}}$
$= \frac{d}{10}$

Time used from Y to X $= \frac{d}{10.5}$

Since total time for the journey $= \frac{5}{4}$
$\Rightarrow \frac{d}{10} + \frac{d}{10.5} = \frac{5}{4}$
$\frac{41}{210}d = \frac{5}{4}$
$d = \frac{5}{4} \times \frac{210}{41}$
$d = 6.4$ km

$\therefore$ Distance between X and Y is 6.4 km.

Example 18
Convert
(i) 14ms^{-1} to km per hour
(ii) 36km/h to m/s.

Solution…

Note:
$1\text{km} \equiv 1000\text{m}$
$1\text{hr} \equiv 3600$ seconds

(i) $14\text{ms}^{-1} = \frac{14\text{m}}{1\text{s}}$
$= \frac{\frac{14}{1000}\text{km}}{\frac{1}{3600}\text{h}}$
$= \frac{14}{1000} \times \frac{3600}{1}$
$= 50.4$ km/h

(ii) $36\text{km/h} = \frac{36\text{km}}{\text{h}}$
$= \frac{36 \times 1000}{3600}$
$= 10\text{m/s}$

Exercise 25 Date:……………………
Convert
(i) 108km/h to ms^{-1}
(ii) 19kmh^{-1} to ms^{-1}
(iii) 144km/h to m/s
(iv) 12ms^{-1} to km/h
(v) 45ms^{-1} to km/h

Exercise 26 Date:……………………
1. A plane flies at an average speed of 750km/h. how far will it fly in:
(a) 25mins (b) 4 hours

2. Comfort walks at 4.25km/h. How far will she walk in three hours?

3. A car travels 60km in 2 hours. What is its average speed?

4. What is the time taken for the following journeys?
(a) 200km at a speed of 50km/h
(b) 400km at a speed of 40km/h

Exercise 27 Date:……………………
1. A train passes a signal post completely in 2 seconds. Its speed is 60km/h. what is the length of the train?

2. Emmanuella walks for 2 hours at w km/h and then for 3 hours at $(w-1)$km/h. The total distance of Emmanuella's journey is 11.5km. Find the value of w.

3. A car runs on the average at 45km to 5 litres of fuel. Calculate how many litres of fuel are required for a journey of 117km.
4. Express 5 hours in seconds leaving your answer in standard form.

5. Kwame rode a bicycle for a distance of $x$km and walked for another $\frac{1}{2}$ hour at a rate of 6km per hour. If Kwame covered a total distance of 10km, find the distance x he covered by bicycle.

Exercise 28 Date:……………………

1. A car consumes a gallon of petrol for every 30km drive. The driver of the car set out on a journey of 420km with 10 gallons of petrol in the fuel tank.
   (i) How many more gallons of petrol will be needed to complete the journey?
   (ii) Find the cost of the petrol for the journey of 420km if a gallon of petrol cost GH¢5.50.

2. A man travelled from Bakwa to Pabam, the distance between the two towns is 51km. at Pabam he covered an additional 40km on official duties. He returned to Bakwa the next day.
   (i) Find the total distance covered by the man.
   (ii) If the car used one litre of petrol to cover 20km, find the amount of petrol used for the whole journey.
   (iii) If a litre of petrol cost ¢522.00, calculate the total cost of petrol used for the journey.

Exercise 29 Date:……………………

1. If $x\text{kmh}^{-1} = y\text{ms}^{-1}$, then $y = ?$.

2. A car travels 120km/h. Find the distance it covers in 15 seconds.

3. Dodo travels at an average speed of 110km/h. What time does he take to cover a distance of 220km?

4. A journey made at an average speed of 40km per hour took $2\frac{1}{2}$ hours. How long will it take if the same journey was made at an average speed of 50km per hour?

5. A car travels 245km at a constant speed in $3\frac{1}{2}$ hours. How far does it travel in 90 minutes?

Exercise 30 Date:……………………

1. A bus travels at a distance of 56km at an average speed of 70km per hour. It travels a further 60km at an average speed of 50km per hour.
   (i) The total time taken;
   (ii) The average speed

2. A cyclist starts a journey at 10:30am and plans to get home 45km away by 3:00pm. At first, he travelled for 2 hours at $7\frac{1}{2}\text{kmh}^{-1}$ and then $x\text{kmh}^{-1}$ for the rest of the time in order to arrive promptly at home. Find the value of x.

Exercise 31 Date:……………………

1. A man drives from Ibadan to Oyo, a distance of 48km, in 45 minutes. If he drives at 72km/h where the surface is good and 48km/h where it is bad, find the number of kilometers of good surface.

2. Towns P and Q are $x$km apart. Two motorists set out at the same time from P to Q at steady speeds of 60km/h and 80km/h. the faster motorist got to Q 30 minutes earlier than the other. Find the value of x.

Exercise 32 Date:……………………

1. Two passenger trains, A and B, 300km apart, start towards each other at the same time. They meet after 2 hours. If train B travels $\frac{8}{7}$ as fast as train A, what is the speed of each train?

2. An aeroplane leaves Barcelona at 10:10pm and reaches Accra 4,415km away at 5:50am, the next morning. Find, correct to the nearest whole number, the average speed of the aeroplane in kmh^{-1}.

3. Two cyclists Musa and Amanda left P at the same time in opposite directions. If their speed are 8km/h and 12km/h respectively;
   (i) how long will it take them to be 40km apart?
   (ii) calculate the distance covered by Musa within the time in (i).

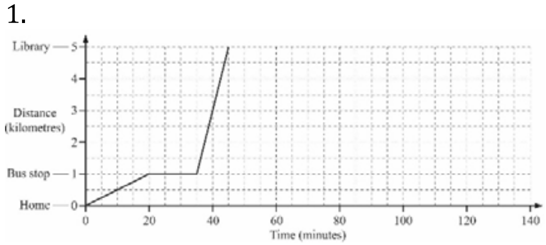
Exercise 33 Date:......................

1. Mr. Mensah's farm is 20km from his house. He uses his car to travel $y$km of the distance from his house and then walks $1\frac{1}{2}$ hours at the rate of 3 km per hour to get to his farm. Find y.

2. A boy has to cover 4km to catch a bus. He walks part of the distance at 3km per hour and runs the rest at 5km per hour. If he takes 1 hour to complete the distance, find the
   (i) distance covered by walking
   (ii) distance covered by running
   (iii) time taken when walking
   (iv) time taken when running

Exercise 34 Date:......................

1.



Kofi travels from home to the library. He walks to the bus stop and waits for a bus to take to the library.

(a) Write down
   i) the distance to the bus stop.
   ii) how many minutes Kofi waits for a bus.
   iii) how many minutes the bus journey takes to the library.

(b) Calculate in kilometers per hour
   i) Kofi's walking speed.
   ii) the speed of the bus
   iii) the average speed for Kofi's journey from home to the library.

(c) Kofi walks in the library for one hour. Then he travels home by car. The average speed of the car is 30km/h. Complete the travel graph.

2. When I walk from house at 4km/h, I will get to the office 30 minutes later than when I walk at 5km/h. Calculate the distance between my house and office.

Exercise 35 Date:......................

1. The distance between two villages P and Q is $x$km. A train running between P and Q arrives 3 minutes behind the scheduled time when it travels at an average speed of 45km/h but arrives 9 minutes ahead of the scheduled time when it arrives at an average speed of 60kmh^{-1}. Calculate:
   (i) The distance x
   (ii) The scheduled time for the journey

2. A man left the sum of ₦1,260,000 to be shared equally among all his surviving children. Three of the children died before their father and so each of the survivors received ₦70,000 more than they would have received if all had lived. How many children survived their father?

3. Melchizedek cycles along Skyline Drive. He cycles 60km at average speed of $x$km/h. He then cycles a further 45km at an average speed of $(x + 4)$km/h. Find the value of x.

4. The distance a train travels on a journey is 600km.
   (a) Write down an expression, in terms of x, for the average speed of the train when
      (i) The journey takes x hours
      (ii) The journey takes $(x + 1)$ hours

   (b) The difference between the average speed in part a(i) and part a(ii) is 20km/h. Find the average speed of the train for the journey in part a(ii). Show all your working.

5.
   (a) A car travels 245km at a constant speed in $3\frac{1}{2}$ hours. How far does it travel in 90 minutes?
   (b) The speed of a car is 63kmh^{-1}. Express the speed in ms^{-1}.
   (c) A boy walks 800m in 20 minutes. Calculate his average speed in km per hour.
   (d) A train travelling at 105kmh^{-1} goes through a tunnel 1575m long.

Calculate in seconds, the time a passenger on a train spends inside the tunnel.

Example 19
A cyclist is riding a bicycle with wheels 1m in diameter. If the wheels make 132 revolutions each minute, find his speed in kilometer per hour $\left[\pi = \frac{22}{7}\right]$

Solution…
Speed $= \frac{\text{distance}}{\text{time}}$

Speed in meter per minute:
$$= \frac{132\times 2\pi r \text{ meters}}{\text{minutes}}$$

Speed in kilometers per hour:
$$= \frac{132\times 2\pi r}{1000\times\frac{1}{60}}$$
$$= \frac{132\times\times 2\times\frac{22}{7}\times\frac{1}{2}\times 60}{1000}$$
$$= 24.88 \text{ kmh}^{-1}$$

Exercise 36 Date:……………………

1. A man walked 5 kilometers, then travelled a certain distance by Nissan Urvan bus and twice as far by train. If the whole journey was 104 kilometers, how far did he travel by bus?

2. Two cars travelled along the same road. The first car travelled at 45km/h for a certain time. The second car travelled at 80km/h for six minutes less than this time, but covered 55 more kilometers. For how long did the second car travel?

3. Two minibuses start from the same station and travel in opposite directions, along the same straight road. The first bus travels at a speed of 72km/h, the seconds at 48km/h. In how many hours will they be 360 kilometers apart?

4. Two trains leave two different stations which are 300km apart. The first train leaves at 10:00am and the second train leaves at 10:30am. They approach each other on parallel tracks and pass at 1:00pm. Each train travels at a constant speed; the one leaving at 10:30am moves at 10lm/h faster than the other. Find their speeds.

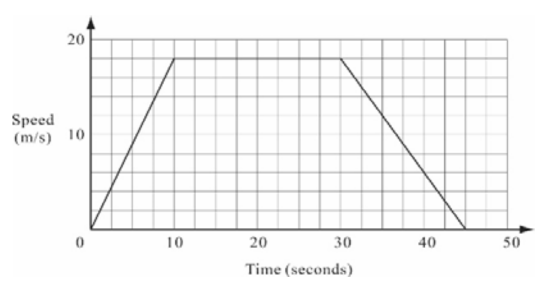
5. A boy walked for 3 hours and cycled for 4 hours, covering a total distance of 87km. A week later he walked for 22 hours and cycled for 5 hours, covering 100km. What were his average speed of walking and his average speed of riding if his walking speed and his cycling speed were constant on both occasions?

6. It takes 9 students two – thirds of an hour to fill 12 tanks with water. How many tanks of water will 4 students fill one – third of an hour at the same rate?

Exercise 37 Date:……………………

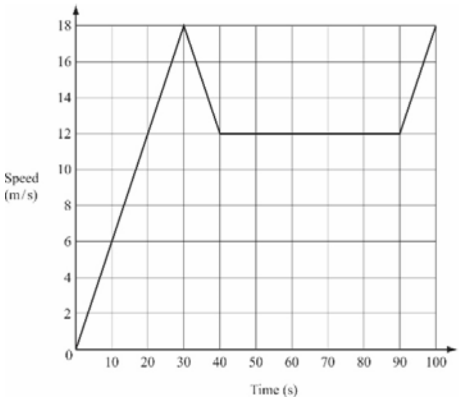
1. A motor boat starts from rest and accelerates uniformly for 10s before it reaches a speed of 15m/s. It maintains this speed for 30s before it slows down uniformly, becoming stationary in a further 20s.
   (a) Draw the speed – time graph.
   (b) Use this graph to find
      (i) the distance travelled
      (ii) the acceleration over the first 10s
      (iii) the deceleration over the last 20s
      (iv) the mean speed for the journey in kilometres per hour
2.
   (c) The top speed of a car is 54 metres per second. Change this speed in kilometers per hour.
   (d) A train left Sydney at 23 : 30 on December 18th and arrived in Brisbane at 02 : 40 on December 19th. How long, in hours and minutes, was the journey?

3. A cyclist is training for a competition and the graph show one part of the training.



(a) Calculate the acceleration during the first 10 seconds.
(b) Calculate the distance travelled in the first 30 seconds.
(c) Calculate the average speed for the entire 45 seconds.

4.



The diagram shows part of a journey by a truck.

(a) The truck accelerates from rest to 18m/s in 30 seconds. Calculate the acceleration of the truck.
(b) The truck then shows down in 10 seconds for some road works and travels through the road works at 12m/s.

At the end of the road works it accelerates back to a speed of 18m/s in 10 seconds.

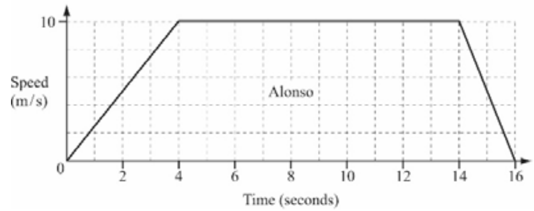
(c) Find the total distance travelled by the truck in the 100 seconds.

5. A holiday in Europe was advertised at a cost of €245. The exchange rate was $\$ = €1.0.6$, calculate the cost of the holiday in dollars, giving your answer correct to the nearest cent.

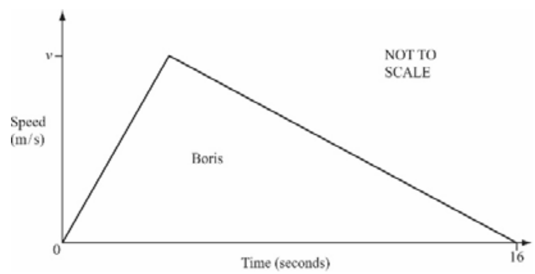
6. Michael changed \$600 into pounds (€) when the exchange rate was $€1 = \$2.40$. He later changed all the pounds back into dollars when the exchange rate was $€ = \$2.60$. How many dollars did he receive?

7. A cyclist left Melbourne on Wednesday 21 May at 09 : 45 to travel to Sydney. The journey took 97 hours. Write down the day, date and time that the cyclist arrived in Sydney.

8. The graphs show the speeds of two cyclists, Alonso and Boris.
Alonso accelerated to 10m/s, travelled at a steady speed then slowed to a stop.



Boris accelerated to his maximum speed, v m/s, and then slowed to a stop.

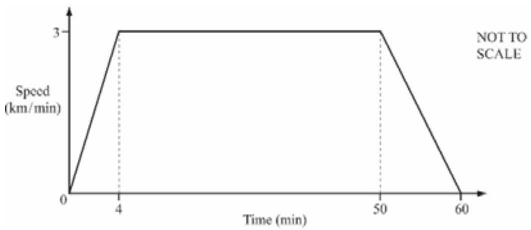


Calculate the maximum speed for Boris. Show all your working.

9. The scale on a map is $1 : 20{,}000$.
(a) Calculate the actual distance between two points which are 2.7 cm apart on the map. Give your answer in kilometres.
(b) A field has an area of 64400 m^2. Calculate the area of the field on the map in cm^2.

10. The scale of a map is $1 : 25{,}000$.
(a) The actual distance between two cities is 80km. Calculate this distance on the map. Give your answer in centimetres.
(b) On the map a large forest has an area of $6cm^2$. Calculate the actual area of the forest. Give your answer in square kilometres.

11.



A train journey takes one hour. The diagram shows the speed – time graph for this journey.

(a) Calculate the total distance of the journey. Give your answer in kilometres.

(b)

(i) Correct 3 kilometres/minute into metres/second.

(ii) Calculate the acceleration of the train during the first 4 minutes. Give your answer in metres/second^2.

12. Gregor changes \$700 into euros (€) when the rate is €1 = \$1.4131. Calculate the amount he receives.

13. The taxi fare in a city is \$3 and then \$0.40 for every kilometre travelled.

(a) A taxi fare is \$9. How far has the taxi travelled?

(b) Taxi fares cost 30% more at night. How much does a \$9 daytime journey cost at night?

14.

(a) Martina changed 200 Swiss Francs (CHF) into euros €. The exchange rate was €1 = 1.14 CHF. Calculate how much Martina received. Give your answer correct to the nearest euro.

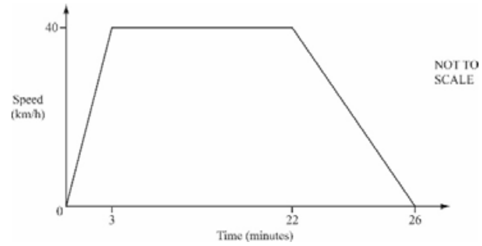
(b) GUY \$ 1.00 = US \$0.01 and EC \$ 1.00 = US \$0.37
Calculate the value of

(i) GUY \$60,000 in US \$.

(ii) US \$ 925 in EC \$.

15. George and his friend Jane buys copies of the same book on the internet. George pays \$16.95 and Jane pays £11.99 on a day when the exchange rate is \$1 = £0.626.
Calculate, in dollars, how much more Jane pays.

16.



The diagram shows the speed – time graph of a train journey between two stations. The train accelerates for 3 minutes, travels at a constant maximum speed of 40km? h, then takes 4 minutes to slow to a stop.
Calculate the distance in kilometres between the two stations.

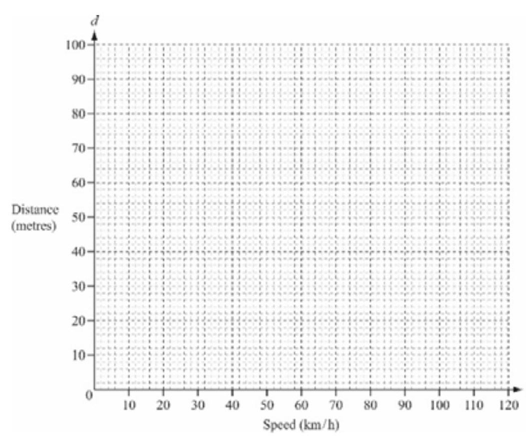
17. The braking distance, d metres, for Alex's car travelling at v km/h is given by the formula.

$$200d = v(v + 40)$$

(a) Calculate the missing values in the table.

v km/h	0	20	40	60	80	100	120
d metres	0		16		48		96

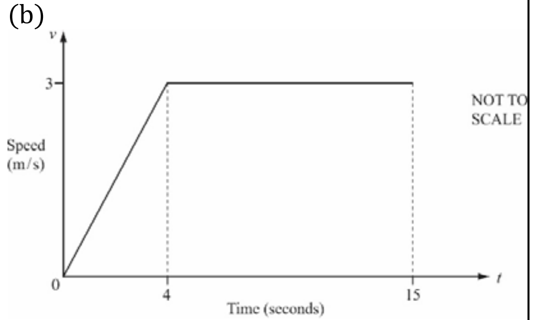
(b) On the grid below, draw the graph of $200d = v(v + 40)$ for $0 \leq v \leq 120$.



(c) Find the braking distance when the car is travelling at 100km/h.

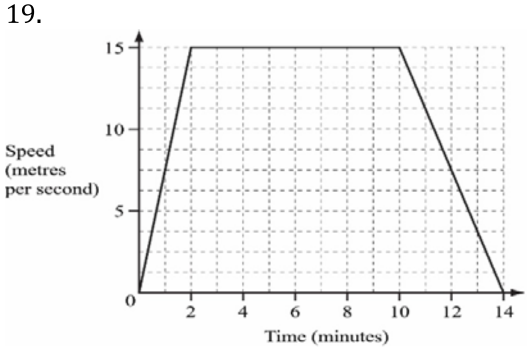
(d) Find the speed of the car when the braking distance is 80m.

18.
(a) The maximum speed of a car is 252km/h. Change this speed into metres per second.
(b)



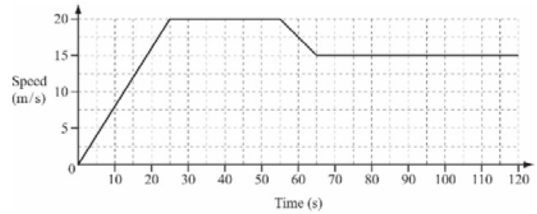
The diagram shows the speed – time graph for 15 seconds of the journey of a cyclist.
(i) Calculate the acceleration of the cyclist during the first 4 seconds.
(ii) Calculate the average speed for the first 15 seconds.

19.



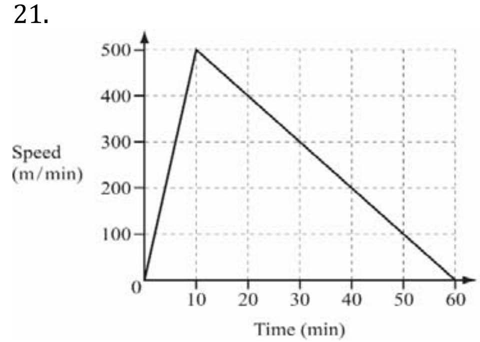
The diagram shows the speed – time graph of a train journey between two stations. The train accelerates for two minutes, travels at a constant maximum speed, then slows to a stop.
(a) Write down the number of seconds that train travels at its constant maximum speed.
(b) Calculate the distance between the two stations in metres.
(c) Find the acceleration of the train in the first two minutes. Give your answer in m/s^2.

20.



The diagram shows the speed – time graph for the first 120 seconds of a car journey.
(a) Calculate the acceleration of the car during the first 25 seconds.
(b) Calculate the distance travelled by the car in the first 120 seconds.

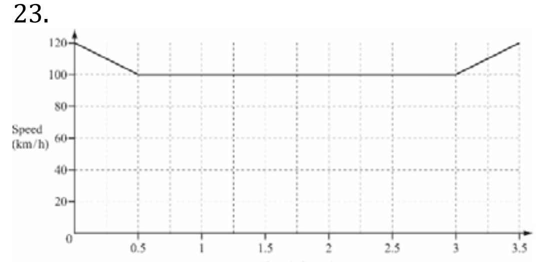
21.



The diagram shows the speed – time graph for a boat journey.
(a) Work out the acceleration of the boat in metres/minute^2.
(b) Calculate the total distance travelled by the boat. Give your answer in kilometres.

22. The scale of a map is 1 : 500,000.
(a) The actual distance between two towns is 172km. Calculate the distance, in centimetres, between the towns on the map.
(b) The area of a lake on the map is 12cm^2. Calculate the actual area of the lake in km^2.

23.



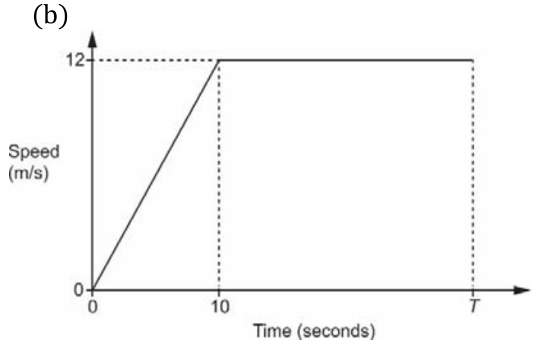
The diagram shows the speed – time graph of a car journey.

The speed of the car is shown in kilometres/hour.

Calculate the distance travelled by the car during the 3.5 minutes shown in the diagram.
Give your answer in kilometres.

24.
(a) A car travels at 108km/h for 20 seconds. Calculate the distance the car travels. Give your answer in metres.

(b)



The diagram shows the speed – time graph for the first T seconds of a car journey.
(i) Find the acceleration during the first 10 seconds.
(ii) The total distance travelled during the T seconds is 480m. Find the value of T.

25.
(a) Change 6.54 kilometres into metres.
(b) Change 7850cm^3 into litres.
(c) Abena takes 65 seconds to run 4000m. Calculate her average speed.
(d) Write the correct symbol, $>, =$ or $<$ in each statement.
(i) 500m………………… 5km
(ii) 50mm ……………… 0.5cm
(iii) 5000cm ……………. 0.05km

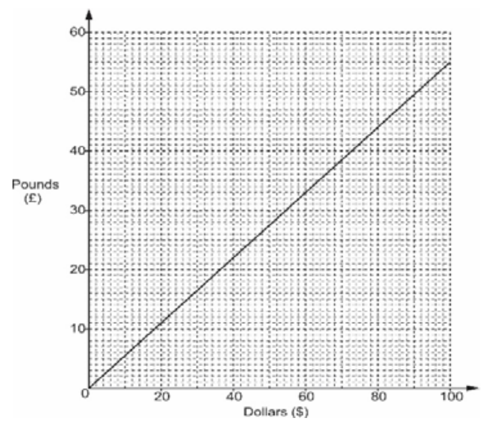
26. The graph shows information about the journey of a train between two stations.

(a)
(i) Work out the acceleration of the train during the first 4 minutes of this journey. Give your answer in km/h^2.
(ii) Calculate the distance, in kilometres, between the two stations.

(b)
(i) Show that 126km/h is the same speed as 35m/s.
(ii) The train has a total length of 220m. At 0930, the train crossed a bridge of length 1400m. Calculate the time, in seconds, that the train took to completely cross the bridge.

(c) On a different journey, the train took 73 minutes, correct to the nearest minute, to travel 215km, correct to the nearest 5km. Calculate the upper bound of the average speed of the train for this journey. Give your answer in km/h.

27.
(a) This graph for converting between dollars ($) and pounds (£).



(i) Use the graph to convert $80 to pounds.

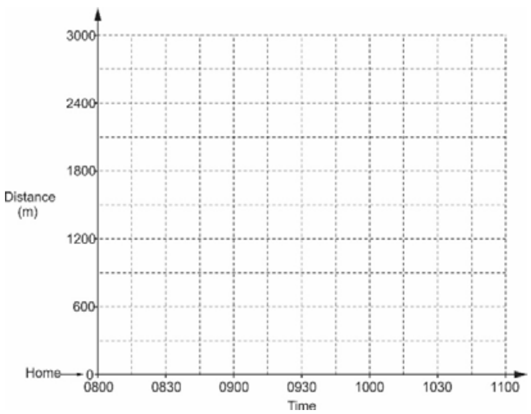
(ii) Taylor changes £100 to dollars. Work out how many dollars he receives.

(b)

α) Samuel leaves home at 0815 and walks at 80 metres per minute, arriving at her friend's house half an hour later.

(i) Work out the distance she walks, in metres, from her home to her friend's house.

(ii) On the grid, show her journey from her home to her friend's house.



β) Samuel stays at his friend's house for $1\frac{1}{2}$ hours. He hen jogs home, arriving 15 minutes later.

(i) On the grid, complete the travel graph for his journey.

(ii) Calculate his average speed, in metres per minute, on his journey home.

28. Kingsley runs 10km at an average speed of $x$km/h. The next day he run 12km at an average speed of $(x-1)$km/h. The time taken for the 10km run is 30 minutes less than the time taken for the 12km run.

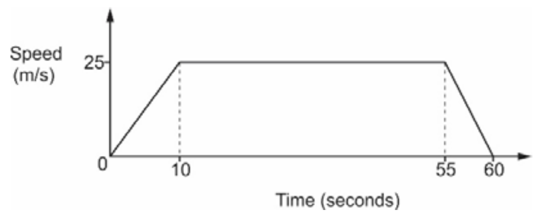
(a)

(i) Write down an equation in x and show that it simplifies to $x^2 - 5x - 20 = 0$.

(ii) Use the quadratic formula to solve the equation $x^2 - 5x - 20 = 0$. Show your working and give your answers to 2 decimal places.

(iii) Find the time that Kingsley takes to complete the 12km run. Give your answer in hours and minutes correct to the nearest minute.

(b) A cheetah runs for 60 seconds. The diagram shows the speed – time graph.



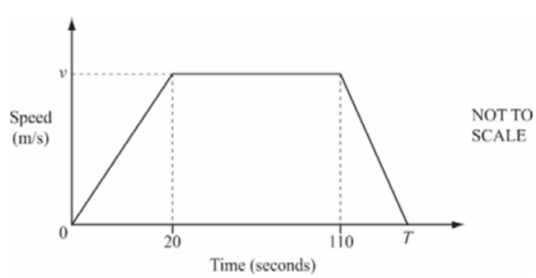
(i) Work out the acceleration of the cheetah during the first 10 seconds.

(ii) Calculate the distance travelled by the cheetah.

29.

(a) Philip cycles along Skyline Drive. He cycles 60km at an average speed of x km/h. He then cycles a further 45km at an average speed of $(x+4)$km/h. Write down an equation in x and solve for the value of x.

(b) The diagram shows the speed – time graph for a car travelling along a road for T seconds.

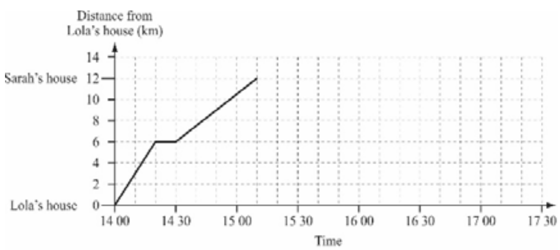


To begin with the car accelerated at 0.75m/s^2 for 20 seconds to reach a speed of v m/s.

(i) Find the speed, V, of the car.
(ii) The total distance travelled is 1.8 kilometres. Calculate the total time, T, of the journey.

(c) Elizabeth runs 22 kilometres correct to the nearest kilometre. She takes $2\frac{1}{2}$ hours, correct to the nearest half hour. Calculate the upper hand bound of Elizabeth's speed.

30.
α)



The travel graph shows Lola's journey from her house to Sarah's house.

(a) Lola stopped at a shop on the way to Sarah's house. For how many minutes did she stop?
(b) Write down the time she arrived at Sarah's house.
(c) Calculate Lola's average speed from leaving the shop, to arriving at Sarah's house. Give your answer in kilometres per hour.

(d) Lola stayed at Sarah's house for 1 hour 20 minutes. She then cycled home without stopping. Her journey took 50 minutes. Complete the travel graph.

β) A car travels at 56km/h. Find the time it takes to travel 300 metres. Give your answer in seconds correct to the nearest second.

PERCENTAGES I

The symbol % means "per cent". This comes from the Latin words per centum, which means out of 100.

$\therefore 3\% = 3 \times \frac{1}{100} = \frac{3}{100}$

$59\% = 59 \times \frac{1}{100} = \frac{59}{100}$

In general, if r is a rational number, $r\% = r \times \frac{1}{100}$

Example 1
Express the following
(i) $\frac{79}{100}$ (ii) $\frac{1}{4}$ (iii) $\frac{2}{5}$

Solution…
Method 1
We write $\frac{n}{100} = \frac{79}{100}$

$n = \frac{79}{100} \times 100$
$= 79$
$\therefore \frac{79}{100} = 79\%$

Method 2
(i) $\frac{79}{100} = \frac{79}{100} \times \frac{100}{100}$
$= \left(\frac{79}{100} \times 100\right) \times \frac{1}{100}$
$= \frac{79}{100} \times 100\%$ $\left(\text{Replace } \frac{1}{100} \text{ by } \%\right)$
$= 79\%$

(ii) **Method 1**
We write $\frac{n}{100} = \frac{1}{4}$
$n = \frac{1}{4} \times 100$
$= 25$
$\therefore \frac{1}{4} = 25\%$

Method 2
$\frac{1}{4} = \frac{1}{4} \times \frac{100}{100}$
$= \left(\frac{1}{4} \times 100\right) \times \frac{1}{100}$
$= 25\%$

(iii) $\frac{2}{5} = \frac{2}{5} \times \frac{100}{100}$
$= \left(\frac{2}{5} \times 100\right) \times \frac{1}{100}$
$= \frac{2}{5} \times 100\%$
$\therefore \frac{2}{5} \equiv 40\%$

Exercise 1 Date:……………………
Express the following as percentages.
(i) $\frac{9}{100}$ (iii) $\frac{5}{8}$ (v) $\frac{7}{4}$
(ii) $\frac{3}{5}$ (iv) $\frac{6}{5}$ (vi) $\frac{5}{12}$

Example 2
Express 70% as a fraction in its simplest form.

Solution…
$70\% = 70 \times \frac{1}{100}$
$= \frac{7}{10}$
$\therefore 70\% \equiv \frac{7}{10}$

Exercise 2 Date:……………………
Express the following percentages as fractions in their simplest form.
(i) 20% (v) 125%
(ii) 50% (vi) $33\frac{1}{3}\%$
(iii) 35% (vii) $62\frac{1}{2}\%$
(iv) 25%

Example 3
Express as a percentage.
(i) 0.07 (ii) 0.5 (iii) 0.124

Solution…
(i) $0.07 = \frac{7}{100} = 7\%$
(ii) $0.5 = \frac{50}{100} = 50\%$
(iii) $0.124 = \frac{12.4}{100} = 12.4\% = 12\frac{2}{5}\%$

Exercise 3 Date:……………………
Express the following decimal numbers as percentages.
1. 0.75 3. 0.435 5. 1.25
2. 0.075 4. 2.75

Exercise 4 Date:……………………
Write the following percentages as decimal numbers.
1. 48% 3. 172% 5. $\frac{1}{4}\%$
2. 93% 4. 108%

Finding Percentages of a Given Quantity

In general, $n\%$ of $k = \frac{n}{100} \times k$

Example 4
Find
(i) 7% of 300
(ii) $12\frac{1}{2}$% of GH¢500.00

Solution…
(i) 7% of 300 $= \frac{7}{100} \times 300 = 21$
(ii) $12\frac{1}{2}$% of GH¢500.00 $= \frac{25}{2} \times \frac{1}{100} \times 500$
$=$ GH¢62.5

Exercise 5 **Date:……………………**
Find:
1. 10% of 40
2. 8% of 74
3. 12% of 500cm
4. 27% of 56g
5. 99% of GH¢1000.00
6. 16% of £125
7. $17\frac{1}{2}$% of \$350
8. 25% of ₦18
9. 18% of £1.50
10. 48% of \$250

Exercise 6 **Date:……………………**
Find
(α)
(i) 20% of 200 sheep.
(ii) 2.5% of 5000 pencils.
(iii) 10% of 160 students.
(iv) 9% of 981 pencils.

(β) In a class of 50 students, 20% have black hair, 10% have blonde hair and 70% have brown hair. Calculate the number of students with
(i) black hair
(ii) blonde hair
(iii) brown hair

Expressing One Quantity as a Percentage of a Similar Quantity

In general, m as a percentage of
$$n = \frac{m}{n} \times 100$$

Exercise 7 **Date:……………………**
1. Express 24 as a percentage of 50
2. Express 14 as a percentage of 14
3. Express 17 as a percentage of 63
4. Express 45 as a percentage of 90
5. Express 20 as a percentage of 35
6. Express 33 as a percentage of 300

Exercise 8 **Date:……………………**
Calculate the value of x in each of the following.
1. 40% of x is 240
2. 24% of x is 84
3. x% of 240 equals 12
4. 85% of x is 765

Exercise 9 **Date:……………………**
1. If 125% of p equals 5% of 400, find the value of p.
2. What percentage of 120 is 48?
3. What percentage of $27\frac{1}{2}$ tonnes is 4.4 tonnes?
4. If 15% of a number m is equal to r, what is y% of m?

Increasing and Decreasing by a Given Percentage
To increase a quantity or amount by a percentage is to make it larger by adding to it.
To decrease a quantity or amount by a percentage is to make it smaller by subtracting from it.

NOTE:
1. 100% equal to the original

2. To increase by a given percentage:
(i) add the percentage increase to 100%
(ii) multiply the amount by this new percentage

3. To decrease by a given percentage:
(i) Subtract the percentage from 100%
(ii) Multiply the amount by this new percentage

Applications of Increasing and Decreasing by a Given Percentage Include:

(i) Discount – a reduction in the cost of an item.
(ii) Mark – ups – increases in the cost of an item.
(iii) Depreciation – a loss in value.
(iv) Appreciation – an increase in an item's worth.

Example 5

Abena's weekly wage of GH¢300 was increased by 15%. Find the new wage.

Solution…

100% corresponds to original wage

$100\% \longrightarrow 300$

$115\% \longrightarrow x$

If more, less divide.

$$x = \frac{115}{100} \times 300$$

∴ New wage = GH¢345.00

Example 6

Decrease 734 by 20%.

Solution…

100% corresponds to original

$100\% \longrightarrow 734$

$80\% \longrightarrow x$

If less, more divide.

$$x = \frac{80}{100} \times 734$$

$$\therefore x = \text{GH¢}587.20$$

Example 7

A car increase in value by 25% to GH¢500.00. Find its value before the rise.

Solution…

Let x be the price before the rise.

$125\% \longrightarrow 500$

$100\% \longrightarrow x$

If less, more divide.

$$x = \frac{100}{125} \times 500$$

$$\therefore x = 400$$

The value before the rise is GH¢400.00.

Exercise 10 Date:……………………

Increase the following by the given percentages.

1. 150 by 25%
2. 70 by 250%
3. 90 by 7%
4. 57 by 2%
5. 9.6 by $12\frac{1}{2}\%$

Exercise 11 Date:……………………

Decrease the following by the given percentage:

1. 120 by 25%
2. 75 by 8%
3. 684 by $7\frac{1}{2}\%$
4. 75 by 42%
5. 90 by 90%

Exercise 12 Date:……………………

(a) If 2% of x equal 15, find
  (i) 1% of x
  (ii) 20% of x
  (iii) 100% of x
  (iv) 5% of x
  (v) 50% of x

(b)
  (i) Increase GH¢56 by 8% and decrease the result by 5%
  (ii) Decrease GH¢97.40 by 5% and then increase the result by 5%
  (iii) Increase 5430 by $12\frac{1}{2}\%$ and then decrease this result by 10%.

PERCENTAGE CHANGE

$$\boxed{\%\text{change} = \frac{\text{change}}{\text{original}} \times 100}$$

Change: "profit", "loss", "appreciation", "depreciation", "increase", "decrease", "error", "discount", etc.

For example,

Percentage increase $= \frac{\text{increase}}{\text{original}} \times 100$

Example 8

A table was GH¢700.00 in 2018 and now GH¢760.00. Find the percentage increase.

Solution…

Percentage increase $= \frac{\text{increase}}{\text{original}} \times 100$

Increase $= 760 - 700 = 60$

Percentage increase $= \frac{60}{700} \times 100 = 8.57\%$

Example 9
Find the percentage decrease for a photo frame which was GH¢14.00 in last year is now GH¢5.00 now.

Solution…
Change $= 14 - 5 = 9$

Percentage decrease $= \frac{9}{14} \times 100 = 64.3\%$

Exercise 13 **Date:……………………**

1. Find the percentage increase for each of the following
   (i) A home was GH¢600,000.00 now GH¢650,000.00.
   (ii) A chocolate bar was GH¢7.00 now GH¢9.30.
   (iii) A bottle of coke was GH¢1.20 now GH¢1.50.

2. Find the percentage decrease for each of the following:
   (i) A textbook was GH¢30.00, now GH¢25.00.
   (ii) A cassette player was GH¢70.00 now GH¢52.00.
   (iii) A computer was GH¢1500.00, now GH¢970.00.

3. A car was sold for GH¢15,500.00 representing a loss of 28% on the cost price. Calculate the cost price of the car.

4. Patience bought a second hand textbook from Stephen for GH¢12.50. If Stephen lost 80% on the deal, what was the original cost of the textbook?

5. A man bought 5 reams of duplicating paper, each of which was supposed to contain 480 sheets. The actual number of sheets in the packets were 435, 420, 405, 415 and 440.
   (a) Calculate correct to the nearest whole number, the average percentage error for the packets of paper.
   (b) If the agreed price for full ream was ₦35.00. Find, correct to the nearest naira, the amount by which the buyer was cheated.

APPLICATIONS OF PERCENTAGES
COMMISSIONS
Commissions are payments made to salesmen or sales representatives who serves as agents for the company for their services. The commission paid to agent is usually a percentage of the total sales or percentages made by the agent.

Example 10
Madam Diana sells books for Stevkon's Series. She earns 20% commission on every book sold. If she sells GH¢1,280.00, find, how much Madam Diana earns.

Solution…
$$\begin{aligned} 20\% \text{ of GH¢}1{,}280 &= \frac{20}{100} \times 1280 \\ &= \text{GH¢}256.00 \end{aligned}$$

∴ Madam Diana earns GH¢256.00

Example 11
Mr. Adu receives GH¢168.00 commission on a GH¢1809.00 sale of calculators. What is his rate of commission?

Solution…
100% corresponds to original
$100\% \longrightarrow 1809$
$x \longrightarrow 168$
If less more divide,
Rate of commission $= \frac{168}{1809} \times 100 = 9.29\%$

Exercise 14 **Date:……………………**

1. A salesman is allowed 15% commission on his sales. What is his commission on sales totaling GH¢575.00.

2. Samuel sold his house through an agent who charged 8% commission on the selling price. If Samuel received $117,760.00 after the sale what was the selling price of the house?

3. An agent sold some property for GH¢2,467,000.00. If he was paid a commission at $17\frac{1}{2}\%$ of the selling price, how much money was he paid?

4. An agent sold a car for GH¢25,000.00. He was paid GH¢3,300.00 for his

services. Express this as a percentage of the selling price of the car.

5. A salesman is paid a salary of GH¢780.00 a month. In addition, he receives $3\frac{1}{2}$ commission on every sale that he makes. If in one month he made sales worth GH¢7,842.00, how much did he earn that month?

Exercise 15 Date:......................

1. Calculate the 8% commission on sales totaling:
   (i) GH¢600.00 (iii) GH¢4750
   (ii) GH¢2200.00 (iv) GH¢35,000.00

2. If the commission in Q1 changes to $8\frac{1}{2}$% on all sales, recalculate the commission.

3. A real estate agent earns commission of 5% on the first GH¢200,000.00, 3% on the next GH¢200,000.00 and 1% on any remaining amount. Calculate the commission earned on properties selling for:
   (a) GH¢150,000.00
   (b) GH¢200,000.00
   (c) GH¢300,000.00
   (d) GH¢400,000.00
   (e) GH¢729,500.00

PROFIT AND LOSS

$$\boxed{\text{Profit} = \text{Selling Price} - \text{Cost Price}}$$

$$\boxed{\text{Profit } \% = \frac{\text{Profit}}{\text{Cost Price}} \times 100}$$

$$\boxed{\text{Loss} = \text{Cost Price} - \text{Selling Price}}$$

$$\boxed{\text{Loss } \% = \frac{\text{Loss}}{\text{Cost Price}} \times 100}$$

Example 12

1. An article which cost GH¢10.00 was sold for GH¢17.00.
   (i) What was the profit?
   (ii) What was the profit percent?

2. An article which cost GH¢12,300 was sold for GH¢11,200. Find the lost percent.

Solution...

1.
(i) $\text{Profit} = S.P - C.P$
$= 17 - 10$
$= GH¢7$

(ii) $\text{Profit } \% = \frac{7}{10} \times 100$
$= 70\%$

2. $\text{Loss} = C.P - S.P$
$= 12{,}300 - 11{,}200$
$= 1{,}100$

$\text{Loss } \% = \frac{1100}{12{,}300} \times 100 = 8.94\%$

Exercise 16 Date:......................

Find the profit or loss percent in each of these cases.

1. The C.P was GH¢400.00 and the profit was GH¢800.00.

2. The C.P was GH¢1,200.00 and the S.P was GH¢1,100.00

3. The C.P was GH¢5,000.00 and the S.P was GH¢4,500.00.

4. The C.P was GH¢800.00 and the S.P was GH¢950.00.

5. The C.P was GH¢150,000.00 and the loss was GH¢15,000.00.

Example 13

A phone cost GH¢1940.00. If it was sold at a profit of 10%, what was the selling price?

Solution...

Method 1

100% $\longrightarrow$ 1940
110% $\longrightarrow$ S.P
If more, less divide

$\therefore \text{S.P} = \frac{110}{100} \times 1940$
$\text{S.P} = \text{GH¢}2{,}134.00$

Method 2
$\text{Profit} = \frac{10}{100} \times 1940 = 194$

$\therefore \text{SP} = \text{CP} + \text{Profit}$
$= 1940 + 194$
$= \text{GH¢}2{,}134.00$

Example 14
An article which cost GH¢1,320.00 was sold at a loss of 20%. Find the selling price.

Solution…

Method 1:
100% corresponds to cost price
$100\% \longrightarrow 1{,}320$
$80\% \longrightarrow \text{S.P}$
If less, more divide.
$\therefore \text{S.P} = \frac{80}{100} \times 1{,}320$
$= \text{GH¢}1056.00$

Method 2:
$\text{Loss } \% = \frac{20}{100} \times 1320 = 264$

$\therefore \text{SP} = \text{GH¢}1{,}320 - 264$
$= \text{GH¢}1056.00$

Exercise 17 **Date:……………………**
Find the SP in each of these cases.
1. The CP was GH¢1,600.00 and the profit was 40%.

2. The CP was GH¢3,300.00 and the profit was 17%.

3. The CP was GH¢2,500.00 and the loss was 24%.

4. The CP was GH¢450.00 and the loss was 33%.

5. The CP was GH¢75.00 and the profit was 13%.

Example 15
A laptop was sold for GH¢1350.00 at a profit of 15%. Find the CP.

Solution…

Method 1
100% corresponds to cost price
$115\% \longrightarrow 1{,}350$
$100\% \longrightarrow \text{C.P}$
If less, more divide.
$\therefore \text{C.P} = \frac{100}{115} \times 1{,}350$
$\therefore \text{C.P} = \text{GH¢}1173.91$

Method 2
Let $\text{C.P} = \text{GH¢}x$
$\text{The profit} = \frac{15}{100} \times x$

But $\text{S.P} = \text{C.P} + \text{P}$
$1350 = x + \frac{15}{100}x$
$1350 \times 100 = x \times 100 + 100 \times \frac{15}{100}x$
$135000 = 100x + 15x$
$115x = 135000$
$x = 1173.91$

$\text{C.P} = \text{GH¢}1173.91$

Exercise 18 **Date:……………………**
Find the CP in each of these cases.
1. The SP was GH¢500.00 and the profit 25%.

2. The SP was GH¢300.00 and the profit 20%.

3. The SP was GH¢1,250.00 and the profit was 18%.

4. The SP was GH¢875.00 and the loss 10%.

5. The SP was GH¢2,500.00 and the loss was 17%.

Example 16
A trader bought m phones for GH¢48,000.00. She found that 40 of them were spoilt. She then sold all the remaining phones. The selling price of one phone was GH¢100.00 more than the cost price.
(a) Find in terms of m
  (i) the cost price of the one phone
  (ii) the number of phones that she sold
  (iii) the selling price of one phone

(iv) an expression for the total sum that she received from the sale

(b) If she made a profit of GH¢12,000.00 from the sales, find
(i) the number of phones she originally bought
(ii) the cost price for a phone

Solution…

(a)

(i) Cost of one phone $= \frac{\text{GH¢}48000}{m}$

(ii) Number of phones sold $= m - 40$

(iii) Selling price of one phone
$= 100 + \text{Cost Price}$
$= \text{GH¢}\left(100 + \frac{48,000}{m}\right)$

(iv) Total sum of received from the sale
$= \text{No. of phones sold} \times \text{SP of one phone}$
$= \text{GH¢}\left[(m-40)\left(100 + \frac{48,000}{m}\right)\right]$

(b)

(i) $\text{Profit} = \text{S.P} - \text{C.P}$

$12,000 = (m-40)\left(100 + \frac{48000}{m}\right) - 48000$

$60,000 = (m-40)\left(\frac{100m+48000}{m}\right)$

$60,000m = (m-4)(100m + 48000)$

$60,000m = 100m^2 + 48,000m - 4000m - 1,920,000$

$100m^2 - 16000m - 1920000 = 0$

$m^2 - 160m - 19200 = 0$

$(m-240)(m+80) = 0$

$m = 240$ or $m = -80$

$\therefore m = 240$ since $m > 0$

$\therefore$ The number of phones she originally bought $= m$

(ii) Cost of one phone

$= \frac{\text{GH¢}48000}{m}$

$= \text{GH¢}\frac{48000}{240} = \text{GH¢}200.00$

Exercise 19 **Date:……………………**

1. Isaac sold an article for ₦6,900.00 and made a profit of 15%. Calculate his percentage profit if he had sold it for ₦6,600.00.

2. A car dealer made a profit of 22.5% by selling a car for GH¢58,000.00. Find, correct **two** decimal places, the percentage profit if the car had been sold for GH¢61,200.00.

3. A shopkeeper buys 40kg of fruits of ₦120.00. He sells 20kg at ₦5.00 per kg, 10kg at ₦3.00 per kg, 5kg at ₦2.00 per kg and the remaining 5kg at 50k per kg. Calculate the:
   (a) amount he releases from the sales
   (b) total profit
   (c) percentage profit on his outlay of ₦120.00

4. A dealer sold a car to a man and made a profit of 15%. The man then sold it to a woman for ₦120,175.00 at a loss of 5%. How much did the dealer buy the car?

5. A salesman bought some plates at ₦50.00 each. If he sold all of them for ₦600.00 and makes profit of 20% on the transaction, how many plates did he buy?

6. A man sold 100 articles at 25 for ₦66.00 and made a gain of 32%. Calculate his gain or loss percent if he sold them at 20 for ₦50.00.

7. Araba purchased x number of apples for GH¢ 48.00. She found out after a day that 40 of the apples were rotten and then sold all the remaining apples. If the selling price of one apple was GH¢ 1.00 more than the cost price.
   (a) Find in terms of x
      (i) the cost price of the one apple
      (ii) the number of apples that she sold
      (iii) the selling price of one apple
      (iv) an expression for the total sum that she received from the sale

   (b) If she made a profit of GH¢12.00 from the sales, find
      (i) the number of apples she originally bought
      (ii) the cost price for a apple

Exercise 20 **Date:……………………**

1. A trader bought 100 tubers of yam at 5 for ₦250.00. She sold them in sets of 4 for ₦290.00. Find her gain percent.

2. Two commodities A and B cost D70 and D80 per kg respectively. If 34.5kg of A is mixed with 26kg of B and the mixture is

sold at D85 per kg, calculate the percentage profit.

3. A trader sold 1,750 articles for GH¢525,000.00 and made a profit of 20%.
   (i) Calculate the cost price of each article.
   (ii) If he wanted 45% profit made on the cost price, how should he have sold each of the articles?

4. A man bought n articles for x cedis each. He sold p of them for $(x + 2)$ cedis each and the reminder for $(x + 1)$ cedis each.
   (i) Find his profit in terms of p and n.
   (ii) If $n = 800{,}000.00$, $p = 640{,}000.00$ and $x = 50$, express his profit as a percentage of the cost.

Exercise 21 Date:......................

1. A trader bought 30 baskets of pawpaw and 100 baskets of mangoes for ₦2,450.00. She sold the pawpaw at a profit of 40% and the mangoes at a profit of 30%. If her profit on the entire transaction was ₦855.00, find the:
   (i) cost price of a basket of pawpaw
   (ii) selling price of the 100 baskets of mangoes

2. In a school, there are 1000 boys and a number of girls. The 48% of the total number of students that were successful in an examination was made up of 50% of the boys and 40% of the girls. Find the number of girls in the school.
3. Madam Kwakyewaa imported a quantity of frozen fish costing GH¢400.00. The goods attracted an import duty of 15% of it cost. She also paid a sales tax of 10% of the total cost of the goods including the import duty and then sold the goods for GH¢660.00. Calculate her percentage profit.

4. Mr. Kofi sold a machine and made a profit of 15%. The buyer later sold it to Mr. Nana at a loss of 10%. If Mr. Nana paid GH¢20,700.00 for the machine, how much did Mr. Kofi buy it?

Exercise 22 Date:......................

1. A man bought 250 oranges for D1,000.00. He kept 20% of the oranges for himself, sold 115 at D6.50 each and the rest at D5.00 each. Calculate his percentage profit.

2. A trader purchased 10 dozen eggs at ₦300.00 per dozen. On getting to his shop, he found that 20 eggs were broken. How much did he sell the remaining eggs if he made a profit of 10%.

3. A publisher prints 30,000 copies of a book at GH¢2.00 each and sold them for GH¢2.76 **each.** The publisher agrees to pay the author 10% of the selling price for the first 6,000 copies sold and $12\frac{1}{2}\%$ of the selling price for all copies sold in excess of 6,000. If 25,380 copies of the book were sold,
   (a) Calculate, correct to the nearest Ghana cedi, the:
      (i) total amount received by the author;
      (ii) net profit the publisher makes after he has paid the author.

   (b) Find, correct to one decimal place, the publisher's net profit as a percentage of the author's total receipt.

4. The population of a village increases by 20% every year. The district Assembly grants the village GH¢15.00 per head at the beginning of every year. If the population of the village was 3,000 in the year 2003, calculate the Assembly's total grant to the village from 2003 to 2007.

Exercise 23 Date:......................

1. A man bought some shirts for GH¢720.00. If each shirt was GH¢2.00 cheaper, he would have received 4 more shirts. Calculate the number of shirts bought.

2. A student plans to spend ₦200.00 on p notebooks but the price of the notebooks had increased by ₦10.00. As a result, the number of notebooks the student could buy was reduced by 1. Find the price of each notebook before the increase.

3. A woman bought 130kg of tomatoes for ¢52,000.00. She sold half of them at a profit of 30%. The rest of the tomatoes began to go bad, she then reduced the selling price per kg by 12%. Calculate
   (a) the new selling price per kg;
   (b) the percentage profit on the whole transaction if she threw away 5 kg of bad tomatoes.

4. A manufacturer finds that the cost of materials and labour to make a certain article are in the ratio 3 : 5. The manufacturer sells to a retailer at a profit of $27\frac{1}{2}\%$ and the retailer sells to customer at a profit of 25%. If the customer pays ¢6,375,
   (a) calculate how much the article cost the manufacturers for materials and labour.
   (b) If there is a 20% rise in the labour cost, but no increase in the cost of materials and the manufacturer decides not to increase the price charged to retailer, calculate the percentage profit which the manufacturer then makes.

DISCOUNT

A discount is a reduction in the selling price of a product. Discounts are usually express as percentage of the selling price. If the discount is r.

The discount $= r\%$ of selling price

$$= \frac{r}{100} \times \text{selling price}$$

NOTE:

(i) Marked Price = Original Price
= Catalogue Price
= 100%

(ii) If discount is $r\%$ then the new price is $(100 - r)\%$ of the original price.

i.e. Dicount Price $= \frac{100-r}{100} \times$ original price

(iii)

$$\boxed{\text{Discount}(d) = \text{Marked Price} - \text{Selling Price}}$$

(iv)

$$\boxed{\text{Percentage Discount} = \frac{\text{Discount}}{\text{Marked Price}} \times 100\%}$$

Example 17

If an article cost GH¢450.00. What is its new value, if there is a discount of 15%?

Solution…

NOTE:

New Value = Discount Price

Method 1:

$100\% \longrightarrow 450$

$85\% \longrightarrow \text{New Value}$

$\therefore$ New Value $= \frac{85}{100} \times 450$
$= \text{GH¢}382.5$

Method 2:

Discount = 15% of original cost
$= \frac{15}{100} \times 450$
$= \text{GH¢}67.5$

$\therefore$ New Value $= 450 - 67.5$
$= \text{GH¢}382.50$

Example 18

Mr. Konadu was given a discount of 17% of the price of a fridge he bought from Hisense. The Marked Price of the Fridge is GH¢1895.00. How much did he pay for it?

Solution…

Method 1:

$100\% \longrightarrow 1895$

$83\% \longrightarrow \text{New Value}$

If less, more divide.

New amt paid $= \frac{83}{100} \times 1895$

New Price = GH¢1572.85

Method 2:

Discount = 17% of the original price
$= \frac{17}{100} \times 1895 = 322.15$

$\therefore$ Amount paid for the fridge
$= 1895 - 322.15 = \text{GH¢}1572.85$

Example 19
A phone marked at GH¢947.00 was sold at GH¢923.00. What is the percentage discount?

Solution…
Discount $(d) = \text{MP} - \text{SP}$
$= 947 - 923$
$= 24$

Discount Percent $(d\%) = \frac{\text{discount}}{\text{M.P}} \times 100\%$
$= \frac{24}{947} \times 100$
$= 2.53\%$

Example 20
At what price should an article be marked to make a profit of 25% on its purchase price GH¢900.00 after being discounted by 25%.

Solution…

Method 1
$100\% \longrightarrow 900$
$125\% \longrightarrow \text{S.P}$
$\text{S.P} = \frac{125}{100} \times 900 = 1125$

$75\% \longrightarrow 1{,}125$
Since there has been a discount of 25% on the marked price,

$100\% \longrightarrow \text{M.P}$
If more, less divide.

$\text{M.P} = \frac{100}{75} \times 1125$
$\text{M.P} = \text{GH¢}1{,}500.00$

Method 2
Here, cost price (CP) = GH¢900.00
Selling Price = 25% increase (profit) on GH¢900.00

$\therefore \text{S.P} = \frac{25+100}{100} \times 900 = 1125$

Now $\text{S.P} = 75\%$ of marked price because 25% discount on M.P

$\therefore \text{S.P} = \frac{75}{100} \times \text{M.P}$

$1125 = \frac{75}{100} \times \text{M.P}$
$\text{M.P} = 1500$

$\therefore$ Marked Price $= \text{GH¢}1{,}500.00$

Exercise 24 **Date:……………………**
1. During a sale, a shop reduced its prices by 16%. Calculate the discount on articles marked at these prices.
   (i) GH¢800.00
   (ii) GH¢2,500.00
   (iii) GH¢5,400.00
   (iv) GH¢3,300.00

2. A manufacture allows a trade discount of 25% on the catalogued price of goods purchased by a retailer. How much did the retailer pay for goods with the following catalogued prices?
   (i) GH¢130.00
   (ii) GH¢570.00
   (iii) GH¢3,750.00
   (iv) GH¢2,986.00

3. A shop sold goods at 10% discount off the marked price. Find the marked price if a customer paid the following for the goods bought.
   (i) GH¢98.00
   (ii) GH¢137.00
   (iii) GH¢775.00
   (iv) GH¢1,350.00

Exercise 25 **Date:……………………**
1. A discount of 15% is allowed on an article. If a customer received ₦600.00 discount, find the actual amount paid on the article.

2. A shop – keeper marks a television set for sale at ₦36,000 so as to make a profit of 20% on the cost price. When he sells it, he allows a discount of 5% off the marked price. Calculate the actual percentage profit.

3. A manufacturer offers distributors a discount of 20% on any article bought and a further discount of $2\frac{1}{2}$ for prompt payment.
   (i) If the marked price of an article is ₦25,000, find the total amount

saved by a distributor for paying promptly.

(ii) If a distributor pays ₦11,750 promptly for an article marked for x, find the value of x.

Exercise 26 Date:......................

1. A television set was marked for sale at GH¢760.00 in order to make a profit of 20%. The television set was actually sold at a discount of 5%. Calculate, correct to 2 significant figures, the actual percentage profit.

2. A radio which a dealer bought for ₦6,000.00 and marked to give a profit of 30% was reduced in a sale by 10%. Find
   (i) the total sale price
   (ii) the percentage profit

3. A shop – owner marked a shirt at a price to enable him make a gain of 20%, during a special sales period. The shirt was sold at 10% reduction to a customer for ₦864.00. What was the original cost to the shop – owner?

Exercise 27 Date:......................

1. A bookshop had 650 copies of a book for sale. The books were marked ₦75 per copy in order to make a profit of 30%. A bookseller bought 300 copies at 5% discount. If the remaining copies are sold at ₦75 each, calculate the percentage profit the bookshop would make on the whole?

2. A trader sold an article at a discount of 8% for GH¢828.00. If the article was initially marked to gain 25%, find the:
   (i) cost price of the article
   (ii) discount allowed

SIMPLE INTEREST

Interest can be defined as money added by a bank to sums deposited by customers. In the same way, when we borrow money from a bank, we pay interest on the loan. The money which is invested or borrowed is called the **principal**.

Interest on money borrowed or invested is paid at definite intervals (monthly, quarterly, half – yearly or yearly). The principal together with its interest for a stated period is called the **amount** for that time. **Per annum** (p.a) means in one year.

NOTE:

$I = \frac{PRT}{100}$

Where $I =$ simple interest
$P =$ the principal
$T =$ time in years
$R =$ rate percent

$$\boxed{A = P + I}$$

Example 21

A man invested GH¢680 in a bank at a rate of 4% in 3 years.
Find.
(i) the simple interest earned
(ii) the amount

Solution…

Given, $P =$ GH¢680.00
$R = 4\%$
$T = 3$ years

(i) $I = \frac{PRT}{100}$
$I = \frac{680 \times 4 \times 3}{100}$
$I =$ GH¢81.6

(ii) Amount $= P + I$
$=$ GH¢680 $+$ GH¢81.6
$=$ GH¢761.6

Example 22

Patience invests her GH¢90.00 for 2 years at $r\%$ per year simple interest. At the end of 2 years the amount of the money she has is GH¢99.00. Calculate the value of r.

Solution…

Given, $P =$ GH¢90.00
$T = 2$ years
$R = r\%$
$A =$ GH¢99.00

$A = P + I$
$A = P + \frac{P \times R \times T}{100}$

$$99 = 90 + \frac{90 \times r \times 2}{100}$$
$$99 - 90 = \frac{90 \times r \times 2}{100}$$
$$9 = \frac{180r}{100}$$
$$9 \times 100 = 180r$$
$$\therefore r = 5$$

Exercise 28 Date:......................

(α) Find the simple interest paid in the following cases:
(a) $P = \text{GH¢}400.00, R = 3\%, T = 5\text{yrs}$
(b) $P = \text{GH¢}945.00, R = 2\%, T = 3 \text{ yrs}$

(β) Calculate how long it will take for the following amounts of interest to be earned at the given rate.
(a) $P = \text{GH¢}500, r = 6\%, \text{S.I} = \text{GH¢}150$
(b) $P = \text{GH¢}400, r = 9\%, \text{S.I} = \text{GH¢}252$

Exercise 29 Date:......................

1. Calculate the rate of interest per year which will earn the given amount of interest:
   (a) $P = \text{GH¢}800, T = 3 \text{ yrs}, I = \text{GH¢}96$
   (b) $P = \text{GH¢}700, T = 5 \text{ yrs}, I = \text{GH¢}160$

2. Find
   a) Simple interest
   b) Amount
   On GH¢1,240.00 for each of the following
   (i) 2 years at 18% p.a
   (ii) 6 months at 20% p.a
   (iii) 3 years at 16% p.a
   (iv) 9 months at 12% p.a
   (v) 8 months at 15% p.a

Exercise 30 Date:......................

1. Find the simple interest on the following:
   (a) GH¢60 for 3 years at 5% per annum
   (b) GH¢1525 for 4 years at $7\frac{1}{2}\%$ per annum
   (c) GH¢750 for $2\frac{1}{4}$ years at 6% per annum

2. In how many years will GH¢312.50 invested at 4% per annum simple interest amount to GH¢500.00?

Exercise 31 Date:......................

1. The simple interest on GH¢600,000.00 for $3\frac{3}{4}$ years is GH¢56,250.00. Find the rate percent per annum.

2. What principal will amount to GH¢1,160,000.00 in 3 years at 15% per annum?

3. A man borrowed a sum of money from a bank at an interest rate of 12%. After 1 years he paid GH¢896,000.00 to settle the loan and the interest. How much did he borrow from the bank?

4. A man invested a certain amount of money in 2 separate projects in the ratio 3: 2. His profit was calculated on interest rates 5% and 4% simple interest, respectively. If after two years, he received a sum of GH¢9,200.00 as his profit for the two projects. Calculate the total amount he invested in the two projects.

Exercise 32 Date:......................

1. Calculate the amount earned in:
   (i) GH¢500 at 7% p.a for 10 months.
   (ii) GH¢1000 at 4% p.a for 200 days.
   (iii) GH¢4500 at 3.75% p.a for 15 months.
   (iv) GH¢50,000 at 6.3% p.a for 30 months.

2. A man invested ₦20,000 in bank A and ₦25,000 in bank B at the beginning of a year. Bank A pays simple interest at a rate of $y\%$ per annum and B pays $1.5y\%$ per annum. If his total interest at the end of the year from the two banks was ₦4,600, find the value of y.

3. A man saves ₦3,000 in a bank P, whose interest rate was $x\%$ p.a and ₦2,000 in another bank Q whose interest rate was $y\%$ p.a. His total interest in one year was ₦640. If he had saved ₦2,000 in P and ₦3,000 in Q for the same period, he would have gained ₦20 as additional interest. Find the values of x and y.

4.
   (a) A manufacturing company requires 3 hours of direct labor to process over ₦87.00 worth of raw materials. If the company uses₦30,450.00 worth of raw materials, what amount should it budget for direct labor at ₦18.25 per hour.

   (b) An investor invested ₦x in bank M at the rate of 6% simple interest per annum and ₦y in bank N at the rate of 8% simple interest per annum. If a total of ₦8,000,000.00 was invested in the two banks and the investor received a total of ₦2,320,000.00 as interest from the two banks after 4 years, calculate the:
      (i) values of x and y.
      (ii) interest paid by the second bank.

Exercise 33 Date:……………………

1. In a sale, normal prices are reduced by 15%. The normal price of a television was \$640. Work out the sale price of the television.
2. An airline increases the prices of its flights by 8%.
   (a) Before the increase, the price of a flight to Cairo was £475. Work out the price of a flight to Cairo after the increase.
   (b) The increase in price of a flight to Cairo after the increase.

3.
   (a) Mr. Glover buys a painting for GH¢675.00. Later, he sells it and makes a percentage profit of 12%. Work out the price for which Mr. Glover sells the painting.
   (b) Abigail sells her car. She makes a loss of GH¢2162.00. Her percentage loss is 23%. Work out the price for which Abigail sells her car.

4.
   (a) Theophilus' savings increased from GH¢155.00 to GH¢167.40. Work out the percentage increase in Theophilus' savings.
   (b) Isaac's savings increased by 4.5%. His savings are now GH¢125.40. What were his savings before the increase?

5. In a sale, all normal prices are reduced by 15%,
   (a) The normal price of a washing machine is 270 dollars.
   Work out the sale of the washing machine.

   (b) The normal price of a food processor is reduced by 13.50 dollars. Work out the normal price of the food processor.

6. Daniel bought 12 boxes of drinks. He paid GH¢15.00 for each box. There were 12 drinks in each box. Daniel sold all of the other drinks at a reduced price. He made an overall profit of 15%. Calculate how much Daniel sold each reduced price drink for.
7. Paul's height was 125cm when his age was 7 years. His height was 153cm when his age was 12 years.
   (a) Work out the percentage increase in Paul's height between the ages of 7 and 12 years.
   (b) Paul's height at the age of 12 years was 85% of his height at the age of 20 years. Calculate Paul's height when his age was 20 years.

8.
   (a) The price of an article is \$45. The price of the article is reduced by 12% in the sale. Calculate the price of the article in the sale.
   (b) Calculate the simple interest on a loan of:
      (i) \$2000 at a rate of 6% per annum over 4 years.
      (ii) € 9600 at a rate of 7.3% per annum over 17 month period.
      (iii) \$30,000 at a rate of 6.8%, per annum over a 5 year 4 month period.
      (iv) GH¢7500 at a rate of 7.6% per annum over a 278 day period.

9. A company makes compost by mixing loam, sand and coir in the following ratio:
   loam : sand : coir = 7 : 2 : 3.
   (i) How much loam is there in a 72 litre bag of the compost?
   (ii) In a small bag of the compost there are 13.5 litres of coir. How much compost is in a small bag?
   (iii) The price of a large bag of compost in \$8.40. This is an increase of 12% on the price last year. Calculate the price last year.

10.
   (a) Last year a golf club charged \$1,650 for a family membership. This year the cost increased by 12%. Calculate the cost of a family membership this year.

   (b) The golf runs a competition. The total prize money is shared in the ratio 1st Prize : 2nd Prize = 9 : 5. The 1st prize is \$500 more than the 2nd prize.
      (i) Calculate the total prize money for the competition.
      (ii) What percentage of the total prize money is given as the 1st prize?

   (c) For the members of the golf club the ratio men : children = 11 : 2.
   The ratio women : children = 10 : 3
      (i) Find the ratio men : women
      (ii) The golf club has 24 members who are children. Find the total number of members.

   (d) The club shop sold a box of golf balls for \$20.40 The shop made a profit of 20% on the cost price. Calculate the cost price of the golf balls.

11. Jane and Kate share \$240 in the ratio 5 : 7
   (a) Show that Kate receives \$140.
   (b) Jane and Kate each spend \$20. Find the new ratio Jane's remaining money: Kate's remaining money. Give your answer in its simplest form.
   (c) Kate invests \$120 for 5 years at 4% per year simple interest. Calculate the total amount Kate has after 5 years.

12. Elizabeth and Samuel each buys a bike Elizabeth buys a city – bike which has a price of GH¢120. She pays 60% of this price and then pays GH¢10 per month for 6 months.
   (i) How much does Elizabeth pay altogether?
   (ii) Work out your answer to part (i) above as a percentage of the original price of GH¢120.
   (iii) Samuel pays GH¢159.10 for a mountain – bike in a sale. The original price had been reduced by 14%. Calculate the original price of the mountain – bike.

13. A trader was charged 2 pesewas per month for every GH¢1.00 he borrowed from a bank.
   (i) At what percentage rate per annum was the interest charged?
   (ii) How much would the trader pay as interest on a loan of GH¢5,000.00 for 6 months.

14. A company buys a car for GH¢27,000.00 and sells it to Mr. Fosu for GH¢36,000.00 after a discount of 10% on the mark price.
   (a) Calculate the
      (i) marked price of the car.
      (ii) percentage profit made by the company.

   (b) If Mr. Fosu sells the car after covering a mileage of 128,000km. Find the
      (i) value of the car if the rate of depreciation is GH¢0.03 per km.
      (ii) range of values for which Mr. Fosu could sell the car so that he does not lose more than GH¢2,000.00 or gain more than GH¢3,000.00 on the depreciated value.

HIRE PURCHASE

Hire purchase is a system of purchasing goods on credit where the buyer makes an initial down payment, with the balance being paid in installments plus interest. This system gives an opportunity to people who cannot afford immediate cash payment for goods to acquire some property.

To calculate the approximate rate of interest, we use the average time $\frac{1+n}{2}$, where n is the number of years or months of installment.

NOTE:

Hire purchase is charged at a simple interest rate.

Example 23

Jones bought a car for GH¢680.00. He later put it up for sale at GH¢880.00. He agreed to sell it to Ruby under the following hire purchase terms:

An initial payment of 20% of the price and the balance paid at 15% simple interest per annum in twelve monthly equal installments. Calculate

(a) the amount paid every month;
(b) the total amount Ruby paid for the car.
(c) the percentage profit Jones made on the cost of the car.

Solution…

(a) Cost price of the car = GH¢680.00
Selling price of the car = GH¢880.00

Initial amount by Ruby $= \frac{20}{100} \times 880.00$
$= \text{GH¢}176$

Amount remaining to be paid by Ruby
$= \text{GH¢}880.00 - \text{GH¢}176.00$
$= \text{GH¢}704.00$

This amount attracts simple interest at a rate of 15% in 12 months (i.e. 1 year)
Interest on balance $= \frac{704 \times 15 \times 1}{100}$
$= \text{GH¢}105.60$

Total balance to be paid by Ruby for the year:
$= \text{GH¢}704.00 + \text{GH¢}105.60$
$= \text{GH¢}809.60$

$\therefore$ Amount to be paid every month
$= \frac{809.60}{12}$
$= \text{GH¢}67.47$

(b) Total amount Ruby paid for the car
= Initial payment +
Total amt paid in 12 months
$= \text{GH¢}176.00 + \text{GH¢}809.60$
$= \text{GH¢}985.60$

(c) Percentage profit Jones made on the car
$= \frac{\text{Profit}}{\text{Cost Price}} \times 100$
Profit = Selling Price − Cost Price
$= \text{GH¢}985.60 - \text{GH¢}680.00$
$= \text{GH¢}305.60$

$\therefore$ Profit Percent $= \frac{305.60}{680} \times 100$
$= 44.94\%$

Exercise 34 Date:……………………

1. The cash price of an article was GH¢60,000.00. A man paid 25% of the cash price as deposit. He then paid GH¢8,165.00 a month for six months.
   (a) How much did he pay altogether for the article?
   (b) Find the interest charged.
   (c) Find also the approximate rate of interest.

2. A man bought an article costing GH¢24,000.00 on hire purchase. He ended up paying $6\frac{1}{4}\%$ more than the cash price. If he made an initial deposit of 25% of the cash price and then paid the rest in six equal monthly instalments, find
   (a) the initial deposit
   (b) the approximate rate of interest
   (c) the amount of each installment

Exercise 35 Date:……………………

1. A woman buys furniture for which the cash price is GH¢1,800.00. She pays $33\frac{1}{3}\%$ cash deposit and agrees to pay GH¢135.00 a month for 10 months. Find the approximate rate of interest.

2. Mr. Adjei bought a refrigerator cost GH¢1,600.00 from a consumer credit union on hire purchase. He made an

initial deposit of GH¢1,000.00 and agreed to pay the rest in 15 monthly installments of GH¢50.00 each. What is the approximate rate of interest charged?

3. A man made 12 monthly payments of GH¢9,500 to repay a loan of GH¢10,000.00, starting one month after the loan was made. Calculate the approximate rate of interest.

4. A company was granted a loan of GH¢7,500.00. The loan was to be repaid in four quarterly installments of GH¢2,200 starting three months after the loan has been granted. Find the approximate rate of interest.

Exercise 36 Date:……………………

1. Mr. Amos bought a car for GH¢25,000.00 in 2009. He paid 40% of the cost and paid the rest in equal monthly installments. He took 8 years to make full payment for the car. Interest was at 18% simple interest. Calculate
   (i) monthly installment
   (ii) total amount he paid for the car
   (iii) percentage increase in the cost of the car

2. The hire purchase price of a car is ₦4,045.00. If 20% is paid as deposit and the rest is paid in 18 equal monthly installment, calculate the:
   (a) amount of deposit
   (b) amount of each monthly installment

PERCENTAGES II

COMPOUND INTEREST
Compound interest means interest is paid not only on the principal amount, but also on the interest itself. It is compounded (or added to).

Better still, in compound interest, the interest earned is added to the principal at the end of each period for which interest is paid. The amount then becomes the principal for the next period. In this case the principal increases after every period, and so does the interest.

Compound interest problems can be solved using the formula:

$A = P(1 + r\%)^n$
Where: r = rate of interest for each time/ period
P = principal
n = number of time periods
A = accumulated amount

i.e. $A = P\left(1 + \frac{r}{100}\right)^n$

Compound Interest $= A - P$
$= P\left(1 + \frac{r}{100}\right)^n - P$
$= P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$

Example 1
A woman deposited GH¢20,000 in a bank at $3\frac{1}{2}\%$ for 2 years per annum. Calculate
(i) The amount
(ii) Compound interest

Solution…
Given, $P =$ GH¢20,000.00
$r = 3\frac{1}{2}\% = \frac{7}{2}\% = 3.5\%$
$n = 2$ years

(i) Amount $= P\left(1 + \frac{r}{100}\right)^n$
$= 20{,}000\left(1 + \frac{35}{100}\right)^2$
$=$ GH¢21,424.50

(ii) Compound interest
$= A - P$
$=$ GH¢21,424.50 $-$ GH¢20,000.00
$=$ GH¢1,424.50

Exercise 1 Date:……………………..
1. A man deposited GH¢8,000.00 in a bank at 12% compound interest per annum. Find his total amount at the end of the third year.

2. Find the compound interest on $300 in 3 years at 4% interest per annum. Give your answer to the nearest dollar.

3. A principal of GH¢5,600.00 was deposited for 3 years at compound interest. If the interest earned was GH¢1,200.00, find correct to 3 significant figures the interest rate per annum.

4. A man invest €1500 for 2 years at compound interest. After one year, his money amounts to €1,560. Find
   (i) rate of interest
   (ii) interest for the second year

Exercise 2 Date:……………………..
1. Abena invests GH¢750.00 for 3 years at a rate of 2% per year compound interest. Calculate the compound interest.

2.
   (a) Samuel invest GH¢500.00 at a rate of 5% per year simple interest. Calculate the interest Samuel has after 3 years.
   (b) Stephen invest GH¢500.00 at a rate of 5% per year compound interest. Calculate how much more interest Stephen has than Samuel after 3 years.

3. Abigail invest GH¢1,225.00 for 3 years at a rate of 4.5% per year compound interest. How much interest does she receive after 3 years?

Exercise 3 Date:……………………..
1. Mr. Antwi invested GH¢800.00 at a rate of 5% per year compound interest. Calculate the total amount he has after 2 years.

2. Elizabeth puts GH¢600.00 into a bank account for 3 years at a rate of 3.4% per

year compound interest. Calculate how much will be in the account at the end of the 3 years.

3. Alex invests $200 for 2 years at a rate of 2% per year simple interest. Kingsley invests $200 for 2 years at a rate of 2% per year compound interest. Calculate how much more interest Kingsley has than Alex.

4. Mr. Anang deposited GH¢13,150.00 at Stevkon's Bank which pays an annual interest of 14% compounded quarterly. How much
   (a) Would Mr. Anang have in the bank at the end of 3 years.
   (b) Interest would he earn in the 3 years.
   (c) Should he deposit in the bank in order to have GH¢5,000.00 at the end of 3 years?

5. Mr. Asare borrowed GH¢800.00 at $12\frac{1}{2}\%$ per annum compound interest. At the end of the first years, he paid GH¢200.00, at the end of the second year he paid another GH¢200.00 and at the end of the third year he paid GH¢250.00. Calculate the
   (a) outstanding balance at the end of the third year
   (b) total amount to be paid at the end of the third year if he had not made any repayment.

Exercise 4 Date:……………………..

1. Jane and Kate share $240 in the ratio 5 : 7.
   (a) Find Kate's share
   (b) Jane and Kate spend $20. Find the new ratio Jane's remaining money : Kate's remaining money. Give your answer in its simplest form.
   (c) Kate invests $120 for 5 years at 4% per year simple interest. Calculate the total amount Kate has after 5 years.
   (d) Jane invests $80 for 3 years at 4% per year compound interest. Calculate the total amount Jane has after 3 years. Give your answer correct to the nearest cent.

2. An investment of $200 for 2 years at 4% per year compound interest is the same as an investment at $200 for 2 years at $r\%$ per year simple interest. Find the value of r.

3. The population of a village increases by 20% every year. The District Assembly grants the village GH¢15.00 per head at the beginning of every year. If the population of the village was 3,000 in the year 2003, calculate the Assembly's total grant to the village from 2003 to 2007.

4. John invested $8000 for 3 years at 5% per annum compound interest. Calculate the value of her investment at the end of 3 years.

5. A bank pays compound interest of 6% per annum on its savings accounts. Elizabeth invests $7500 for 3 years. Calculate the total interest gained after 3 years.

6. Samuel invested an amount of money at 4% per annum compound interest. At the end of 2 years, the value of his investment was GH¢3380.00. How much of the GH¢3380.00 was interest?

7. An investment of $25,000 is made for 4 years at 8.2% per annum compounded yearly. Find
   (a) the final value of the investment
   (b) the interest earned.

8. GH¢8,000 is invested for 10 years at 8% per annum compound interest. Find
   (a) the final value of the investment.
   (b) the amount of the interest earned.
   (c) the simple interest rate needed to be paid for the same return on the investment.

9. A man deposited ¢80,000.00 in a bank of 12% compound interest per annum. Find his total amount at the end of the third year.

10.
   (a) Emmanuel's parents deposited GH¢1,000 into a savings account as

a college fund when he was born. How much will Emmanuel have in this account after 18 years at a yearly simple interest rate of 8%?

(b) If you deposit GH¢7,500 into an account paying 9% annual compounded interest. How much money will be in the account after 7 years?

11. Mrs. Konadu invests GH¢1,000,000 for 5 years at an annual compound interest rate of 2.5%. How much will she have by the end of the period?

DEPRECIATION

The monetary value of an asset decreases over time due to use, wear and tear or obsolescence. This decrease is measured as **depreciation**.

NOTE:

While compound interest increases the initial value each year by a given percentage, depreciation decreases the initial value each year by a given percentage.

Depreciation problems can be solve using the formula:

$V = V_0\left(1 - \frac{d}{100}\right)^n$

Where V = depreciation value

V_0 = original value

d = rate of depreciation

n = number of calculations

Example 2

The cost of a phone is GH¢1,850.00. This price depreciates each year by 15% of its value at the beginning of the year. What will be the value of the phone after 3 years?

Solution…

Given, V_o = GH¢1,850

$d = 15\%$

$n = 3$

$\mathrm{V} = \mathrm{V}_0\left(1 - \frac{d}{100}\right)^n$

Depreciation value after 3 years

$\mathrm{V} = 1{,}850\left(1 - \frac{15}{100}\right)^3$

V = GH¢1,136.13

Exercise 5 **Date:……………………..**

1. A motor cycle cost $960. Its value depreciates each year at 12%. How much is it worth after a year use?

2. Judith bought a computer that had a value of $1500. At the end of each year, the value of her computer had depreciated by 40% of its value at the start of that year.
   Calculate the value of her computer at the end of 3 years.

3. The value of a printing machine depreciated each year by 8% of its value at the beginning of that year. If the value of a new machine is GH¢54,000.00, find its value at the end of the third year.

Exercise 6 **Date:……………………..**

1. A car costs ₦300,000.00. It depreciates by 25% in the first year and 20% in the second year. Find its value after 2 years.
2. A car costs GH¢60,000.00. Its value depreciates by 25% for the first year, 20% the second year and 15% the third year. Find its value at the end of the third year.
3. The table below gives information on the values and the rates of depreciation in value of two motor vehicles.

Motor Vehicle	Intial Value	Yearly Rate of Depreciation	Value after one year
Taxi	$40,000	12%	$ p
Private Car	$25,000	q %	$21,250

Calculate
(i) the values of p and q
(ii) the value of the Taxi after 2 years

4. Tomas has two cars.
   (i) The value, today, of one car is \$21,000.00. The value of this car decreases exponentially by 18% each year. Calculate the value of this car after 5 years. Give your answer correct to the nearest.

   (ii) The value, total, of the other car is \$15,000. The value of this car increases exponentially by x% each year. After 12 years the value of the car will be \$42,190.00. Calculate the value of x.

INCOME TAX
Income tax is the tax deducted from an employee's salary before the salary is paid.

NOTE:
1. Tax free income or (pay)
   = sum of all non – taxable allowances

2. Taxable income or (pay)
   = salary − Tax Free Pay (allowance)

3. Total deduction
   = Tax + Social Security Contribution etc

4. Tax
   = Rate of Tax × corresponding amt to be traded

5. Take Home Pay = Net income
   = Annual salary − Total deduction

6. Net Monthly Pay $= \frac{\text{Net Annual Income}}{12}$

Example 3
A man has a wife, 6 children and his total income in 1985 was ₦8,500.00. He was allowed the following tax free:

Personal	**₦120.00**
Wife	₦300.00
Each child	**₦250.00 for a maximum of 4**
Dependent relatives	₦400.00
Insurance	₦250.00

The rest was taxed as follows, the;

First	**₦2000.00 at 10%**
Next	₦2000.00 at 15%
Next	₦2000.00 at 20%
Next	₦2000.00 at 25%

Calculate:
(a) his tax free pay
(b) his taxable income
(c) His annual net pay
(d) His net monthly pay

Solution…
(a) Tax free
$= \text{sum of all non} - \text{taxable allowances}$
$= 120 + 300 + (4 \times 250) + 400 + 250$
$= ₦2,070.00$

(b) Taxable income
$= \text{Total income} - \text{Tax Free Pay}$
$= 8,500.00 - 2,070.00$
$= ₦6,430.00$

(c)

Amt	Tax Amt(₦)	Remaining Amt
₦2000	$\frac{10}{100} \times 2000 = 200$	$6430 - 2000 = 4430$
₦2000	$\frac{15}{100} \times 2000 = 300$	$4430 - 2000 = 2430$
₦2000	$\frac{20}{100} \times 2000 = 400$	$2430 - 2000 = 430$
₦430	$\frac{25}{100} \times 430 = 1007.5$	$2430 - 200 = 430$
	Total Tax = ₦1007.5	

∴ Annual Net Pay
$= \text{Annual Salary} - \text{Total Deductions}$
$= 8,500 - 1,007.5$
$= ₦7,492.5$

(d) Net Monthly Pay $= \frac{7492.5}{12} = ₦624.375$

Exercise 7 **Date:……………………..**
1. A man has a wife and 6 children and his total income in a year was GH¢850.00. He was given the following tax free allowances.

Personal	GH¢120
Wife	GH¢30
Children	GH¢25 per child for a maximum of 4 children
Medical	GH¢40

The rest was taxed as follows:

First	GH¢200 at 10%
Next	GH¢200 at 15%
Next	GH¢200 at 20%
Remainder at 25%	

Calculate his:
(a) taxable income
(b) monthly tax

2. In a certain year the annual income was calculated as follows:

Amount	Rate of Tax
First GH¢1,000.00	GH¢4 in every GH¢100
Next GH¢2,000.00	GH¢8 in every GH¢100
Next GH¢4,000.00	GH¢20 in every GH¢100
Next GH¢6,000.00	GH¢32 in every GH¢100
Remainder	GH¢48 in every GH¢100

Calculate the tax paid by Mr. Obeng whose annual salary was GH¢16,400,000.00 in that year,

Exercise 8 **Date:……………………..**

1. A man whose annual basic salary is ₦750,000.00 is allowed the following tax reliefs.

Personal allowance – 20% of basic salary
Wife allowance – ₦70,000.00
Children allowance – ₦30,000.00 per child up to 4 children
Dependent relations – ₦100,000.00

(a) If the man has four children, calculate his taxable income.
(b) If he pays tax at the rate of 35 kobo in the Naira on the remaining taxable income, calculate his monthly tax.
(c) Calculate his net annual income.
(d) Calculate his net monthly income.

2. In a certain country, the annual income tax payable by an individual is as follows:

Amount	Rate of Tax
First ¢140,700	Free of Tax
Next ¢100,000	5%
Next ¢150,000	15%
Next ¢200,000	25%
Next ¢300,000	35%
Next ¢350,000	45%
Next ¢400,000	55%

Mr. Hermosa's annual salary is ¢1,390,700. Calculate
(a) his taxable income
(b) his annual income tax
(c) the percentage of his income that went into tax, correct to two significant figures.

3. A worker is given a tax – free allowance of GH¢5,000.00 and he pays 20 pesewas in the Ghana cedi as tax on the rest of his income. If his net income is GH¢21,000.00, calculate the:
(i) taxable income
(ii) income tax

Exercise 9 **Date:……………………..**

1. In a certain country, the annual tax payable by an individual in 1997 was assessed at the following rates:
For the first \$200.00 – Nil
For the next \$300.00 – 10%
For the next \$500.00 – 15%
For the next \$800.00 – 20%
The remaining amount – 30%
(a) Calculate the income tax payable by Mr. Vonda whose annual income was \$2,800.00.
(b) If Miss Nyenge paid a monthly tax of \$29.00, calculate her annual income.

2. A man on a bank salary of GH¢60,000.00 per annum has three children. He is allowed the following reliefs:

Personal allowance – 40% of basic salary;
Children allowance – GH¢1,500.00 each Up to a maximum of 4 children
Dependents – 25% of basic salary
Insurance – 10% of basic salary
Calculate his
(a) taxable income
(b) monthly tax if tax is charged at the rate of 12 pesewas in the Ghana cedi.

3. A man earns ₦150,000 per annum. He is allowed a tax free pay of ₦40,000. If he pays 25 kobo in the Naira as tax on his taxable income, how much has he left?

4. A man was allowed 20% of his income as tax-free. He then paid 25 kobo in the Naira on the remainder. If he paid ₦1,200.00 as tax, calculate his total income.

5. Mr. Ansu's salary was ¢3,450,000 per annum. He contributed 5% of his salary per annum to a Social Security Fund on which he did not pay any tax. In addition, he was allowed ¢240,000.00 per annum free of tax. After these deductions, Mr. Ansu paid tax at 17.5% on the first 70% of his taxable income. He also paid tax at the rate of 45% on the remaining 30% of his taxable income. Calculate
   (a) Mr. Ansu's annual contribution to the Social Security Fund
   (b) the annual amount on which he paid tax
   (c) his annual income tax
   (d) the percentage of his income that went into tax, correct to three significant figures.

6. A housing estate consists of 100 houses each rented at GH¢150.00 per month and 108 flats each rented at GH¢110.00 per month. If all were rented out in a year,
   (a) Find the total annual rent collected
   (b) Calculate the half year tax, if income paid on rent is 8% per annum.

VALUE ADDED TAX (VAT)
Value Added Tax (VAT) is a tax on sales of goods and services by the government.

CALCULATIONS ON VAT
If a commodity cost, say P, without VAT i.e. VAT exclusive (this is the basic cost) and the VAT rate is x%, then the VAT is given by,

$$\boxed{\text{VAT} = \frac{x}{100} \times \text{P}}$$

The cost of the commodity with VAT inclusive now becomes

$$\boxed{\text{P} + \frac{x}{100} \times \text{P}}$$

Sometimes, sellers add the VAT to their commodity as part of the marked price. If one wants to know the VAT element, then we have

$$\boxed{\text{VAT} = \frac{x}{100 + x} \times \text{PI}}$$

Where PI is the cost with VAT inclusive.

Example 4
Mr. Konadu bought a phone for GH¢1,200.00 excluding VAT. If the VAT rate is $12\frac{1}{2}$%, calculate how much he paid to the cashier.

Solution…

Method 1
100% corresponds to 1,200
100% $\longrightarrow$ 1,200
112.5% $\longrightarrow$ x

If more, less divide.
$x = \frac{112.5}{100} \times 1,200$
$x =$ GH¢1,350.00

Method 2
The Basic Cost = GH¢1,200.00
VAT rate at $12\frac{1}{2}\% = 12.5\%$
VAT paid $= \frac{12.5}{100} \times 1,200 =$ GH¢150

Total amount paid = VAT + Basic Cost
= GH¢150 + GH¢1,200
= GH¢1,350.00

Example 5
A woman bought an article at GH¢1,940.00 with VAT inclusive. Calculate the VAT component if the VAT rate is $17\frac{1}{2}$%.

Solution…

Method 1
117.5% corresponds to 1,940
117.5% $\longrightarrow$ 1,940
17.5% $\longrightarrow$ x

If less, more divide.
$x = \frac{17.5}{117.5} \times 1{,}940$
$x =$ GH¢288.94

Method 2
Given, Cost + VAT = GH¢1,940.00
VAT = 17.5%

100% + 17.5% = 117.5%
117.5% ≡ 1,940

But VAT $= \frac{x}{100+x} \times \mathrm{PI}$

VAT at 17.5% $= \frac{17.5}{100+17.5} \times 1{,}940$
= GH¢288.94

Example 6
The VAT rate of a country is 9%. Patience bought an article that cost her GH¢3,420.00 with VAT inclusive. Calculate:
(i) the VAT component
(ii) the actual cost of the goods without VAT

Solution…
Method 1
(i) 109% corresponds to 3,420
109% $\longrightarrow$ 3,420
9 % $\longrightarrow$ x

If less, more divide.
$x = \frac{9}{109} \times 3{,}420$
$x =$ GH¢282.39

(ii) 109% $\longrightarrow$ 3,420
100% corresponds to the actual cost.

100% $\longrightarrow$ x
$x = \frac{100}{109} \times 3{,}420$
∴ Actual cost = GH¢3137.61

Method 2
(i) VAT $= \frac{x}{100+x} \times \mathrm{PI}$
Given, PI = GH¢3,420.00
$x = 7\%$

∴ VAT $= \frac{9}{109} \times 3{,}420$
= GH¢282.39

(ii) The actual cost = GH¢3,420 − GH¢282.39
= GH¢3137.61

Example 7
The VAT rate of a country is 13%. A man bought an item with a price tag of GH¢2,880.00 VAT exclusive. What price did he pay for the item?

Solution…
Method 1
100% corresponds to 2,880
100% $\longrightarrow$ 2,880
113 % $\longrightarrow$ x

If more, less divide.
$x = \frac{113}{100} \times$ GH¢2,880
$x =$ GH3,254.40

Method 2
The VAT paid = 13% of the basic cost
$= \frac{13}{100} \times$ GH¢2,880
= GH¢374.40

VAT inclusive cost = GH¢374.40 + GH¢2,880
= GH¢3,254.40

Exercise 10 **Date:……………………..**
1. Mrs. Mensah paid GH¢150,000 as 12.5% VAT on goods she purchased. Find the cost of the goods that Mrs. Mensah bought.
2. Sampson bought a phone for GH¢1,350.00 excluding VAT. If the VAT rate is 7%, calculate how much he paid to the cashier.
3. The Value Added Tax (VAT) paid by a man on a deep freezer was GH¢90.00. If VAT was charged at 15%,
   (i) What was the price of the deep freezer?
   (ii) How much did the man pay including VAT?

Exercise 11 Date:…………………….

1. The price of a dress including Value Added Tax (VAT) is GH¢220.00. If the VAT is 10%, calculate the
   (i) price of the dress, excluding VAT
   (ii) VAT charged

2. The VAT rate of a country is $17\frac{1}{2}$%. A company bought goods that cost them GH¢17,384.00 with VAT inclusive. Calculate:
   (i) the Vat component
   (ii) the actual cost of the goods without VAT

Exercise 12 Date:…………………….

1. The VAT rate of a country is $13\frac{1}{2}$%. A man bought an item with price tag GH¢18,416.00 VAT exclusive. What price did he pay for the item?

2. A man bought a commodity at GH¢7,350.00 with VAT inclusive. Calculate the VAT component if the VAT rate is $17\frac{1}{2}$%.

3. A deep freezer is tagged GH¢3,940.00 + VAT. If the VAT rate is $17\frac{1}{2}$%. Find
   (i) the VAT
   (ii) the purchase price of the deep freezer.

ELECTRICAL POWER TARIFFS

It is a tariff charged by Electricity Company of Ghana for the amount of electricity used at anywhere it is supplied.

Exercise 13 Date:…………………….

1. The monthly electricity charges in a country are scheduled as follows:

First 50 units	GH¢5.00
Next 100 units	GH¢0.12 per unit
Next 150 units	GH¢0.15 per unit
Next 300 units	GH¢0.24 per unit
Remaining units	GH¢0.36 per unit

(a) How much did Mr. Konadu pay for using 930 units in a month?

(b) A man paid GH¢114.00 for electricity consumed in a month. How many units of electricity did he consume?

2. The monthly electricity charges in a country are calculated as follows:
   First 50 units – ¢4,000.00
   Next 100 units – ¢120.00 per unit
   Next 150 units – ¢150.00 per unit
   Next 300 units – ¢220.00 per unit
   Remaining – ¢350.00 per unit

   (a) How much did Mr. Owusu pay for using 720 units in a month?
   (b) A man paid ¢73,260.00 for electricity consumed in a month. How many units of electricity did he consume?

WATER TARIFFS

It is a tariff charged by the Ghana Water Company for the amount of water consumed.

Exercise 14 Date:…………………….

1. Ghana Water Company Tariff for January 2008.

First 10 litres	50p	per litre
Next 20 litres	75p	per litre
Next 30 litres	GH¢1	per litre
Next 50 litres	GH¢1.25p	per litre
Remaining litres	GH¢1.50p	per litre

(a) Mr. Koranteng used 310 litres of water in January 2008. How much did he pay for the water?

(b) Mr. Addo paid GH¢740.00 in a month. Calculate the number of litres he used in that month.

2. In a household, the meter reading for water at the end of October, 1999 was 7848 thousand litres. The meter reading at the end of November, 1999 was 8908 thousand litres.
   The household was charged for the consumption at the following rates:
   - The first 10 thousand litres at ¢500.00 per thousand litres.
   - The next 30 thousand litres at ¢1,300.00 per thousand litres.
   - The next 40 thousand litres at ¢1,820.00 per thousand litres.

Calculate

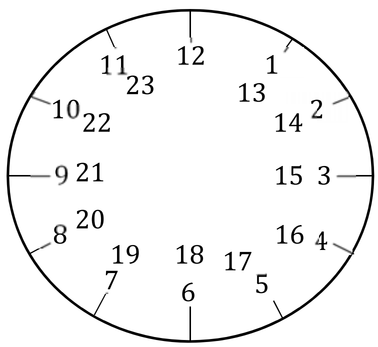
(a) the consumption at the end of November.

(b) the total charge for the consumption.

MODULO ARITHMETIC

It is a system of arithmetic for integers where numbers "wrap around" after they reach a certain value (modulus). Better still it is a system of arithmetic in which after division only remainders are considered.

For example, our time is divided into 24hrs number from 0 to 23.



Clearly from the diagram,
$13 = 1 +$ some multiple of 12
$= 1 + 1(12)$

i.e. the remainder when 13 is divided by 12 is 1 and it is written as

$13 \equiv 1 \text{mod } 12$ (read as 13 is congruent to 1 mod 12).

$14 \equiv 2 + 1(12)$
i.e. the remainder when 14 is divided by 12 is 2 and it is written as $14 \equiv 2\text{mod}12$.

In general,
Let $n > 1$, $a, b \in \mathbb{Z}$, we say that a is congruent to b. modulo n if and only if n divides $(a - b)$.
n is the modulus of the congruence. This can be written mathematically as:

$$\boxed{a \equiv b \bmod n}$$

i.e. b is the remainder when n divides a.

Example 1
1. $\frac{18}{13} = 1\text{R}5$
$\Longrightarrow 18 \equiv 5\text{mod}13$
i.e. 5 is the remainder when 18 is divided by 13.

2. Also $\frac{17}{5} = 3R2$
$17 = 2\text{mod}5$
2 is the remainder when 17 is divided by 5.
Clearly in modulo the idea of remainder is paramount (important).

3. $\frac{16}{4} = 4R0$
$\Longrightarrow 16 \equiv 0\text{mod}4$

Note:
If the remainder is less than the modulus, we leave it as it is.

Example 2
Simplify the following in the moduli given
1. 9mod5 2. 18mod 7 3. 5 mod 8

Solution…
1. $\frac{9}{5} = 1R4$ $\therefore 9 \equiv 4\text{mod}5$
2. $\frac{18}{7} = 2R4$ $\therefore 18 \equiv 4\text{mod}7$
3. Since $5 < 8$ $\therefore 5\text{mod}8 = 5$

Exercise 1 **Date:………………..**
Simplify the following
1. 27(mod7)
2. 66(mod4)
3. 111(mod13)
4. 221(mod12)
5. 391(mod15)

Example 3
Simplify $17 + 3 + 46(\text{mod}15)$

Solution…
$17 + 3 + 46 = 66$
$\therefore \frac{66}{15} = 4R6$
$\therefore 17 + 3 + 46 \equiv 6\text{mod}15$

Exercise 2 **Date:………………..**
Find the sum of the following
1. $47 + 37(\text{mod}9)$
2. $3 + 4 + 1 + 2 + 3(\text{mod}5)$
3. $14 + 67 + 35(\text{mod}7)$
4. $893 + 412(\text{mod}4)$

MODULO OF A NEGATIVE NUMBER

The modulo of a negative number is obtained by adding the modulo to the number until we get a positive number.

Example 4

Simplify the following.

1. $-2\text{mod}5$
2. $-8\text{mod}3$
3. $-12\text{mod }5$

Solution…

1. $-2\text{mod}5$
   $-2+5=3$
   $\therefore -2 \equiv 3\text{mod}5$

2. $-8\text{mod}3$
   $-8+3=-5$
   $-5+3=-2$
   $-2+3=1$
   $\therefore -8 \equiv 1\text{mod}3$

3. $-12\text{mod}5$
   $-12+5=-7$
   $-7+5=-2$
   $-2+5=3$
   $\therefore -12 \equiv 3\text{mod}5$

Addition, Subtraction and Multiplication of Congruencies

If $a \equiv b(\text{mod}m)$ and $c \equiv d(\text{mod}n)$

Then,

(i) $a+c \equiv b+d(\text{mod}m)$
(ii) $a-c \equiv b-d(\text{mod}m)$
(iii) $ac \equiv bd(\text{mod}m)$

Example 5

$25 \equiv 1\text{mod}4$………………….. (1)
$9 \equiv 1\text{mod}4$…………………… (2)

$(1)+(2) \Longrightarrow 25+9 \equiv 2\text{mod}4$

Exercise 3 Date:……………….

1. What is the necessary and sufficient condition for $a \equiv b(\text{mod}n)$ to hold?
2. If $\text{A} \div \text{B} = \text{R}$ remainder 3, write this in congruence notation form.
3. Evaluate $-1[3-(1-5)](\text{mod}2)$
4. Determine whether the statement is true or false? Show working.
   $4 \equiv -1[1+2-1+5-(-5-5)](\text{mod}3)$

Exercise 4 Date:……………….

1. If $6 \times 7 \equiv m(\text{mod}8)$. Find m.
2. If $23+14 \equiv a(\text{mod}5)$, find a.
3. In what modulus is it true that $9+8=5$?
4. Evaluate $8+6(\text{mod}5)$
5. Evaluate $6-36(\text{mod}9)$
6. Find x such that $5+1+3 \equiv 2 \text{ mod } x$.

Example 6

Draw an addition $\oplus$ table for arithmetic modulo 5.

Using your table evaluate;

(a)
(i) $2 \oplus 4$
(ii) $(3 \oplus 2) \oplus 1$
(iii) $(2 \oplus 3) \oplus (3 \oplus 4)$

(b) Find the truthset of $4 \oplus x = 2$

Solution…

$\oplus$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(a) From the table,

(i) $2 \oplus 4 = 1$

(ii) $(3 \oplus 2) \oplus 1$
$= 0 \oplus 1$
$= 1$

(iii) $(2 \oplus 3) \oplus (3 \oplus 4)$
$= 0 \oplus 2$
$= 2$

(b) When $x=0, 4 \oplus 0 = 4$
$x=1, 4 \oplus 1 = 0$
$x=2, 4 \oplus 2 = 1$
$x=3, 4 \oplus 3 = 2$
$x=4, 4 \oplus 4 = 3$

$\{x : x = 3\}$

Example 7

(a) Draw a table for multiplication $\otimes$ modulo 7 on the set $P = \{2, 3, 4, 5, 6\}$.

(b) Use your table to find on the set P, the truthset of $n \otimes (n \otimes 6) = 3$.

Solution…

(a)

$\otimes$	2	3	4	5	6
2	4	6	1	3	5
3	6	2	5	1	4
4	1	5	2	6	3
5	3	1	6	4	2
6	5	4	3	2	1

(b) $n \otimes (n \otimes 6) = 3$

When $n = 2$,
$2 \otimes (2 \otimes 6) = 2 \otimes 5 = 3$

When $n = 3$,
$3 \otimes (3 \otimes 6) = 3 \otimes 4 = 5$

When $n = 4$,
$4 \otimes (4 \otimes 6) = 4 \otimes 3 = 5$

When $n = 5$,
$5 \otimes (5 \otimes 6) = 5 \otimes 2 = 3$

When $n = 6$,
$6 \otimes (6 \otimes 6) = 6 \otimes 1 = 6$

$\therefore \{n : n = 2, 5\}$

Exercise 5 Date:………………..

(a) Copy and complete the multiplication $\otimes$ table for modulo 8 on the set $\{2, 3, 4, 5, 6, 7\}$.

$\otimes$	2	3	4	5	6	7
2	4	6	0	2	4	6
3	6		4		2	5
4				4	0	4
5		7		1		3
6	4	2	0	6	4	
7	6		4		2	1

(b) From the table, find
 (i) the truthset of the equation $n \otimes n = 1$.
 (ii) the value of $(3 \otimes 5) \otimes 6$.

Exercise 6 Date:………………..

(a) Draw a table for multiplication $\otimes$ in modulo 8 on the set $\{3, 5, 7\}$.

(b) Using your table
 (i) Evaluate $(7 \otimes 3) \otimes 5$ and $7 \otimes (3 \otimes 5)$.
 (ii) Find the truthset of $n \otimes n = 1$.

Exercise 7 Date:………………..

1.
 (a) Draw a table of multiplication $\otimes$ in modulo 8 on the set $\{2, 3, 5, 7\}$.
 (b) Use your table to find the solution set of
  (i) $3 \otimes n = 5$ (ii) $n \otimes n = 1$

2.
 (a) Copy and complete the multiplication table modulo 5 on the set $\{1, 2, 3, 4\}$.

$\otimes$	1	2	3	4
1	1		3	
2		4	1	
3	3	1		2
4	4	3		1

 (b) From the table
  (i) Solve the expression $2n \otimes 4 = 3$
  (ii) Find the value of n for which $2 \otimes (3 \otimes n) = 2$.

Exercise 8 Date:………………..

1. Copy and complete the following table for multiplication modulo 11.

$\otimes$	1	5	9	10
1	1	5	9	10
5	5			
9	9			
10	10			

Use the table to
 (i) Evaluate $(9 \otimes 5) \otimes (10 \otimes 10)$
 (ii) Find the truthset of
  (α) $10 \otimes m = 2$
  (β) $n \otimes n = 4$

2.
 (a) Construct a table for multiplication in modulo 7 on the set $\{2, 3, 5, 6\}$. Use the table to solve the following equations.
  (i) $m \otimes m = 2$

(ii) $n \otimes (n \otimes 6) = 3$

(b) Draw the multiplication table for the set $\{1, 2, 3, 4, 5, 6\}$ in (mod7).
(i) Find the identity element
(ii) Find the inverse element
(iii) Find the value of n for which $5 \otimes (3 \otimes n) = n$.

Exercise 9 **Date:………………**

1.
(a) Draw a table of multiplication $\otimes$ in modulo 8 on the set $\{2, 3, 5, 7\}$.
(b) Use your table to find
(i) $3 \otimes n = 5$
(ii) $n \otimes n = 1$

2.
(a) Draw the table for
(i) Addition $\oplus$ modulo 7;
(ii) Multiplication $\otimes$ modulo 7 on the set $\{0, 1, 2, 3, 4\}$.

(b) From your tables evaluate
(i) $m \otimes m = 2$
(ii) $m \oplus (m \otimes 4) = 5$
(iii) $m \otimes (m \oplus 3) = 0$

3.
(i) Draw the multiplication $\otimes$ table for arithmetic modulo 7
(ii) Using the table
(α) State with reasons whether or not the operation is commutative.
(β) Evaluate $(4 \otimes 6) \otimes (5 \otimes 4)$
(γ) Find the truthset of $n \otimes n \equiv n$.

4.
(a) Draw up tables of addition (mod6) and multiplication (mod6) for the set $\{0, 1, 2, 3, 4, 5\}$.
(b)
(i) State with reasons whether or not the operation is commutative.
(ii) State with reasons whether or not the operation is associative.
(iii) Find the identity element
(iv) Find the inverse element.

5.
(i) Copy and complete the tables of addition $\oplus$ and multiplication $\otimes$ in modulo 5.

$\oplus$	1	2	3	4
1	2	3	4	0
2	3			
3	4			2
4	0			

$\otimes$	1	2	3	4
1	1	2	3	4
2	2			
3				2
4				1

(ii) Use the tables to find;
(a) $4 \otimes 2 \oplus 3 \otimes 4$
(b) m such that $m \otimes m = m \oplus m$
(c) n such that $3 \oplus n = 2 \otimes n$

6. An operation $*$ is defined on the set $X = \{1, 3, 5, 6\}$ by $m * n = m + n + 2(\text{mod}7)$, where $m, n \in X$.
(i) Draw a table for the operation.
(ii) Using the table, find the truthset of
(α) $3 * n = 3$ (β) $n * n = 3$

7. The operation Δ is defined on the set $T = \{2, 3, 5, 7\}$ by $x \,\Delta\, y = (x + y + xy)$ mod8.
(i) Construct modulo 8 table for the operation Δ on the set T.
(ii) Use the table to find
(α) $2 \,\Delta\, (5 \,\Delta\, 7)$ (β) $2 \,\Delta\, n = 5 \,\Delta\, 7$

CONGRUENCE EQUATIONS

Example 8
Find the truthset of $3x + 1 \equiv 4\text{mod}6$.

Solution…
Congruent class of mod6 $= 0, 1, 2, 3, 4, 5$

$x \equiv$	0	1	2	3	4	5	(mod6)
$3x + 1 \equiv$	1	4	0	4	1	4	(mod6)

$\therefore \{x = 1, 3, 5\}$

Example 9
Find the truthset such that $x^2 \equiv 1\text{mod}6$.

Solution…
Congruent class of $6 \equiv 0, 1, 2, 3, 4, 5$.

$x \equiv$	0	1	2	3	4	5	(mod6)
$x^2 \equiv$	0	1	4	3	4	1	(mod6)

$\therefore \{x: x = 1, 5\}$

Exercise 10 **Date:……………….**
Find the truthset of the following such that
1. $3x + 1 \equiv 7(\text{mod}9)$
2. $2x + 5 \equiv 1(\text{mod}7)$
3. $x^2 \equiv 4(\text{mod}6)$
4. $x + 7 \equiv 2(\text{mod}13)$
5. $x + 31 \equiv 17(\text{mod}37)$

Exercise 11 **Date:……………….**
Solve the congruence equations.
1. $2x \equiv 3(\text{mod}9)$
2. $4x \equiv 6(\text{mod}10)$
3. $3x \equiv 4(\text{mod}6)$
4. $x^2 \equiv -1(\text{mod}13)$
5. $x^2 \equiv -1(\text{mod}7)$

Exercise 12 **Date:……………….**
1. If x is a positive integer, determine the least value of x for which
   (i) $2x \equiv 3(\text{mod}8)$
   (ii) $2x + 1 \equiv 4(\text{mod}7)$
   (iii) $5 + x \equiv 2(\text{mod}7)$

2. Find the solution set in each case.
   (i) $3x + 4 \equiv 0(\text{mod}5)$
   (ii) $x^2 \equiv 3(\text{mod}5)$
   (iii) $x^2 + 1 \equiv 0(\text{mod}5)$
   (iv) $4 + x \equiv 2(\text{mod}6)$
   (v) $x^3 \equiv 1(\text{mod}8)$

3.
   (a) List all the integers of x in the set $\{x \in \mathbb{Z}: 1 \le x \le 100, x \equiv (6\text{mod}11)\}$

   (b) Solve the simultaneous congruence equations.
   $x \equiv 1(\text{mod}2)$
   $x \equiv 2(\text{mod}3)$

4. Solve the congruence equations.
   (i) $2x \equiv 5(\text{mod}9)$
   (ii) $x^3 \equiv 6(\text{mod}7)$
   (iii) $x^4 \equiv 1(\text{mod}13)$
   (iv) $x^2 + 2x + 3 \equiv 0(\text{mod}6)$

5. List all the integers x in the set $\{x \in \mathbb{Z}: 1 \le x \le 50, x \equiv 3\text{mod}7\}$

INDICES

a^n means a is multiplied by itself n times.

$a^n = a \times a \times a \times \ldots \times a$ (n factors)

Example 1

1. $3^4 = 3 \times 3 \times 3 \times 3 = 81$

2. $\left(\frac{2}{3}\right)^5 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

RULES OF INDICES

Rule 1

$$\boxed{a^m \times a^n = a^{m+n}}$$

Example 2

Express the following products as powers.

(i) $2^4 \times 2^3$

(ii) $3^{2x-y} \times 3^{3y-x}$

Solution…

(i) $2^4 \times 2^3 = 2^{4+3} = 2^7$

(ii) $3^{2x-y} \times 3^{3y-x}$
$= 3^{2x-y+3y-x}$
$= 3^{x+2y}$

Exercise 1 Date:……………………

(a) Express the following products as powers.
1. $7 \times 7 \times 7 \times 7$
2. $3 \times 3 \times 2 \times 3 \times 2 \times 2$
3. $x \times y \times x \times y \times x \times y$
4. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

(b) Express the following powers as products.
1. 6^3
2. 2^6
3. $(0.7)^4$
4. $\left(\frac{2}{3}\right)^6$
5. $\left(\frac{1}{2}\right)^5$

Exercise 2 Date:……………………

Express each of the following as a single power.
1. $y^3 \times y^{13} \times y^{17}$
2. $r^3 \times r^{15} \times r^7$
3. $2y^2 \times 2y^4$
4. $24q^2 \times 8q^{-3}$
5. $2^6 \times 2^2$
6. $4x^3y \times 5x^2y$
7. $5xy^8 \times 3x^6y^{-5}$
8. $3x^2y^3 \times x^4y$

Rule 2

$$\boxed{\frac{a^m}{a^n} = a^{m-n}} \quad \text{or} \quad \boxed{a^m \div a^n = a^{m-n}}$$

where, $m > n$.

Example 3

Express each of the following as a single power with a positive exponent.

1. $\frac{a^6}{a^3}$
2. $\frac{2^7}{2^4}$
3. $\left(\frac{1}{4}\right)^4 \div \left(\frac{1}{4}\right)^{-2}$

Solution…

1. $\frac{a^6}{a^3} = a^{6-3} = a^3$
2. $\frac{2^7}{2^4} = 2^{7-4} = 2^3$
3. $\left(\frac{1}{4}\right)^4 \div \left(\frac{1}{4}\right)^{-2} = \left(\frac{1}{4}\right)^{4-(-2)}$
$= \left(\frac{1}{4}\right)^{4+2}$
$= \left(\frac{1}{4}\right)^6$

Exercise 3 Date:……………………

Simplify
1. $6p^3 \times 2p^2 \times 4p^4 \times 2p^5$
2. $5a^3c^2 \times 2a^2c^7$
3. $4x^3y \times 5x^2y$
4. $\frac{(3x^6)^2}{x^{-4}}$
5. $18x^2y^6 \div 2xy^2$
6. $a^3b^7 \div a^6b^2$
7. $8a^3 \times 9b^2$
8. $2a^2 \times 3b^3 \times 6c \times 10a^3b^4c^5$
9. $\frac{2}{3}ap^3 \times \frac{3}{4}pa^2 \times \frac{1}{2}aq^2 \times \frac{2}{5}qa^2$

Exercise 4 Date:……………………

Express each of the following in the simplest form with a positive exponent.
1. $2^6 \div 6^2$
2. $8^7 \div 8^3$
3. $\frac{3^2 \times 3^5 \times 3^7}{3^3 \times 3^6}$
4. $\frac{b^7 \times b^{10} \times b^5}{b^3 \times b^{20} \times b^7}$
5. $\frac{m^{13} \times m^{17} \times m^{15}}{m^{23} \times m^{27}}$

Rule 3

$$\boxed{(a^m)^n = a^{mn}}$$

Example 4
Simplify
1. $(4^3)^2$
2. $(6^2)^5$

Solution…
1. $(4^3)^2 = 4^{3\times2} = 4^6$
2. $(6^2)^5 = 6^{2\times5} = 6^{10}$

Example 5
Express $\frac{4b^3\times3b^4}{b^2\times(b^3)^2}$, where $b \neq 0$ in its simplest form with a positive exponent.

Solution…

$$\frac{4b^3\times3b^4}{b^2\times(b^3)^2} = \frac{12b^{3+4}}{b^2\times b^6}$$
$$= \frac{12b^7}{b^8}$$
$$= \frac{12}{b}$$

Exercise 5 Date:……………………..
Write each of the following in the simplest form with positive exponents.
1. $(2a)^3 \times (3a^2)^2$
2. $(a^2)^3 \times (a^3)^3$
3. $(y^3)^2 \div (y^2)^4$
4. $(x^2)^3 \div (x^3)^4$
5. $\frac{12ab^2}{48a^2b}$
6. $\frac{(3a)^2}{7b} \div \frac{a^3}{14b^2}$
7. $\frac{9a^3b^2}{2c^4} \div \frac{3a^2b^4}{4ac^2}$

Rule 4

$$\boxed{(ab)^n = a^nb^n}$$

Example 6
Simplify the following
1. $(2x)^2$
2. $\left(2\sqrt{5}\right)^2$
3. $(3a^2bc^2)^3$

Solution…
1. $(2x)^2 = 2^2x^2 = 4x^2$
2. $(2\sqrt{5})^2 = 2^2 \times \left(\sqrt{5}\right)^2$
   $= 4 \times 5$
   $= 20$
3. $(3a^2bc^2)^3 = 3^3(a^2)^3b^3c^{2\times3}$
   $= 27a^6b^3c^6$

Exercise 6 Date:……………………..
Simplify these expressions.
1. $(2x^2y)^3$
2. $(3x^3y^4)^2$
3. $(8x^2y^3)^2$
4. $(5x^2y^5)^3$
5. $(2x^2y^3)^4 \times (3x^2y^3)^3$

Rule 5

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$$

Example 7
Simplify

(i) $\left(\frac{2}{3}\right)^4$ (ii) $\left(\frac{3}{4}\right)^5$

Solution…

(i) $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

(ii) $\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5} = \frac{243}{1024}$

Example 8
Simplify the following using positive indices.
1. $\frac{(2^3\times3)^4}{(3^2\times2)^3}$
2. $\left(\frac{2}{3}\right)^7 \times \left(\frac{3}{2}\right)^4$
3. $(2a^2)^2 \times (3a)^2$
4. $\frac{(2a)^3}{(a^2b)^2}$

Solution…
1. $\frac{(2^3\times3)^4}{(3^2\times2)^3} = \frac{2^{3\times4}\times3^4}{3^{2\times3}\times2^3}$
   $= \frac{2^{12}\times3^4}{3^6\times2^3} = \frac{2^{12-3}}{3^{6-4}} = \frac{2^9}{3^2}$
2. $\left(\frac{2}{3}\right)^7 \times \left(\frac{3}{2}\right)^4 = \frac{2^7}{3^7} \times \frac{3^4}{2^4}$
   $= \frac{2^{7-4}}{3^{7-4}} = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$
3. $(2a^2)^2 \times (3a)^3$
   $= 2^2a^{2\times2} \times 3^3a^3$
   $= 4a^4 \times 27a^3$

$= 4 \times 27 \times a^4 \times a^3$
$= 108a^{4+3}$
$= 108a^7$

4. $\frac{(2a)^3}{(a^2b)^2} = \frac{2^3a^3}{a^{2\times2}b^2}$
$= \frac{8a^3}{a^4b^2} = \frac{8}{a^{4-3}b^2} = \frac{8}{ab^2}$

Exercise 7 Date:……………………

Write the following in their simplest forms using positive exponents.

1. $\frac{(3a)^2\times(4a^2)^2}{(2a^2)^3\times a^3}$
2. $\frac{(4^2\times3)^3}{(3^2\times4)^2}$
3. $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{2}\right)^2$
4. $\frac{(a^3)^2\times(ab^2)^2}{(b^3)^2\times(ab)^2}$
5. $\frac{(bc)^3\times(b^2c)^2}{(b^3)^2\times(c^3)^3}$

Zero Index

$a^0 = 1$ i.e any non – zero number to the power zero (0) is 1, where a is any real number and $a \neq 0$.
For example, $2^0 = 1, (-100)^0 = 1$.

Example 9

Find the value of
(a) $2x^0 + (k)^0$
(b) $\left(\frac{k}{3}\right)^0 - \left(\frac{1}{k}\right)^0$
For all $x, k \in \mathbb{R}/0$

Solution…

(a) $2x^0 + k^0 = 2(1) + 1 = 3$
(b) $\left(\frac{k}{3}\right)^0 - \left(\frac{1}{k}\right)^0 = 1 - 1 = 0$

Negative Indices

$a^{-n} = \frac{1}{a^n}$, where $a \neq 0$.

Example 10

Find the value of
(a) 5^{-2} (b) $\left(\frac{2}{3}\right)^{-3}$

Solution…

(a) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
(b) $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3} = \frac{1}{\frac{2^3}{3^3}} = \frac{3^3}{2^3} = \frac{27}{8}$

Exercise 8 Date:……………………

Calculate the values of these expressions.

1. 2^{-3}
2. 10^{-2}
3. $\left(\frac{1}{2}\right)^3$
4. $\left(\frac{2}{3}\right)^2$
5. $\left(\frac{5}{3}\right)^4$
6. $\left(\frac{2}{5}\right)^{-3}$
7. $\left(\frac{2}{3}\right)^{-4}$

Exercise 9 Date:……………………

Simplify the following expressions.

1. $5^{-3} \times 5^{-2} \times 5^{-8}$
2. $4^7 \times 4^{-5} \times 4^{-2}$
3. $a^6 \times a^{-4} \times a^2$
4. $\frac{(4^2)^{-2}\times(4^3)^3}{(4^{-3})^2\times(4^2)^{-2}}$
5. $\frac{(2b^2)^{-3}\times(4b)^3}{(2^3b)\times b^2}$
6. $\frac{(ab)^{-3}\times(a^{-2}b)^2}{(ab^{-1})^{-2}\times(a^3b)^{-2}}$

Fractional Indices

1. $\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$

Example 11

Evaluate the following
(a) $16^{\frac{1}{2}}$ (b) $27^{\frac{1}{3}}$

Solution…

(a) $16^{\frac{1}{2}} = 4^{2\times\frac{1}{2}} = 4$
(b) $27^{\frac{1}{3}} = 3^{3\times\frac{1}{3}} = 3$

Example 12

Find the value of $\frac{6^{\frac{1}{2}}\times96^{-\frac{1}{4}}}{216^{-\frac{1}{4}}}$.

Solution…

$\frac{6^{\frac{1}{2}}\times96^{-\frac{1}{4}}}{216^{-\frac{1}{4}}} = 6^{\frac{1}{2}} \times \left(\frac{96}{216}\right)^{-\frac{1}{4}}$
$= 6^{\frac{1}{2}} \times \left(\frac{216}{96}\right)^{\frac{1}{4}} = 6^{\frac{1}{2}} \times \left(\frac{9}{4}\right)^{\frac{1}{4}}$
$= (2 \times 3)^{\frac{1}{2}} \times \left(\frac{3}{2}\right)^{2\times\frac{1}{4}}$
$= 3^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times \frac{3^{\frac{1}{2}}}{2^{\frac{1}{2}}}$
$= 3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$
$= 3^{\frac{1}{2}+\frac{1}{2}}$
$= 3$

2. $$\boxed{a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m}$$

Example 13
Evaluate

(a) $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$ (b) $\left(\frac{1}{64}\right)^{-\frac{5}{6}}$

Solution…

(a) $\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\frac{3}{2}\right)^{3\times\frac{2}{3}} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$

(b) $\left(\frac{1}{64}\right)^{-\frac{5}{6}} = (64)^{\frac{5}{6}} = 2^{6\times\frac{5}{6}} = 2^5 = 32$

Exercise 10 Date:……………………

Simplify each of the following

1. $81^{\frac{1}{4}}$
2. $27^{\frac{2}{3}}$
3. $125^{-\frac{1}{3}}$
4. $\left(\frac{1}{16}\right)^{-\frac{3}{4}}$
5. $64^{\frac{2}{3}}$

Exercise 11 Date:……………………

Evaluate these expressions without using decimals.

1. 3^{-4}
2. 5^{-3}
3. $\left(\frac{3}{4}\right)^2$
4. $\left(\frac{27}{64}\right)^{\frac{1}{3}}$
5. $\left(\frac{16}{625}\right)^{\frac{1}{4}}$

Exercise 12 Date:……………………

Simplify the following

1. $(3p^2)^5$
2. $18x^2y^6 \div 2xy^2$
3. $\left(\frac{5}{m}\right)^{-2}$
4. $(64q^{-2})^{\frac{1}{2}}$
5. $(27x^{12})^{\frac{1}{3}}$
6. $x^{\frac{2}{3}} \div x^{-\frac{4}{3}}$

Exercise 13 Date:……………………

Simplify

1. $81^{\frac{3}{4}}$
2. $\left(\frac{8}{y^6}\right)^{-\frac{1}{3}}$
3. $\left(\frac{8}{a^{12}}\right)^{\frac{1}{3}}$
4. $\left(\frac{1}{81}\right)^{-\frac{3}{4}}$
5. $\sqrt[3]{27t^{27}}$
6. $\left(\frac{16}{9x^4}\right)^{-\frac{3}{2}}$

Exercise 14 Date:……………………

Simplify the following

1. $\frac{(3x^6)^2}{x^{-4}}$
2. $\left(\frac{x^3}{64}\right)^{\frac{2}{3}}$
3. $\left(\frac{27}{8}\right)^{-\frac{4}{3}}$
4. $\left(\frac{49}{16}\right)^{-\frac{3}{2}}$
5. $\left(\frac{16}{81}x^{16}\right)^{\frac{1}{2}}$
6. $(3\sqrt{x})^6$
7. $\frac{3y^4\times(10y)^3}{2y^5}$
8. $\frac{(4x)^2\times 2x^3}{x}$
9. $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$
10. $\left(\frac{125}{27}\right)^{-\frac{1}{3}}$

Exercise 15 Date:……………………

(α) Express each of the following in the form 3^n.

(i) $\frac{1}{9}$ (ii) $\sqrt[3]{3}$ (iii) $3^{16}\times 9^{15}$

(β) Express each of the following in the form 8^p.

(i) $\sqrt{8}$ (ii) $\frac{1}{64}$ (iii) $2^6\times 2^3$

(γ)

(a) Express each of the following in the form 7^k.

(i) $\sqrt[4]{7}$ (ii) $\frac{1}{7\sqrt{7}}$ (iii) $7^4\times 49^{10}$

(b) Express $2^{40}\times 4^{30}$ in the form 2^n.

Exercise 16 Date:……………………

(a) Evaluate the following

(i) 6^0
(ii) $2^{-1}\times 32^{\frac{4}{5}}$
(iii) 4^{-2}
(iv) $\left(3\sqrt{2}\right)^2$
(v) $(1^3+2^3+3^3)^{\frac{1}{2}}$
(vi) $\left(4^{\frac{1}{2}}+9^{\frac{1}{2}}+16^{\frac{1}{2}}\right)^2$

(b) Simplify the following

(i) $16^{\frac{1}{4}}\times 125^{\frac{1}{3}}\times 27^{-\frac{1}{3}}$
(ii) $27^{\frac{1}{2}}\times 64^{\frac{1}{3}}\div 81^{\frac{1}{4}}$
(iii) $36^{\frac{1}{2}}\times 64^{-\frac{1}{2}}$
(iv) $\left(\frac{p^4}{16}\right)^{0.75}$
(v) $\left(\frac{x^{-2}}{27}\right)^{-\frac{1}{2}}$

(c) Simplify the following

(i) $\sqrt{\frac{8^2\times 4^{n+1}}{2^{2n}\times 16}}$

(ii) $\frac{49^{\frac{2}{3}}\div 49^{\frac{1}{6}}}{\left(\frac{1}{7}\right)^{-\frac{7}{6}}\times\left(\frac{1}{7}\right)^{\frac{1}{6}}}$

(iii) $\frac{27^{-\frac{1}{2}}\times 81^{\frac{3}{4}}}{9^{\frac{1}{2}}}$

(iv) $32^{-\frac{1}{5}}\times\left(\frac{1}{2}\right)^{-3}$

Exercise 17 Date:.........................

Simplify the following

1. $\frac{1}{3^{5n}}\times 9^{n-1}\times 27^{n+1}$
2. $2\div\left(\frac{64}{125}\right)^{\frac{-2}{3}}$
3. $\left(\frac{4}{25}\right)^{-\frac{1}{2}}\times 2^4\div\left(\frac{15}{2}\right)^{-2}$
4. $(\sqrt{5})^{-2}\times(75)^{\frac{1}{2}}\times(12)^{-\frac{1}{2}}$

Exercise 18 Date:.........................

1. Simplify $\sqrt{\left(\frac{x^3y^5}{xy^7}\right)}$, where $x>0$ and $y>0$.
2. Simplify $\frac{\left(x^2y^{-3}z\right)^{3/4}}{x^{-1}y^4z^5}$
3. Simplify the following

   (i) $\sqrt{\left(\frac{-1}{64}\right)^{\frac{-2}{3}}}$

   (ii) $-\left(\frac{-1}{27}\right)^{\frac{-2}{3}}$

   (iii) $\left[\left(\frac{81}{16}\right)^{\frac{3}{4}}\right]^{\frac{-2}{3}}$

   (iv) $\frac{\left(x^{\frac{3}{2}}+x^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)}{\left(x^{\frac{3}{2}}-x^{\frac{1}{2}}\right)^2}$

4. Find in its simplest form, the product of $a^{\frac{1}{3}}+b^{\frac{2}{3}}$ and $a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}$.
5. Evaluate each of the following without the use of a calculator.

   (i) $16^{-\frac{3}{2}}$ (iv) $\left(\frac{81}{256}\right)^{\frac{3}{4}}$

   (ii) $\left(4\sqrt{5}\right)^2$ (v) $\left(\frac{25}{49}\right)^{-\frac{1}{2}}$

   (iii) $\frac{9^{-\frac{1}{2}}}{27^{\frac{2}{3}}}$

6. Given that $x^y=z$, find

   (i) z when $x=9$ and $y=-\frac{1}{2}$

   (ii) z when $x=64$ and $y=-\frac{1}{3}$

   (iii) x when $y=-\frac{1}{2}$ and $z=4$

   (iv) y when $z=2$ and $x=\frac{1}{2}$

7. Simplify the following expressions.

   (i) $(3xy^4)^3$

   (ii) $(16a^6b^2)^{\frac{1}{2}}$

   (iii) $(125x^6)^{\frac{2}{3}}$

   (iv) $3^2q^{-3}\div 2^3q^{-2}$

   (v) $\frac{q^2+q^2}{q^{\frac{1}{4}}\times q^{\frac{1}{4}}}$

   (vi) $\frac{16y^{10}\times 4y^{-4}}{32y^7}$

   (vii) $(3125t^{125})^{\frac{1}{5}}$

Exercise 19 Date:.........................

Simplify the following

1. $(x^3)^4$	11. $b^3\times b^2$
2. 4^{-2}	12. $\frac{c^4}{c^8}$
3. x^0	13. $(32y^{15})^{\frac{2}{5}}$
4. $x^7\times x^3$	14. $x^3\div\frac{3}{x^5}$
5. $\frac{(3x^6)^2}{x^{-4}}$	15. $\sqrt[3]{2744}$
6. $2^{-4}\times 2^5$	16. $32x^8\div 8x^{32}$
7. $3y^6\times 5y^{-2}$	17. $(256w^{256})^{\frac{1}{4}}$
8. $(27x^{12})^{\frac{1}{3}}$	18. $16^{-\frac{1}{4}}$
9. $r^5\times r^8$	19. $x\left(2x^{-\frac{1}{4}}\right)^4$
10. $6uw^{-3}\times 4uw^6$	20. $25^{-\frac{3}{2}}$

Solving Exponential Equations

We can solve an exponential equation by:

Case 1

Expressing both sides in terms of the same base and equating the exponents.

i.e. $\boxed{a^m=a^n\Longrightarrow m=n}$

Example 14

Solve the equation $3^x=9$.

Solution…

$3^x=9$

$3^x=3^2$ (making the base the same)

$x=2$ (Equating the exponent)

Example 15
Solve for x.
1. $2^3 \times 2^x = 2^7$
2. $5^x \div 5^9 = 5^7$
3. $a^5 \times a^x = a^6$

Solution…
1. $2^3 \times 2^x = 2^7$
$2^{3+x} = 2^7$
$3 + x = 7$
$x = 7 - 3 = 4$

2. $5^x \div 5^9 = 5^7$
$5^{x-9} = 5^7$
$x - 9 = 7$
$x = 7 + 9$
$x = 16$

3. $a^5 \times a^x = a^6$
$a^{5+x} = a^6$
$5 + x = 6$
$x = 6 - 5$
$x = 1$

Case 2:
Expressing both sides in terms of the same index and then equating the base.

$$\boxed{a^n = b^n \Longrightarrow a = b}$$

Example 16
Solve the following equations.
1. $x^5 = 32$
2. $(x-1)^3 = 64$
3. $x^{-2} = \frac{1}{81}$
4. $(0.25)^{x+1} = 16$

Solution…
1. $x^5 = 32$
$x^5 = 2^5$
$x = 2$

2. $(x-1)^3 = 64$
$(x-1)^3 = 4^3$
$x - 1 = 4$
$x = 4 + 1 = 5$

3. $x^{-2} = \frac{1}{81}$
$x^{-2} = \frac{1}{9^2}$
$x^{-2} = 9^{-2}$
$x = 9$

4. $(0.25)^{x+1} = 16$
$\left(\frac{1}{4}\right)^{x+1} = 4^2$
$4^{-1(x+1)} = 4^2$
$-(x+1) = 2$
$-x - 1 = 2$
$-x = 2 + 1$
$-x = 3$
$x = -3$

Example 17
Solve the following equations.
(a) $5^x = 1$
(b) $7^x = 1$

Solution…
(a) $5^x = 1$
$5^x = 5^0$ (since $5^0 = 1$)
$x = 0$

(b) $7^x = 1$
$7^x = 7^0$ (since $7^0 = 1$)
$x = 0$

Exercise 20 **Date:……………………..**
Solve the following equations.
1. $3^m = 81$
2. $9^x = 3$
3. $729^x = 81$
4. $8^x = 32$
5. $8^{x-2} = 4^{3x}$
6. $x^3 = 27$
7. $(x+1)^5 = 243$
8. $(2x+3)^3 = 125$
9. $(x-1)^5 = \frac{32}{243}$
10. $\frac{1}{5^{-y}} = 25(5^{4-2y})$

Exercise 21 **Date:……………………..**
Solve the following equations.
1. $(36p^4)^{\frac{1}{2}} = 24$
2. $t^{-\frac{1}{3}} = \frac{1}{2}$
3. $t^{-3} = 64$
4. $(8p^6)^{\frac{1}{3}} = 8$
5. $(y^{-2})^2 = \frac{1}{81}$
6. $x^{\frac{1}{3}} = 2$
7. $(25k^2)^{\frac{1}{2}} = 15$
8. $27^x = 9^{x-1}$
9. $\left(\frac{1}{9}\right)^{y-1} = \sqrt{3}$
10. $\frac{1}{27} \times 3^{-y} = 18^{2y}$

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 22 Date:……………………..

Solve the following equations.

1. $5^{\frac{r}{3}} = 125$
2. $2^n = 1024$
3. $4^{2n-3} = 16$
4. $2^m = 0.125$
5. $(0.25)^y = 32$
6. $2^{4m} \times 2^{2m} = 512$
7. $\left(\frac{3}{8}\right)^{\frac{3}{8}} \times \left(\frac{3}{8}\right)^{\frac{1}{8}} = p^q$
8. $3^{4x-2} = \left(\frac{1}{27}\right)^{x+3}$

Exercise 23 Date:……………………..

Solve the equations below.

1. $4^w = \frac{1}{16}$
2. $x^{10} \div x^k = x^3$
3. $3^t = \sqrt[5]{3}$
4. $2^r = \frac{1}{16}$
5. $5^7 \div 5^9 = p^2$
6. $\left(\frac{1}{2}\right)^x = 8$
7. $\left(\sqrt{2}\right)^{5x+4} = 4^{2x-1}$

Exercise 24 Date:……………………..

Solve for n:

1. $3^n = 1$
2. $25^n = 1$
3. $\left(\frac{1}{4}\right)^{2-n} = 1$
4. $2 \times 2^{1-n} = 1$
5. $81^{-\frac{3}{4}} \times 3^{3n} = 1$
6. $y^{10} \times y^n = 1$

Exercise 25 Date:……………………..

Solve the following.

1. $2^{x+2} \times 8^x = 1$
2. $2^{3x} = 0.25$
3. $4^{2-x} \times 16^{x+1} = 64$
4. $2^x + 2^{(x-1)} = 48$
5. $3^x + 3^{x+1} = 36$

Exercise 26 Date:……………………..

Solve the following equations.

1. $\frac{64^n \times 2}{16^{1-n}} = 4^{2n}$
2. $\frac{1}{2}(81^n) = \frac{1}{54}$
3. $\frac{\left(\frac{1}{3}\right)^x \times 9^{3\frac{1}{2}}}{3} = 27^{\frac{2}{3}}$
4. $8^x = 2\sqrt{2}$
5. $\left(\frac{1}{27}\right)^{2x-1} = 243\sqrt{3}$
6. $4^{n+2} = \frac{1}{\sqrt{16^{n-1}}}$

Exercise 27 Date:……………………..

Solve for x.

1. $16^{3x} = \frac{1}{4}(32^{x-1})$
2. $16^{3x-1} = 8^{x+2}$
3. $4^{4x-2} = \left(\frac{1}{256}\right)^{2x+3}$
4. $\frac{3^{2x+1}}{3^{2x-4} \times 3^{6-7x}} = 27^x$
5. $\frac{8^x}{\sqrt{8}} = \frac{1}{64}$
6. $2^{x^2-5x} = \frac{1}{64}$
7. $7^{x^2} - 49^{6-2x} = 0$
8. $2^{\sqrt{x}} = 8^x$
9. $4\left(2^{x^2}\right) = 8^x$

Example 18

Solve the equation.

$2^{2x+1} - 9(2^x) + 4 = 0$

Solution…

$2^{2x+1} - 9(2^x) + 4 = 0$

$2^x \times 2^1 - 9(2^x) + 4 = 0$

$2(2^x)^2 - 9(2x^x) + 4 = 0$

Let $2^x = a$

$2a^2 - 9a + 4 = 0$

$(2a-1)(a-4) = 0$

$2a - 1 = 0$ or $a - 4 = 0$

$a = \frac{1}{2}$ or $a = 4$

When $a = \frac{1}{2}$,

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$x = -1$

When $x = 4$,

$2^x = 4$

$2^x = 2^2$

$x = 2$.

Example 19

Solve $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$

Solution…

$x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$

$\left(x^{\frac{1}{3}}\right)^2 - x^{\frac{1}{3}} - 6 = 0$

Let $x^{\frac{1}{3}} = a$
$a^2 - a - 6 = 0$
$(a-3)(a+2) = 0$
$a - 3 = 0$ or $a + 2 = 0$
$a = 3$ or $a = -2$

When $a = 3$,
$x^{\frac{1}{3}} = 3$
$\left(x^{\frac{1}{3}}\right)^3 = 3^3$
$x = 27$

When $x = -2$,
$x^{\frac{1}{3}} = -2$
$\left(x^{\frac{1}{3}}\right)^3 = (-2)^3$
$x = -8$

Exercise 28 Date:……………………..
Find the truthset of the following.
1. $3^{2x} - 5(3^x) + 6 = 0$
2. $2^{2x+2} - 9(2^x) + 2 = 0$
3. $2^{2x+2} - 5(2^x) + 1 = 0$
4. $5^{2x} - 5^{x+1} + 2 = -2(5)^x$
5. $2^x + 32(2^{-x}) - 12 = 0$
6. $2 \cdot 3^{2x+1} + 3(3^{x-1} - 4) = 0$
7. $3x^{1/2} + 5 - 2x^{-1/2} = 0$
8. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$
9. $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 10 = 0$
10. $3x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$
11. $(3x-2)^4 - 5(3x-2)^2 + 4 = 0$

Example 20
Solve the equations.
$2^{x+y} = 8; 2x + y = 4.$

Solution…
$2^{x+y} = 8$
$2^{x+y} = 2^3$
$x + y = 3$……………………… (1)

$2x + y = 4$……………………… (2)
$(2) - (1)$; $\Longrightarrow x = 1$

Put $x = 1$ into (1)
$1 + y = 3$
$y = 2$

Example 21
If $3^m \times 3^n = 243$ and $3^m \div 3^{2n} = 9$, write down two equations connecting m and n. Hence, find the values of m and n.

Solution…
$3^m \times 3^n = 243$
$3^{m+n} = 3^5$
$m + n = 5$…………………………… (1)

$3^m \div 3^{2n} = 9$
$3^{m-2n} = 3^2$
$m - 2n = 2$………………………… (2)

$(1) - (2)$;
$\Longrightarrow 3n = 3 \Longrightarrow n = 1$

Put $n = 1$ into (1)
$m + 1 = 5$
$m = 4$

Exercise 29 Date:……………………..
Solve the equations simultaneously;
$5x - 4y = 6$; $3^{3(y-x)} = \frac{1}{27}$

Exercise 30 Date:……………………..
Solve for x and y in the following equations.
$2^{(x+4y)} = 1$, $2^{(x+8y)} = \frac{1}{4}$

Exercise 31 Date:……………………..
Solve the equations
$2^{x+y} = 8$; $2x + y = 4$

Exercise 32 Date:……………………..
Solve the simultaneous equations;
1. $2^{3m} \times 32^n = 64$
   $9^m \div 3^n = 81$
2. $4^{2x+y} = \frac{1}{256}$
   $2^x \cdot 4^y = 8$
3. $8^x \div 2^y = 64$
   $3^{4x} \times \left(\frac{1}{9}\right)^{y-1} = 81$
4. $2^{4x} \times 4^y \times 8^{x-y} = 1$
   $3^{x+y} = \frac{1}{3}$
5. $\frac{4^x}{256^y} = 1024$
   $3^{2x} \times 9^y = 243$

6. $8^{q-1} \times 2^{2p+1} = 4^7$
   $9^{p-q} \times 3^q = 81$
7. $2^{x+y} = 16$ and $4^{x-y} = \frac{1}{32}$
8. $8 = 2^{(x+y)}$ and $1 = 3^{x-y}$
9. $9^x \times 3^{2y} = \frac{1}{729}$ and $2^{-x} \times 4^{-y} = \frac{1}{8}$
10. If $3^x \times 9^y = 243$ and
    $3^x \div 3^{2y} = \frac{1}{27}$, find $(x+y)$.

Exercise 33 Date:……………………..

1. If $5^x = n$, express 25^{x+1} in terms of n
2. If $3^{-n} = x$, find
   (i) 3^n (ii) 3^{n+1}
3. Solve the following
   (i) $x^2 - 121 = 0$
   (ii) $2\left(\frac{1}{8}\right)^x = 32^{x-1}$
   (iii) $\left(\frac{1}{125}\right)^x - \left(\frac{1}{25}\right)^{\frac{3}{4}} = 0$
   (iv) $3^{2x-1} = \frac{1}{27}$
   (v) $(0.25)^x = 32$
   (vi) $2^{3x} = 0.25$
   (vii) $\frac{2^{1-y} \times 2^{y-1}}{2^{y+2}} = 8^{2-3y}$
   (viii) $16^n = \sqrt[3]{2^2}$
4. Solve for x
   (i) $8 \times \left(\frac{1}{2}\right)^x = 1$
   (ii) $3 \times \left(\frac{1}{2}\right)^x = 12$
   (iii) $3^{2x+1} \times 9^{-x} = \left(\frac{1}{3}\right)^{x+1}$
   (iv) $\left(\frac{3}{2}\right)^x = \left(\frac{8}{27}\right)^{-\frac{2}{3}}$
   (v) $\frac{1}{9^{(x-1)}} = 27^{(4-3x)}$
   (vi) $\frac{1}{81^{(x-2)}} = 27^{(1-x)}$
   (vii) $3 \times 9^{1+x} = 27^{-x}$
   (viii) $9^{2x} = \frac{1}{3}(27^x)$
   (ix) $2(2^{2x+3}) - 4^{2x+1} = 0$
   (x) $8 \times 2^x = \left(\frac{1}{4}\right)^{3x-5}$
5. Find in its simplest form the product of $a^{\frac{1}{3}} + b^{\frac{2}{3}}$ and $a^{\frac{2}{3}} - a^{\frac{1}{2}}b^{\frac{2}{3}} + b^{\frac{4}{3}}$.
6. Solve the equation.
   (i) $\frac{2^{x-3}}{8^{-x}} = \frac{32}{4^{\frac{1}{2}x}}$
   (ii) $\frac{9^{2y}}{3^{7-y}} = \frac{3^{4y+3}}{27^{y-2}}$
   (iii) $3^{x+1} + 3^{2-x} = 28$
   (iv) $\frac{4^x}{2^{5-x}} = \frac{2^{4x}}{8^{x-3}}$
   (v) $\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^x}$

Exercise 34 Date:……………………..

(α) Solve the following
1. $8^{x+1} = \frac{1}{4}$
2. $9^{(2-x)} = 3$
3. $9^{2x} = \frac{1}{3}(27^x)$
4. $2 \times 4^{1-x} = 8^{-x}$
5. $3 \times 9^{1+x} = 27^{-x}$
6. $\frac{9^{2x-3}}{3^{x+3}} = 1$
7. $(y^{-2})^2 = \frac{1}{81}$
8. $2^x \times 8^{1-x} = \frac{1}{4}$
9. $12 \times 3^{-x} = \frac{4}{3}$
10. $9^{2-x} = \left(\frac{1}{3}\right)^{2x+1}$
11. $10^p = 0.1$
12. $t^{-\frac{1}{3}} = \frac{1}{2}$
13. $(25k^2)^{\frac{1}{2}} = 15$
14. $16^n = \sqrt[3]{2^2}$
15. $\frac{2^{1-y} \times 2^{y-1}}{2^{y+2}} = 8^{2-3y}$

(β) Simplify the following
1. $\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \frac{\sqrt{100}}{\sqrt{81}}$
2. $\left(\frac{27}{125}\right)^{-\frac{1}{3}} \times \left(\frac{4}{9}\right)^{\frac{1}{2}}$
3. $\left(\sqrt{3}\right)^{1-x} \times 9^{1-2x}$
4. $\left(\frac{1}{16}\right)^{-\frac{1}{2}} + \left(\frac{8}{27}\right)^{\frac{2}{3}}$
5. $100^{\frac{3}{2}}$
6. $27^{-\frac{2}{3}}$

Exercise 35 Date:……………………..

1. Simplify the following
   (i) $\frac{a^{3/2}}{b^3} \div \frac{a^{-1}}{b^2}$
   (ii) $\left(\frac{27c^3}{d^3}\right)^{-\frac{1}{3}}$
   (iii) $\sqrt{\frac{x^{-2}y^2}{25x^4}}$
   (iv) $\frac{6x^2y^{-2}}{\sqrt[3]{8x^{-3}}}$

2. Solve the following
   (i) $25^{4-x} = \left(\frac{1}{125}\right)^{x+2}$
   (ii) $16^{3x-1} = 8^{x+2}$
   (iii) $3x^{\frac{2}{3}} = 192$
   (iv) $9x^{-\frac{2}{3}} = 16$
   (v) $8(2^{x+1}) = 2\sqrt{2^x}$
   (vi) $4^{x^2+24} = 16^x$
   (vii) $3^{x^2+2} = 27^x$

3. Simplify
   (i) $\frac{12a^4}{a^3b} \div \frac{4b^2}{a^5b^4}$
   (ii) $\frac{2y}{x} \div \frac{2y^2}{x^4y^3}$
   (iii) $\frac{(2xy)^4}{2xy^3}$
   (iv) $\frac{2d^2 \times 5ad^2}{4d^2}$
   (v) $\frac{(3a^3x^4)^2}{3a^3x^2}$
   (vi) $\frac{8x^{12}y^6}{4x^4y^3}$

4. Simplify
   (i) $\left(\frac{64}{27}\right)^{-\frac{2}{3}}$
   (ii) $(5x^2y^3)^2 \div (125x^9y^6)^{\frac{1}{3}}$
   (iii) $\left(\frac{27a^3}{8b^6}\right)^{-\frac{4}{3}}$
   (iv) $\sqrt[3]{\left(\frac{64}{343}\right)}$

5. Solve the following
   (i) $(x-2)^6 = \frac{64}{729}$
   (ii) $(x+1)^{\frac{1}{4}} = \frac{4}{3}$
   (iii) $32^{x^2} = 4^{2-x}$
   (iv) $2^{8-x^2} = 128^{-x}$
   (v) $9^{x^2+1} = 243^{-x}$
   (vi) $3^{x^2+4x} = \frac{1}{81}$

Exercise 36 **Date:……………………..**

1. Solve the following
   (i) $5^{2x+1} = \frac{1}{25}$
   (ii) $4^{3x+1} = 64$
   (iii) $2^x \times 4^{x+1} = 8$
   (iv) $2 \times 4^{x+1} = 16^{2x}$
   (v) $16^{x+2} = \frac{1}{4}$
   (vi) $27^{x^2+x} = 3^{3x^2} \times 9$

2.
   (α) Write down the values of p, q and r given that:
   (i) $64 = 8^p$
   (ii) $\frac{1}{64} = 8^q$
   (iii) $\sqrt{8} = 8^r$

   (β) Evaluate the following
   (i) $32^{\frac{3}{5}}$
   (ii) $(2\sqrt{5})^2$
   (iii) $8^{\frac{5}{3}}$
   (iv) $\left(3^{\frac{1}{2}} + 3^{-\frac{1}{2}}\right)\left(3^{\frac{3}{2}} - 3^{\frac{1}{2}}\right)$
   (v) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$

   (γ) If $x^p = (\sqrt{x})^3$, $x^q = \frac{1}{x^2}$ and $x^r = \frac{x^P}{x^q}$,
   (i) Find the value of r
   (ii) Evaluate x^r when $x = 4$

3.
   (a) Given that $4^{3x+1} = 8^{y+1}$, express y in terms of x.
   (b) If $5^{-x} = 10$, determine the value of $\frac{2^{x-1}+2^{x+1}}{5\times10^x}$.
   (c) Given that $(1 + 2^2 + 3^3 + 4^4)^{\frac{1}{2}} = d\sqrt{2}$, find the value of the integer d.
   (d) Given $m + \frac{1}{m} = 3$
   (i) Determine the value of $m^2 - 1 + \frac{1}{m^2}$.
   (ii) Hence determine the value of $m^3 + \frac{1}{m^3}$.

   (e) Simplify the following
   (i) $x^2\left(4x^{-\frac{1}{2}}\right)^2$
   (ii) $\dfrac{\left(10^{\frac{1}{3}} \times 8^{-\frac{1}{2}}\right)^{-3}}{\left(25^{\frac{1}{4}} \times 4^{\frac{1}{8}}\right)^{-2}}$
   (iii) $\frac{(16x^3y^{-5}z^{-2})^3}{(2x^{-3})^{10}(y^2z^3)^{-1}}$
   (iv) $\frac{16\left(16(xy^2)^2\right)^{-3}}{(8y^{-1}z)^{-5}} \times (2xz^2)^3$
   (v) $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$
   (vi) $\frac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2}$

(vii) $\left(\frac{16x^{-\frac{5}{6}}}{81\sqrt{x}}\right)^{-\frac{3}{4}}$

Exercise 37 Date:……………………..

1. Solve the following
   (i) $9^{2x+1} = 81^{3x+2}$
   (ii) $5 \times 5^{x+1} = 25$
   (iii) $\frac{1}{8^{2-3y}} = 2^{y+1}$
   (iv) $\left(\frac{1}{27}\right)^{2x-3} = 81^{5-3x}$
   (v) $3^{4x-2} = \left(\frac{1}{27}\right)^{x+3}$
   (vi) $\frac{1}{5^y} = 25(5^{4-2y})$
   (vii) $\frac{\sqrt[3]{64}}{2^x} = \frac{1}{\sqrt{32}}$

2.
   (a) Solve the following
   (i) $4\left(2^{x^2}\right) = 8^x$
   (ii) $\frac{32^{x^2-1}}{4^{x^2}} = 16$
   (iii) $2^{x^2-5x} = \frac{1}{64}$
   (iv) $3^{7x-2} = 81\sqrt{3}$

   (b) Solve the simultaneous equations $\frac{5^x}{25^{3y-2}} = 1$ and $\frac{3^x}{27^{y-1}} = 81.$

   (c) Given that $\frac{\sqrt{a^{4/3} - b^{-2/5}}}{a^{-1/3}b^{3/5}} = a^p b^q$. Find the values of p and q.

3.
   (a) Simplify that following
   (i) $\sqrt[3]{\frac{8}{27}} - \left(\frac{4}{9}\right)^{-1/2}$
   (ii) $\frac{(ab^2)^3 \times (a^2b)^2}{(a^2b^2)^2}$
   (iii) $\frac{x^{3n+1}}{x^{2n+\frac{5}{2}}\ \sqrt{x^{2n-3}}}$

   (b) Simplify $\sqrt{x^8y^{10}} \div \sqrt[3]{x^3y^{-6}}$, giving your answer in the form $x^a y^b$, where a and b are integers.

   (c)
   (i) Show that $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}}$ can be written in the form $(t-2)^p(qt+r)$, where p, q and r are constants to be found.
   (ii) Hence solve the equation $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}} = 0.$

   (d) Simplify the following
   (i) $\left(\frac{x}{2y}\right)^3 \times \frac{2x}{3} \div \frac{2}{9y^2}$
   (ii) $\frac{3x}{4y} \times \left(\frac{2x^2}{y^3}\right)^2 \div \frac{6x^3}{5y^5}$
   (iii) $\frac{\sqrt{a^3b^2}}{6a^3} \times \frac{3a^5b}{(a^3b^2)^2} \div \frac{ab}{\sqrt{a^3b^2}}$
   (iv) $\frac{13\sqrt{x^2+y^3}}{7x^5y^6} \div \frac{36\sqrt{x^2+y^3}}{35(x^2+y^2)^2} \times \frac{216(x^2y)^2}{52(y+x^2)^3}$

LOGARITHM

A logarithm ('log' for short) is an index.

Consider, $6^2 = 36$.

This can be written using logarithm notation as $\log_6 36 = 2$.

The number 6 is called the base of the logarithm.

Similarly, the result $5^3 = 125$ can be written as $\log_5 125$.

In this case, the base is 5.

Generally,
$a^b = c$ is written as $\log_a c = b$.

Example 1
Write in logarithmic form.
(i) $49 = 7^2$ (iii) $3^{-2} = \frac{1}{9}$
(ii) $y^4 = 3$

Solution…
(i) If $49 = 7^2$, then $\log_7 49 = 2$
(ii) If $y^4 = 3$, then $\log_y 3 = 4$
(iii) If $3^{-2} = \frac{1}{9}$, then $\log_3 \left(\frac{1}{9}\right) = -2$

Exercise 1 **Date:……………………**
Rewrite the following relations in a form involving logarithms.
1. $64 = 8^2$
2. $1000 = 10^3$
3. $7 = 49^{\frac{1}{2}}$
4. $n = \left(\frac{1}{2}\right)^{-3}$
5. $m = 2^b$

Example 2
Evaluate the following
1. $\log_2 8$
2. $\log_2 64$
3. $\log_{\frac{1}{2}} 4$

Solution…
1. Let $\log_2 8 = x$
$$8 = 2^x$$
$$2^3 = 2^x$$
$$\therefore x = 3$$
2. Let $\log_2 64 = x$
$$64 = 2^x$$
$$2^6 = 2^x$$
$$x = 6$$
3. Let $\log_{\frac{1}{2}} 4 = x$
$$4 = \left(\frac{1}{2}\right)^x$$
$$2^2 = 2^{-x}$$
$$x = -2$$

Example 3
Solve the following
1. $\log_x 16 = 2$
2. $\log_x \left(\frac{1}{9}\right) = -2$

Solution…
1. $\log_x 16 = 2$
$16 = x^2$
$4^2 = x^2$
$\therefore x = 4$
(We cannot define logs for negative numbers, so $x > 0$ and we take the positive square root to give $x = 4$)

Exercise 2 **Date:……………………**
Find x if
1. $\log_x 64 = 3$
2. $\log_{\frac{1}{3}} x = -2$
3. $\log_{\frac{1}{2}} 8 = x$
4. $\log_3 x = -4$
5. $\log_{100} x = \frac{1}{2}$

Laws Of Logarithm
Let $\mathrm{P} = a^m$, then $m = \log_a P$

Let $\mathrm{Q} = a^n$, then $n = \log_a \mathrm{Q}$

1. $\mathrm{P} \times \mathrm{Q} = a^m \times a^n$
$$= a^{m+n}$$

Then $\log_a(P \times Q) = m + n$
$\therefore \log_a(\mathrm{P} \times \mathrm{Q}) = \log_a \mathrm{P} + \log_a \mathrm{Q}$

Law 1

$$\boxed{\log_a(\mathrm{P} \times \mathrm{Q}) = \log_a \mathrm{P} + \log_a \mathrm{Q}}$$

2. $\frac{P}{Q} = \frac{a^m}{a^n} = a^{m-n}$
$\frac{P}{Q} = a^{m-n}$
$\log\left(\frac{P}{Q}\right) = m - n$
$\log\left(\frac{P}{Q}\right) = \log_a P - \log_a Q$

Law 2

$$\boxed{\log\left(\frac{P}{Q}\right) = \log_a P - \log_a Q}$$

3. $P^n = (a^m)^n$
$P^n = a^{mn}$
$\log_a P^n = mn$
$\log_a P^n = n \log_a P$

Law 3

$$\boxed{\log_a P^n = n \log_a P}$$

Properties of Logarithm

1. $\log_a a = 1$
For example, $\log_{10} 10 = 1, \log_3 3 = 1$

2. $\log_a 1 = 0$
i.e. the logarithm of one (1) to any base is zero (0).
For example, $\log_{10} 1 = 0, \log_3 1 = 0$

Example 4

Express each of the following in terms of $\log x$, $\log y$.

(a) $\log xy$
(b) $\log\left(\frac{x}{y}\right)$
(c) $\log(x^2y)$
(d) $\log(\sqrt{x})$
(e) $\log\left(\frac{1}{x^2}\right)$
(f) $\log(x\sqrt{y})$
(g) $\log\left(\frac{x^3}{y}\right)$

Solution…

(a) $\log xy = \log x + \log y$
(b) $\log\left(\frac{x}{y}\right) = \log x - \log y$
(c) $\log(x^2y) = \log x^2 + \log y$
$= 2\log x + \log y$
(d) $\log(\sqrt{x}) = \log x^{\frac{1}{2}} = \frac{1}{2}\log x$
(e) $\log\left(\frac{1}{x^2}\right) = \log 1 - \log x^2$
$= 0 - 2\log x$
$= -2\log x$
(f) $\log(x\sqrt{y}) = \log x + \log\sqrt{y}$
$= \log x + \log y^{\frac{1}{2}}$
$= \log x + \frac{1}{2}\log y$
(g) $\log\left(\frac{x^3}{y}\right) = \log x^3 - \log y$
$= 3\log x - \log y$

Example 5

If $\log_{10} 4 = 0.6021$ and $\log_{10} 5 = 0.6990$, find

(i) $\log_{10} 20$
(ii) $\log_{10}\left(\frac{5}{4}\right)$
(iii) $\log_{10} 64$
(iv) $\log_{10}\sqrt{5}$

Solution…

(i) $\log_{10} 20 = \log_{10} 4 \times 5$
$= \log_{10} 4 + \log_{10} 5$
$= 0.06021 + 0.6990$
$= 1.3011$

(ii) $\log_{10}\left(\frac{5}{4}\right) = \log_{10} 5 - \log_{10} 4$
$= 0.6990 - 0.6021$
$= 0.0969$

(iii) $\log_{10}\sqrt{5} = \log_{10} 5^{\frac{1}{2}}$
$= \frac{1}{2}\log_{10} 5$
$= \frac{1}{2} \times 0.6990$
$= 0.3495$

Exercise 3 Date:......................

1. Given that $\log_{10} 5 = 0.6990$ and $\log_{10} 3 = 0.4770$, find $\log_{10} 45$.

2. Given that $\log_{10} 6 = 0.7782$, find without using tables or calculators, the value of $\log_{10} 600$.

3. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, calculate without using mathematical tables or calculator the value of
(i) $\log_{10} 54$ (ii) $\log_{10} 0.24$

4. If $\log 5 = 0.6990$, $\log 7 = 0.8451$ and $\log 8 = 0.9031$, evaluate $\log\left(\frac{35\times49}{40\div56}\right)$.

Exercise 4 Date:......................

1. Given that $\log_{10} 2 = 0.3010$, $\log_{10} 7 = 0.8451$ and $\log_{10} 3 = 0.4771$, without using mathematical tables or calculator find the value of:

(i) $\log_{10} 3.6$ (ii) $\log_{10} 0.9$

2. Given that $\log_{10} 2 = 0.3010$, $\log_{10} 7 = 0.8451$ and $\log_{10} 5 = 0.6990$, evaluate without using logarithm tables.
   (a) (i) $\log_{10} 35$ (ii) $\log_{10} 2.8$
   (b) Given that $N^{0.8942} = 2.8$, use your result in a(ii) to find the value of N.

3. Given that $\log p = x$ and $\log q = y$, express $\log\left(\frac{pq}{1000}\right)$ in terms of x and y.

4. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 5 = 0.6990$, evaluate correct to 3 significant figures, $\log_{10} 50 - \log_{10} 40$.

5. Given that $\log_{10} a = p + q$ and $\log_{10} b = p - q$, express
   (a) $\log_{10} a^3b^3$ (b) $\log_{10}\left(\frac{a}{100b}\right)$,
   in terms of p and q.

Exercise 5 **Date:......................**

1. Given that $\log_2 a = 3$, $\log_2 b = -2$ and $\log_2 c = 5$, evaluate $\log_2 8\sqrt{\left(\frac{a^5 b}{c^3}\right)}$

2. Given that $\log_p X = 9$ and $\log_p Y = 6$.
   (i) $\log_p \sqrt{x}$ (iii) $\log_p(XY)$
   (ii) $\log_p\left(\frac{1}{X}\right)$ (iv) $\log_Y X$

3. Given that $\log_8 p = x$ and $\log_8 q = y$, express in terms of x and/or y.
   (i) $\log_8 \sqrt{p} + \log_8 q^2$
   (ii) $\log_8\left(\frac{q}{8}\right)$
   (iii) $\log_2(64p)$

4. Given that $\log_{10} a = x$, $\log_{10} b = y$, $\log_{10} c = z$, express $\log_{10}\sqrt{\left(\frac{10a}{b^5 c}\right)}$.

5.
   (a) Given that $u = \log_4 x$, find in simplest form in terms of u
      (i) x
      (ii) $\log_4\left(\frac{16}{x}\right)$
      (iii) $\log_x 8$

   (b) Given that $\log_2 p = a$, $\log_8 q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b.

Example 6

Simplify the following
1. $\log_3 27 + \log_4 16 - \log_8 64$
2. $2\log_{10} 5 - \frac{1}{2}\log_{10} 16 + 4\log_{10} 2$
3. $\log_3 27 + 2\log_3 9 - \log_3 243$
4. $\log_{10}\sqrt{35} + \log_{10}\sqrt{2} - \log_{10}\sqrt{7}$

Solution...

1. $\log_3 27 + \log_4 16 - \log_8 64$,
   $= \log_3 3^3 + \log_4 4^2 - \log_8 8^2$
   $= 3\log_3 3 + 2\log_4 4 - 2\log_8 8$
   $= 3 + 2 - 2 = 3$

2. $2\log_{10} 5 - \frac{1}{2}\log_{10} 16 + 4\log_{10} 2$
   $= \log_{10} 5^2 - \log_{10} 16^{\frac{1}{2}} + \log_{10} 2^4$
   $= \log_{10}\left[\frac{5^2}{\sqrt{16}} \times 2^4\right]$
   $= \log_{10}\left[\frac{25}{4} \times 16\right]$
   $= \log_{10} 100$
   $= \log_{10} 10^2$
   $= 2\log_{10} 10$
   $= 2 \times 1 = 2$

3. $\log_3 27 + 2\log_3 9 - \log_3 243$
   $= \log_3 3^3 + 2\log_3 3^2 - \log_3 3^5$
   $= 3\log_3 3 + 2 \times 2\log_3 3 - 5\log_3 3$
   $= 3 \times 1 + 4 \times 1 - 5 \times 1$
   $= 2$

4. $\log_{10}\sqrt{35} + \log_{10}\sqrt{2} - \log_{10}\sqrt{7}$
   $= \log_{10}\left[\frac{\sqrt{35}\times\sqrt{2}}{\sqrt{7}}\right]$
   $= \log_{10}\frac{\sqrt{70}}{\sqrt{7}}$
   $= \log_{10}\left(\frac{70}{7}\right)^{\frac{1}{2}}$
   $= \frac{1}{2}\log_{10} 10$
   $= \frac{1}{2} \times 1 = \frac{1}{2}$

Exercise 6 **Date:......................**

Simplify the following.

1. $\log_{10}\sqrt[3]{5^8}$
2. $\frac{\log_m 256}{\log_m 64}$
3. $\frac{\log_5 8}{\log_5 \sqrt{8}}$
4. $\log_2\left(\sqrt{2}\right)^6$
5. $\frac{\log_{\frac{1}{2}} 8}{\log_{\frac{1}{4}} 32}$
6. $\log_{0.25} 8$

Exercise 7 **Date:......................**

LOGARITHM

Without using tables or calculators, evaluate

1. $\log_{10} 6 + \log_{10} 45 - \log_{10} 27$
2. $\log_{10}\left(\frac{75}{10}\right) - 2\log_{10}\left(\frac{5}{9}\right) + \log_{10}\left(\frac{100}{243}\right)$
3. $\frac{1}{2}\log_{10}\left(\frac{25}{4}\right) - 2\log_{10}\left(\frac{4}{5}\right) + \log_{10}\left(\frac{320}{125}\right)$
4. $\log_5\left(\frac{3}{5}\right) + 3\log_5\left(\frac{15}{2}\right) - \log_5\left(\frac{81}{8}\right)$

Exercise 8 **Date:......................**

Evaluate:

1. $2\log_{10} 5 - \frac{1}{2}\log_{10} 16 + 4\log_{10} 2$
2. $2\log_{12} 4 - \frac{1}{2}\log_{12} 81 + 4\log_{12} 3$
3. $\frac{\log_3 27 - \log_{\frac{1}{4}} 64}{\log_3\left(\frac{1}{81}\right)}$
4. $\log_{10}\left(\frac{30}{16}\right) - 2\log\left(\frac{5}{9}\right) + \log\left(\frac{400}{243}\right)$

Example 7

Solve the following

1. $3\log_{10} x + \log_{10} 3 = \log_{10} 81$
2. $\log k - \log(k-2) = \log 5$

Solution…

1. $3\log_{10} x + \log_{10} 3 = \log_{10} 81$

$$\begin{aligned}\log_{10} x^3 + \log_{10} 3 &= \log_{10} 81\\ \log_{10} 3x^3 &= \log_{10} 81\\ 3x^3 &= 81\\ x^3 &= 27\\ x^3 &= 3^3\\ x &= 3\end{aligned}$$

2. $\log\left(\frac{k}{k-2}\right) = \log 5$

$$\begin{aligned}\frac{k}{k-2} &= 5\\ k &= 5(k-2)\\ k &= 5k-10\\ k-5k &= -10\\ -4k &= -10\\ k &= \frac{5}{2} = 2\tfrac{1}{2}\end{aligned}$$

Exercise 9 **Date:......................**

Solve the following.

1. $\log_2 x = \log_8 4$
2. $\frac{1}{2}\log_{10}\frac{1}{x} = 1$
3. $\log_4 x = -3$
4. $\frac{\log_{10} x}{\log_{10} 64} = \frac{1}{2}$
5. $\log_4 16 = \log_x 16$
6. $2\log_x\left(3\frac{3}{8}\right) = 6$

Exercise 10 **Date:......................**

Solve the following

1. $\log_{10}(2x+1) - \log_{10}(3x-2) = 1$
2. $\log_{10}(8x+1) - \log_{10}(2x+1) = \log_{10}(x+2)$
3. $\log_3(4x+1) - \log_3(3x-5) = 2$
4. $3\log_{10} 2 - 2\log_{10} 3 = 1 + \log_{10}\left(\frac{1}{x}\right)$
5. $(y-1)\log_{10} 4 = y\log_{10} 16$
6. $\log(x+12) = 1 + \log(2-x)$

Exercise 11 **Date:......................**

Solve the following

1. $\log_{10} x - \frac{1}{2}\log_{10} 2 = \frac{1}{2}\log_{10}(x+4)$
2. $\log_{\sqrt{3}} 9 = x$
3. $\log_6(x-5) + \log_6 x = 2$
4. $\log_2(4x-8) - \log_2(x-5) = 4$
5. $\log_2 x + \log_2(x-2) = 3$
6. $\log_7(17y+15) = 2 + \log_7(2y-3)$

Exercise 12 **Date:......................**

Solve the following

1. $\log_{10}(20x-10) - \log_{10}(x+3) = \log_{10} 5$
2. $\log_7(17y+15) = 2 + \log_7(2y-3)$
3. $\log_2(x+5) = 5 - \log x$
4. $\log_5(8y-6) - \log_5(y-5) = \log_4 16$
5. $\log(2x^2+5x-2) = 1$
6. $\log x - \frac{1}{2}\log 2 = \frac{1}{2}\log(x+4)$
7. $\log_{81} x = -0.5$
8. $1 + \log(3x-1) = \log(2x+1)$

Exercise 13 **Date:......................**

Solve the following

1. $(\log_3 y)^2 + \log_3 y^2 = 8$
2. $\log_2(x+8) - \log_8 2 = \log_8 4 + \log_2(2x+1)$
3. $2\log x - \log\left(\frac{x+10}{2}\right) = 1$
4. $2\log_3 y - \log_3(y+4) = 2$
5. $(\log_5 y)^2 - 7(\log_5 y) + 12 = 0$
6. $\log_a n = \log_a 3 + \log_a(2n-1)$
7. $2\log_a n - \log_a(5n-24) = \log_a 4$

Exercise 14 **Date:......................**

Evaluate the following

1. $\frac{\log\sqrt{243} - \log\sqrt{27}}{\log 81}$
2. $\log_2 8 + \log_k k^3 + \log_3 9$
3. $\frac{\log 27}{\log 6 - \log 2}$
4. $3\log_{10} 2 + \log_{10} 20 - \log_{10} 1.6$
5. $\frac{\log a^3 - \log a}{\log a}$
6. $\frac{1}{3}\log\left(\frac{125}{8}\right) - 2\log\frac{2}{5} + \log\frac{80}{125}$

Example 8

LOGARITHM

Solve the equations
$\log(y-x)=0; \quad 2\log y=\log(21+x)$

Solution…
$\log(y-x)=0$
$y-x=10^0$
$y-x=1$…………………… (1)

$2\log y=\log(21+x)$
$\log y^2=\log(21+x)$
$y^2=21+x$……………………… (2)

$(1)+(2)$
$21+y=1+y^2$
$y^2-y-20=0$
$y^2-5y+4y-20=0$
$y(y-5)+4(y-5)=0$
$(y-5)(y+4)=0$
$y-5=0$ or $y+4=0$
$y=5$ or $y=-4$

When $y=5$,
$5-x=1$
$x=4$

When $y=-4$,
$-4-x=1$
$x=-5$
The solutions are
$x=4, y=5$ or $x=-5, y=-4$

Exercise 15 **Date:……………………**
Solve the following simultaneous equations
1. $\log x+\log y=1$
   $x+y=11$

2. $\log(x-2)+\log 2=2\log y$
   $\log(x-3y+3)=0$

3. $\log_{10} x+\log_{10} y=4$
   $\log_{10} x+2\log_{10} y=3$
4. $xy=10$ and
   $2(\log_{10} x-1)=\log_{10} y$

APPLICATION USING LOGARITHM TABLE
Logarithm tables are used to carry out computations involving multiplication and division. This method was used in the olden days when there were no calculators.

If we take a number such as 300, we can write it in standard form as:
$$300=3\times 10^2$$

Hence $\log_{10} 300=\log_{10} 3+\log_{10} 10^2$
$=0.4771+2$
$=2.4771$

We call the number 2 before the decimal point **characteristic** of $\log_{10} 300$ and the decimal 0.4771, we call it the **mantissa** of $\log_{10} 300$.

In general, for a positive number m, we can write it in standard form as: $m=\mathrm{k}\times 10^n$, where $1\le \mathrm{k}<10$ and n is an integer.
Then

$$\begin{aligned}\log_{10} m&=\log_{10}(\mathrm{k}\times 10^n)\\&=\log_{10}\mathrm{k}+\log_{10}10^n\\&=\log_{10}\mathrm{k}+n\\&=n+\log_{10}\mathrm{k}\end{aligned}$$

Example 9
Find the characteristic of the common logarithm for each of the following number.

1. 726	3. 0.000451
2. 0.0726	4. 9983700

Solution…
1. $726=7.26\times 10^2$
   $\therefore$ Characterisic $=2$

2. $0.0726=7.26\times 10^{-2}$
   $\therefore$ Characteristic $=-2$

3. $0.000451=4.51\times 10^{-4}$
   $\therefore$ Characteristic $=-4$

4. $9983700=9.983700\times 10^6$
   $\therefore$ Characterisic $=6$

Exercise 16 **Date:……………………**
Find the characteristic of the common logarithm of each of the following numbers.

1. 3.85	4. 0.0000681
2. 45.6	5. 837614400
3. 0.00724	

Example 10
If $\log_{10} 3.5=0.5441$, find the characteristics and mantissae of the common logarithms of the following numbers.

1. 3.5	3. 35000
2. 350	4. 0.0035

Solution…

LOGARITHM

1. $3.5 = 3.5 \times 10^0$
   $\therefore \log 3.5 = \log 3.5 + \log_{10} 10^0$
   $= \log 3.5 + 0$
   $= 0.5441 + 0$
   $\therefore$ Characteristic $= 0$
   $\therefore$ Mantissa $= 0.5441$

2. $350 = 3.5 \times 10^2$
   $\therefore \log 350 = \log 3.5 + 2$
   $= 0.5441 + 2$
   $\therefore$ Characteristic $= 2$
   $\therefore$ Mantissa $= 0.5441$

3. $35000 = 3.5 \times 10^4$
   $\therefore \log 35000 = \log 3.5 + 4$
   $= 0.5441 + 4$
   Charactristic $= 4$
   Mantissa $= 0.5441$

4. $0.0035 = 3.5 \times 10^{-3}$
   $\log 0.0035 = \log 3.5 + (-3)$
   $= 0.5441 + (-3)$

   $\therefore$ Characteristic $= -3$
   Mantissa $= 0.5441$

Note:
We note in example (10) that all the mantissae are all the same. Furthermore in part (4) we have a negative characteristic. Moreover care is taken not to add a negative characteristic to any mantissa.

For example, since
$\log 0.0035 = \log 3.5 + (-3)$
$= -3 + \log 3.5$
$= -3 + 0.5441$

We write
$\log_{10} 0.0035 = \bar{3}.5441$

The bar on $\bar{3}$ indicates a negative characteristic, but the mantissa remains positive.

Exercise 17 Date:......................
If $\log_{10} 7.055 = 0.8485$, write down the common logarithms of the following numbers.

1. 70.55
2. 7055
3. 0.7055
4. 0.0007055
5. 70550
6. 7055000
7. 0.07055

Example 11
Use the table of common logarithms to obtain logarithms of the following numbers.

1. 38.43
2. 6384
3. 0.000539

Solution…

1. $38.43 = 3.843 \times 10^1$
   $\log 38.43 = \log 3.843 + 1$
   $= 0.5847 + 1$
   $= 1.5847$

2. $6384 = 6.384 \times 10^3$
   $\log 6384 = \log 6.384 + 3$
   $= 0.8051 + 3$
   $= 3.8051$

3. $0.000539 = 5.39 \times 10^{-4}$
   $\log 0.000539 = \log 5.39 + (-4)$
   $= -4 + 0.7316$
   $= \bar{4}.7316$

Exercise 18 Date:......................
Find the common logarithms of the following numbers.

1. 8345
2. 0.0063
3. 4476
4. 0.000481
5. 687993

ANTILOGARITHMS
$\log_{10} 3 = 0.4771$

We say that 3 is the antilogarithm of 0.4771 and we write antilog $0.4771 = 3$.

In general if $n = 10^a$, we write
$a = \log_{10} n$ or anti $\log_{10} a = n$

Example 12
Find the antilogs of the following numbers.

1. 1.3671
2. $\bar{2}.5781$

Solution…

1. anti $\log 1.3671 = 23.2863$

2. anti $\log 0.5781 = 3.785$
   $\therefore$ anti $\log \bar{2}.5781 = 10^{-2} \times 3.785$
   $= 0.0378$

Exercise 19 Date:......................
Find the antilogs of the following numbers.

1. 2.8510
2. 4.5195
3. $\bar{3}.9765$
4. $\bar{1}.0899$
5. $\bar{2}.5781$

Example 13

LOGARITHM

Use logarithms to evaluate the following.

1. 8.96×9.587
2. $\frac{5.34\times67.4}{2.7}$
3. $\sqrt{0.9567}$
4. $(0.5921)^{\frac{1}{3}}$

Solution…

1. Let $m = 8.96 \times 9.587$

$\log m = \log 8.96 \times 9.587$
$= \log 8.96 + \log 9.587$
$= 0.9523 + 0.9817$
$\log m = 1.934$
$m = 10^{1.934}$
$m = 95.9014$
$\therefore 8.96 \times 9.587 = 85.9014$

2. Let $m = \frac{5.34\times67.4}{2.7}$

$\log m = \log\left(\frac{5.34\times67.4}{2.7}\right)$
$= \log 5.34 + \log 67.4 - \log 2.7$
$= 0.7275 + 1.8287 - 0.4314$
$\log m = 2.1248$
$m = 10^{2.1248}$
$m = 133.3$

3. Let $m = \sqrt{0.9567}$

$m = (0.9567)^{\frac{1}{2}}$
$\log m = \frac{1}{2}\log(0.9567)$
$\log m = \frac{1}{2}[\log 9.567 \times 10^{-1}]$
$\log m = \frac{1}{2}[(-1) + 0.9808]$
$\log m = \frac{1}{2}[\bar{1}.9808]$

In this example, we had to evaluate $\frac{1}{2} \times \bar{1}.9808$. When doing this great care must be taken.

We note that,
$\bar{1}.9808 = -1 + 0.9808$
$= -1 - 1 + 1 + 0.9808$
$= -2 + 1.9808$

$\therefore \frac{1}{2} \times \bar{1}.9808 = \frac{1}{2}[-2 + 1.9808]$
$= -1 + 0.9904 = \bar{1}.9904$

$\therefore \log m = \bar{1}.9904$
$m = 10^{-1} \times 9.7814$
$m = 0.9781$

$\therefore \sqrt{0.9567} = 0.9781$

4. Let $m = (0.5921)^{\frac{1}{3}}$

$\log m = \frac{1}{3}\log 0.5921$
$\log m = \frac{1}{3}[\log 5.921 \times 10^{-1}]$
$\log m = \frac{1}{3}[(-1) + 0.7724]$
$\log m = \frac{1}{3}[-3 + 3 - 1 + 0.7724]$
$\log m = \frac{1}{3}[-3 + 2.7724]$
$\log m = -1 + 0.9241$
$\log m = \bar{1}.9241$
$m = 10^{-1} \times 8.3965$
$m = 0.8397$

Example 14

Use logarithm table to evaluate $\frac{15.05\times\sqrt{0.00695}}{6.92\times10^2}$.

Solution…

Number	log
15.05	1.1775……………. (1)
$\sqrt{0.00695}$	$\bar{2}.9210$……………. (2)
Add eqns 1 and 2	0.0985………………. (3)
6.92×10^2	2.8401………………. (4)
Subtract eqn 3 from 4	$\bar{3}.2584$

Antilog of $\bar{3} = \frac{1}{10^3}$
Antilog of $0.2584 = 1.8130$

$\therefore$ Antilog $\bar{3}.2504 = 10^{-3} \times 1.8130$
$= 0.0018130$
$\cong 0.00181$ (3 s. f)

Exercise 20 **Date:……………………**

1. Evaluate, using logarithm tables only $\sqrt[3]{0.246 \times 1.023}$
2. Use logarithm table to evaluate $\sqrt{\frac{0.897\times3.536}{0.00249}}$, correct to 3 significant figures.

Exercise 21 **Date:……………………**

1. Use logarithm table to evaluate $\frac{(3.68)^2\times6.705}{\sqrt{0.3581}}$.
2. Using logarithm table, evaluate $\frac{3\sqrt{1.376}}{4\sqrt{0.007}}$.
3. $\log_{10}(3x - 1) - \log_{10} 2 = 3$

LOGARITHM

4. Evaluate $\frac{\log 81}{\log \frac{1}{3}}$.

5. $\log_{10}(8m+1) - \log_{10}(2m+1) = \log_{10}(m+2)$

6. $\log_2(12x-10) = 1 + \log_2(4x+3)$

7.
   (a) Given that $\log_{10} 5 = 0.699$ and $\log_{10} 3 = 0.477$, find $\log_{10} 45$, without using mathematical tables.
   (b) Hence solve $x^{0.8265} = 45$.

8. Find the value of the following.
   (i) $\log_2 8 + \log_{25} 625 - \log_{16} 256$
   (ii) $\frac{\log_2 8 + \log_2 16 - 4\log_2 2}{\log_4 16}$

9. Solve the following
   (i) $\log_{10}(5x+2) - \log_{10}(x-2) = 2$
   (ii) $\log_{\sqrt{x}} 8 = 2$
   (iii) $\frac{1}{2} + \log_4(x^2+x-5) = \log_4(x^2+x+10)$
   (iv) $\log_2 4x + \log_2(x+4) = 7$

10.
   (i) Simplify $\frac{2\log_3 6 - \log_3 9}{\log_9 27 - \log_9 3}$, leaving the answer in the form $p\log_3 q$, where p and q are constants.
   (ii) Given that $1 + \log_3 x = \log_{27} y$, express y in terms of x.

11. Given that $\log 2x + 2\log(y+1) = \log(x+1)$
   (a) Express y in terms of x
   (b) Determine the possible value of y, if $x = 8$.

12.
   (α) Given that $3\log_2 m = n$ and $\log_2 4m = n + 4$, find the values of m and n.
   (β) (i) Solve the equation $2\log_9 3 + \log_5(7y-3) = \log_2 8$.
   (ii) $\log(5x+10) + 2\log 3 = 1 + \log(4x+12)$

Exercise 22

1.
   (a) Given that $\log_{10} N = \bar{2}.7526$, find the value of N in standard form.
   (b) Use logarithm tables to calculate $\sqrt[3]{0.4276}$.
   (c) Use logarithm tables to evaluate $\frac{86.19 \times (0.0462)^2}{\sqrt{0.846}}$.

2. Solve the following simultaneous equations.
   (a) $\log_2 x - \log_2 y = 2$
   $\log_2(x-2y) = 3$

   (b) $\log_2(x+3) = 2 + \log_2 y$
   $\log_2(x+y) = 3$

3. Given that $\log_{10} x = \bar{1}.3010$ and $\log_{10} y = 1.6021$, find $\log_{10}\sqrt{\frac{x}{y}}$.

Exercise

1. Solve these equations
   (i) $\log_9(x-2) = 2$
   (ii) $\log_3(2x-1) = 3$
   (iii) $\log_{\frac{1}{2}}(3-x) = 5$

2. If $m = \log_x 4$ and $n = \log_x 8$, find expressions in terms of m and n for
   (i) $\log_4 8$ (iii) $\log_x 16$
   (ii) $\log_x 2$ (iv) $\log_8 32$
3. Solve
   $3(\log_3 x)^2 + \log_3 x^5 - \log_3 9 = 0$.

4. Solve for x and y in each of the following pairs of equations.
   (i) $\log_y x = 2$ and $\log_2 x = \log_y(y+1)$
   (ii) $\log_x y = 2$ and $xy = 8$
   (iii) $\log_y(y^2+1) = \log 2$ and $\log x = \log(y+3)$
   (iv) $\log(x-y) + \log 2 = 2\log y$ and $\log(x-3y+3) = 0$
   (v) $\log_3 x = y = \log_9(2x-1)$

5.
   (i) Find the value of $-\log_p p^2$.
   (ii) Find $\log\left(\frac{1}{10^n}\right)$
   (iii) Show that $\frac{\log 20 - \log 4}{\log_5 10} = (\log y)^2$ where y is a constant to be found.
   (iv) Solve $\log_r 2x + \log_r 3x = \log_r 600$.
   (v) Solve $\log_2(x+1) + \frac{2}{\log_2(x+1)} = 3$.

LOGARITHM

SIMULTANEOUS EQUATIONS

GRAPHICAL SOLUTION OF SIMULTANEOUS LINEAR EQUATION IN TWO VARIABLES

To solve a pair of simultaneous linear equations graphically, we can draw both graphs on the same set of axes. The point of intersection of the two linear equations give the solution of the simultaneous equation.

Exercise 1 Date:…………………

Solve the following simultaneous equations using the graphical method.

1. $2x + 5y = 10$
   $x = 14$

2. $2x + 5y = 10$
   $y = 3$

3. $2x + 5y = 10$
   $x - 2y = 4$

Exercise 2 Date:…………………

Solve the following simultaneous equations using the graphical method.

1. $x - y = 2$
   $y - 2x = 2$

2. $3x + 4y = 12$
   $x + y = 2$

3. $3x + 2y = 5$
   $2x - y = 8$

Exercise 3 Date:…………………

Solve the following pairs of equations graphically.

1. $x - 4y = 0$
   $5x + 7y = 0$

2. $2x + 3y = 6$
   $x + y = 2$

3. $x + 2y = 3$
   $x - y = 1$

Exercise 4 Date:…………………

1. Using a scale of 2cm to 1 unit on the $x -$ axis and 2cm to 2 units on the $y -$ axis, draw the graphs for the straight lines:
   $y + 2x = 1$ and $y - 3x = 11$.
   On the same graph sheet. From your groups, find the coordinates of the point of intersection.

2.
   (i) Using a scale of 2cm to 1 unit on both axes, on the same graph sheet, draw the graphs of $y - \frac{3x}{4} = 3$ and $y + 2x = 6$.
   (ii) From your graph, find the co – ordinates of the point of intersection of the two graphs.
   (iii) Show on the graph sheet, the region satisfied by the inequality $y - \frac{3}{4}x \geq 3$.

ELIMINATION METHOD

In this method, we get rid of or eliminate one of the variables by either adding or subtracting the two equations.
In order to use this method, follow the following steps.

(1) Have in mind the variable you want to eliminate and check whether their coefficients are the same in both equations. If they are not the same, make sure they are the same by multiplying one or both of the equation(s) by a constant.

(2) Add the two equations if the signs of the variable you want to eliminate in the two equations are different (i.e. one is positive and the other is negative). Subtract one equation from the other if they have the same signs (i.e. either both are positive or both are negative).

(3) After eliminating one variable, solve the remaining equation for the other variable and substitute the value obtained into any of the equations to get the value of the eliminated variable.

(4) Check your solutions.

Example 1

Solve the equations $x + y = 6$ and $x - y = 2$ simultaneously.

Solution…

$x + y = 6$……………………….. (1)
$x - y = 2$……………………….. (2)

We will eliminate y therefore we will add the equations since they have the same coefficients but with different signs.

i.e. $(1) + (2) \Longrightarrow 2x = 8 \Longrightarrow x = 4$

Put $x = 4$ $\quad (1) \Longrightarrow 4 + y = 6$
$y = 6 - 4 = 2$

Check: $4 + 2 = 6$
$4 - 2 = 2$

Example 2
Solve the equations $x + 3y = 2$ and $2x + 5y = 10$ simultaneously.

$x + 3y = 2$………………….. (1)
$2x + 5y = 10$……………….. (2)

$2 \times (1) \Longrightarrow 2x + 6y = 4$…………………… (1)

$(3) - (2) \Longrightarrow y = -6$

Put $y = -6$ into (1)
$\Longrightarrow x + 3(-6) = 2$
$\Longrightarrow x = 2 + 18 = 20$

Check: $20 + 3(-6) = 2$
$2(20) + 5(-6) = 10$

SUBSTITUTION METHOD
This method means to replace or to put in place of something. In order to use this method, follow the following steps.

(1) Make one of the variables the subject in one equation. (If possible the simplest equation)

(2) Substitute this expression for the unknown into the other equation.

(3) Solve the resulting equation to find one unknown variable.

(4) Solve for the other unknown variable.

(5) Check your answers.

Example 3
Solve the simultaneous equations $x - y = 1$ and $2x - y = 6$.

Solution…
$x - y = 1$……………………………... (1)
$2x - y = 6$……………………………. (2)

From (1) $\quad x = 1 + y$…………..….. (3)
Substitute (3) into (2) $\Longrightarrow 2(1 + y) - y = 6$
$2 + 2y - y = 6$
$2y - y = 6 - 2$
$y = 4$

Put $y = 4$ into (3) $\Longrightarrow x = 1 + 4 = 5$

Check:
$5 - 4 = 1$
$2(5) - 4 = 6$

Example 4
Solve the equations:
$\frac{4t}{3} + \frac{3s}{2} = 4$; $\frac{t}{2} + \frac{s}{4} + 1 = 0$.

Solution…
$\frac{4}{3}t + \frac{3s}{2} = 4$
Multiply through by 6
$6 \times \frac{4}{3} + 6 \times \frac{3s}{2} = 4 \times 6$
$8t + 9s = 24$…………………………...(1)

$\frac{t}{2} + \frac{s}{4} + 1 = 0$
Multiply through by 4
$4 \times \frac{t}{2} + 4 \times \frac{s}{4} + 4 \times 1 = 0 \times 4$
$2t + s + 4 = 0$
$2t + s = -4$………………………………... (2)

From (2):
$s = -4 - 2t$………………………………. (3)

Put (3) into (1)
$\Longrightarrow 8t + 9(-4 - 2t) = 24$
$\Longrightarrow 8t - 36 - 18t = 24$
$\Longrightarrow 8t - 18t = 24 + 36$
$\Longrightarrow -10t = 60$
$\Longrightarrow t = -6$

Put $t = -6$ into (3)
$\Longrightarrow s = -4 - 2(-6) = 8$

Check:
$\frac{4}{3}(-6) + \frac{3}{2}(8) = 4$
$\frac{-6}{2} + \frac{8}{4} + 1 = 0$

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 5 Date:....................

Solve the equations:

1. $5x + 3y = 11$
   $x - 3y = 5$

2. $4x + y = 9$
   $4x - 3y = 5$

3. $\frac{m}{2} + \frac{n}{7} = 9$
   $\frac{m}{7} - \frac{n}{2} = -5$

Exercise 6 Date:....................

Solve the equations:

1. $8x - 7y = -52, x = 2y - 11$
2. $2p - 3q = 4, 3p + 2q = 19$
3. $\frac{5}{6}x - \frac{3}{4}y = 2, \frac{1}{2}x - \frac{2}{3}y = \frac{5}{2}$

Exercise 7 Date:....................

Solve the simultaneous equations.

1. $\frac{4}{3}x + \frac{5}{6}y = 1$ and $\frac{1}{3}x - \frac{5}{12}y = \frac{3}{2}$
2. $\frac{p}{2} - \frac{q}{3} = 1$ and $\frac{p}{4} - \frac{q}{9} = 1$
3. $\frac{5m}{6} + \frac{n}{4} = 7$ and $\frac{2m}{3} - \frac{n}{8} = 3$

Exercise 8 Date:....................

Solve the simultaneous equations

1. $3k - 6 = 2l$ and $\frac{k}{4} - \frac{l}{9} = 1$
2. $\frac{x+y+1}{x-y-1} = 2$ and $\frac{x+y-1}{x-y-1} = -1$.
3. $\frac{x+1}{3} = \frac{2-y}{2} - \frac{5}{6}$ and $3x - \frac{1}{2} = \frac{y-3}{3} - \frac{1}{4}$

Exercise 9 Date:....................

Solve the following simultaneous equations.

1.
   (a) $2x - y = 9$
   $7x + 2y = 26$

   (b) $3x - 2y = 15$
   $2x + y = 17$

   (c) $3x + y = 19$
   $5x - y = 13$

   (d) $3x + y = 18$
   $4x - 2y = 34$

   (e) $3x + y = 5$
   $5x + y = 9$

2.
   (a) $3x + 5y = 9$
   $x + 3y = 4$

   (b) $5x + 9y = -4$
   $12x - 2y = 44$

   (c) $2x - y = 7$
   $3x + y = 3$

   (d) $5x + 2y = 16$
   $3x - 4y = 7$

3.
   (a) $9x + 2y = 8$
   $5x + 6y = -20$

   (b) $2x + 3y = 7$
   $x + 5y = 0$

   (c) $2x - y = \frac{9}{2}$
   $x + 4y = 0$

   (d) $3x - 2y = 21$
   $4x + 5y = 5$

4. Given that $2p - m = 6$ and $2p + 4m = 1$, find the value of $(4p + 3m)$

5. Solve the equations $x + y = \frac{3}{2}$ and $x - y = \frac{5}{2}$ hence use your result to find the value of $(2y + x)$

6. Solve the simultaneous equations
   $\frac{2x+y}{2} = 7$
   $\frac{2x-y}{2} = 17$

7. Solve the simultaneous equations
   $2x + \frac{1}{2}y = 1$
   $6x - \frac{3}{2}y = 21$

8. Solve the simultaneous equations
   $\frac{4}{3}x + \frac{5}{6}y = 1$
   $\frac{1}{3}x - \frac{5}{12}y = \frac{3}{2}$

9. Solve the simultaneous equation
   $\frac{3x-2y}{6} + \frac{9x+y}{2} = -\frac{5}{3}$
   $\frac{2x+3y}{2} - \frac{7x-4y}{3} = \frac{4}{9}$

10. Given that $m + n = 5$ and $m - n = 4$, find the value of $m^2 - n^2$.

Exercise 10 Date:…………………

Solve the simultaneous equations.

1. In the simultaneous equations
   $px + qy = 5$
   $qx + py = -10.$
   p and q are constants. If $x = 1$ and $y = -2$ is a solution of the equations. Find p and q.

2. $\frac{1}{x} + \frac{1}{y} = 5$ and $\frac{1}{y} - \frac{1}{x} = 1$

3. Using the substitution
   $p = \frac{1}{x},\ q = \frac{1}{y},$
   Solve the simultaneous equations
   $\frac{1}{x} - \frac{5}{y} = 7;\ \frac{2}{x} + \frac{1}{y} = 3$

Exercise 11 Date:…………………

Solve the simultaneous equations.

1. $\frac{3}{x} - \frac{4}{y} = \frac{1}{3}$ and $\frac{2}{x} - \frac{5}{y} = 1.$

2. $\frac{2}{3x} - \frac{2}{y} = -8$ and $\frac{9}{2x} - \frac{5}{y} = \frac{11}{2}$

3. $3(2m)^{-1} - 3(4n)^{-1} = 1$ and
   $4(3m)^{-1} - 11(9n)^{-1} = 2$

Exercise 12 Date:…………………

Solve the following simultaneous equations.

1. $0.4x + 2y = 10;\quad 0.3x + 5y = 18$

2. $\frac{a+1}{b+1} = 2;\quad \frac{2a+1}{2b+1} = \frac{1}{3}$

3. $\frac{c+d}{c-d} = \frac{1}{2};\quad \frac{c+1}{d+1} = 2.$

4. $\frac{3x-2y}{5} + \frac{6x-3y}{3} = x + 5$
   $\frac{2x-3y}{3} + \frac{4x-3y}{3} = y + 5$

5. $\frac{2}{x} - \frac{1}{y} = 3;\quad \frac{4}{x} + \frac{3}{y} = 16$

6. $\frac{2}{x} - \frac{3}{y} = 1;\quad \frac{8}{x} + \frac{9}{y} = \frac{1}{2}$

7. $\frac{2}{x} + \frac{3}{y} = \frac{7}{2}$
   $\frac{7}{x} - \frac{4}{y} = -\frac{9}{4}$

SOLVING WORD PROBLEMS INVOLVING SIMULTANEOUS LINEAR EQUATIONS

There are some problems which are stated in words with two unknown quantities. Such problems can be represented algebraically using two letters or symbols, say x and y, to denote the unknown quantities. Since there are two unknowns, it means we need two algebraic equations which are simultaneously related to determine the values of x and y.

Example 5

A family of three adults and two children paid GH¢8.00 for a journey. Another family of four adults and three children paid GH¢11.00 as the fare for the same journey. Calculate the fare for:

(i) an adult
(ii) a child
(iii) a family of four adults and five children.

Solution…

Let GH¢x be the fare for an adult and GH¢y be the fare for a child.

Given, three adults and two children paid GH¢8.00

$\Longrightarrow 3x + 2y = 8$………………… (1)

Also, four adults and three children paid GH¢11.00

$\Longrightarrow 4x + 3y = 11$…………………… (2)

From (1) $3x = 8 - 2y$

$x = \frac{8-2y}{3}$……………… (3)

Put (3) into (2)

$\Longrightarrow 4\left(\frac{8-2y}{3}\right) + 3y = 11$

Multiply through by 3

$$3 \times 4\left(\frac{8-2y}{3}\right) + 3 \times 3y = 3 \times 11$$
$$4(8-2y) + 9y = 33$$
$$32 - 8y + 9y = 33$$
$$-8y + 9y = 33 - 32$$
$$y = 1$$

Put $y = 1$ into (3)

$\Longrightarrow x = \frac{8-2(1)}{3} = 2$

(i) Fare for an adult = GH¢2.00
(ii) Fare for a child = GH¢1.00

(iii) A family of four adults and five children
$= 4x + 5y$
$= 4(2) + 5(1)$
$=$ GH¢13.00

Example 6
A number with two digits is equal to four times the sum of its digits. The number formed by reversing the order of the digits is 27 greater than the given number. What is the number?

Solution…
Let x be the units digit and y the tens digit.
$\therefore$ The number $= yx = 10y + x$

The number formed by reversing the order of the digits i.e. $xy = 10x + y$

Now, $10y + x = 4(x + y)$
$10y + x = 4x + 4y$
$10y - 4y + x - 4x = 0$
$6y - 3x = 0$
$2y - x = 0$……………… (1)

Given, the number formed by reversing the order of the digits is 27 greater than the given number.
$10x + y - (10y + x) = 27$
$9x - 9y = 27$
$x - y = 3$…………………… (3)

$(1) + (2) \Longrightarrow y = 3$

Put $y = 3$ into (3)
$x - 3 = 3 \Longrightarrow x = 6$

Hence, the number $yx = 36$.

Example 7
An aeroplane flies against the wind for a distance of 768km in 4 hours. It returns with the wind in 3 hours. Assuming the speeds of the wind and the aeroplane are the same for both journeys, find
(a) The speed of the aeroplane when there is no wind.

(b) The speed of the wind.

Solution…
Let $x$km/h be the speed of the aeroplane and $y$km/h be the speed of the wind.

Speed $= \frac{\text{Distance}}{\text{Time}}$

When it flies against the wind, the aeroplane real speed decreases by the speed of the wind.

Then $\frac{768}{x-y} = 4$
$\Longrightarrow 768 = 4(x - y)$
$\Longrightarrow x - y = 192$……………………….. (1)

When it flies with the wind, the aeroplane's speed will be increased by the speed of the wind.

$\frac{768}{x+y} = 3$
$\Longrightarrow 768 = 3(x + y)$
$\Longrightarrow x + y = 256$………………………… (2)

$(1) + (2) \Longrightarrow 2x = 448 \qquad \therefore x = 224$
Put $x = 224$ into (1)
$224 - y = 192$
$224 - 192 = y$
$\therefore y = 32$

(a) The speed of the aeroplane is 244km/h.

(b) The speed of the wind is 32km/h.

Exercise 13 **Date:…………………**
1. The sum of two numbers is 8 and their product is – 33. Find the numbers.

2. The ratio of the present ages of Kweku and Yaw is 2:9. Four years ago, the sum of their ages was 47 years. How old is Yaw now?

3. The sum of ages of Elizabeth and Bernice is 35. The sum of twice Elizabeth and three times Bernice age is 89. Find their present ages.

Exercise 14 Date:.....................

1. In a class of 52 students, 16 are science students. If $\frac{1}{3}$ of the boys and $\frac{1}{4}$ of the girls are Science students.
   (i) How many boys are in the class?
   (ii) What is the percentage of girls in the class

2. The sum of ages of a woman and her daughter is 46. In four years' time, the ratio of their ages would be 7: 2. Find their present ages.

3. The sum of ages of two brothers is 38 years. Four years ago, the age of the elder brother was the square of the younger brother's age. Find their ages.

Exercise 15 Date:.....................

1. Two numbers are in the ratio 6:1. If the first is decreased by 4 and the second is increased by 6, the resulting numbers are in the ratio 4:9. Find the original fraction.
2. Tickets for a film show were sold at GH¢4.50 to the general public and GH¢3.75 to students. 400 people attended the show and GH¢1680 was collected in tickets sales.
   (a) How many tickets were sold to students?
   (b) Mr. Mensah was issued with 25 tickets to be sold to the general public and 20 tickets to be sold to students. How much did Mr. Mensah collect after selling all the tickets issued?

3. A shop is sending out a bill for an amount less than £100. The accountant interchanges the two digits and so overcharges the customer by £45. Given that the sum of the two digits is 9, find how much the bill should be.

Exercise 16 Date:.....................

1. The total cost of 60 apples and 100 eggs is GH¢108. The cost of 72 apples is the same as that of 30 eggs. Find how much 12 apples and 20 eggs will cost.
2. A mixture consists of $x$kg of coffee at D200.00 per kg and y kg of another brand of coffee at D220.00 per kg. If the total mass of the mixture is 20kg and the total cost is D4,240.00, calculate:
   (i) the values of x and y
   (ii) correct to the nearest whole number, the percentage profit if the mixture is now sold for D250.00 per kg

3. Three times the age of Felicia is four more than the age of Asare. In three years, the sum of the ages will be 30 years. Find their present ages.

Exercise 17 Date:........................

1. A transport company has a total of 20 vehicles made up of tricycles and taxicabs. Each tricycle carries 2 passengers while each taxicab carries 4 passengers at a time, how many tricycles does the company have?

2. Ten boys and twelve girls collected donations for a project. The total amount collected by the boys was ₦600.00 greater than that collected by the girls. If the average collection of the boys was ₦100.00 greater than the average collection of the girls, how much was collected by the two groups?

3. A man saved ₦3,000 in a bank P, where interest rate was $x\%$ per annum and ₦2,000 in another bank Q whose interest rate was $y\%$ per annum. His total interest in one year was ₦640. If he had saved ₦2,00 in P and ₦3,000 in Q for the same period, he would have gained ₦20 as additional interest. Find the values of x and y.

Exercise 18 Date:.....................

1. Three friends went to a bookshop. The first bought 3 biros and 4 pencils for ₦30.00. The second bought 2 biros and 5 pencils for ₦27.00. If the third bought a biro and a pencil only, how much did he pay?

2. A train fare for a school child is half the fare of a teacher. The total fare for 120

children and 15 teachers for an excursion is GH¢180.00.
(i) Find the fare for a child
(ii) How many children will go on the excursion with 20 teachers for a total fare of GH¢240.00?

3. Two tanks X andf Y are filled to capacity with petrol. Tank X holds 600 litres more than tank Y. If 100 litres of petrol were pumped out of each tank, tank X would then contain 3 times as much petrol as tank Y. Find the capacity of each tank.

4. A T – shirt costs 5 times as much as a singlet. For GH¢800, a trader can buy 32 more singlets than T – shirts. How much does a T – shirt cost?

5. Kofi bought six books and ten pencils from a store. Ama bought three books and twenty – two pencils of the same kind from that store. If each of them paid GH¢17,000.00 for the items, find the cost of
(i) each pencil;
(ii) each book;
(iii) two books and four pencils.

Exercise 19 Date:…………………….

1. The sum of the ages of a father and his son is 60 years. The father's age is three times that of the son. Find their ages.

2. If a certain number with two digits is divided by the sum of the digits, the quotient is 6 and the remainder is 5. The difference between the given number and the number formed by reversing the digits is 18. Find the given number.
3. Two thirds of one number is two more than one half of another number. The sum of the numbers is 129. Find the numbers.
4. The value of a fraction is expressed as $\frac{p}{q}$ is $\frac{2}{3}$. If 3 is subtracted from the numerator and added to the denominator, its value becomes $\frac{3}{7}$. Find the values of p and q.
5. Two number are in the ratio 4 to 7. The sum of the three – quarters of the smaller number and one – fifth of the larger number is 22. Find the numbers.

6. The sum of two numbers is 37. Five time the smaller number exceed four times the larger by 5. Find the numbers.

7. If 3 is added to both the numerator and the denominator of a certain fraction, the result is $\frac{3}{5}$. If 1 is subtracted from both the numerator and denominator the result is $\frac{1}{2}$. What is the fraction?

8. The price of admission to Uncle Ebo Whyte Show was GH¢100.00 for adults and GH¢50.00 for children. The total amount raised from the sale of 600 tickets was GH¢50,000.00. Find
(a) the number of adults admitted
(b) the number of children admitted

9. There are five more boys than there are girls in a class. If there were one more girl in the class, the ratio of boys to girls would be 5 to 4. How many girls are there in the class?

10. A number is made up of two digits. The sum of the digits is 11. If the digits are interchanged, the original number is increased by 9. Find the number.

11. If two is added to the numerator of a fraction, the value of the fraction becomes $\frac{1}{2}$. If 2 is subtracted from the numerator of the original fraction, the value of the fraction becomes $\frac{1}{6}$. Find the original fraction.
12. A tank weighs 5.6kg when it is $\frac{1}{4}$ filled with water. If it weighs 10.4kg when it full, what will be the weight when it is empty?

13. The perimeter of a rectangle is 80m. The length of the rectangle is 10 more than its width. Find the length and breadth of the rectangle.

14. Four years ago, a man was six times as old as his son but in five years' time he will be only three times as old as his son. What are their ages?

15. One day flying with the wind, a bird was able to reach 240km per hour. Another day, when the speed of the wind was only half its previous value, flying against the wind, the bird could reach only 48km per hour. Find the speeds of the wind and the bird's rate of flying when there was no wind.

16. Flying directly against a wind of a given speed, a plane can travel 320km in 2 hours. Flying against a wind of twice the given speed, the same plane can cover 450km in 3 hours. Find the speed of
    (a) the plane when there is no wind
    (b) the wind on the first journey.

17. Find the equation of the parabola of the form $y = ax^2 + bx + c$ whose graph passes through the points $(1, 0)$, $(2, 0)$ and $(0, 4)$.

18.
    (α) Find the truth set of the simultaneous equations.
    $3x - 2(y + 3) = 2$
    $2(x - 3) + 4 = 3y - 5$

    (β) Solve the simultaneous equations
    $\frac{x}{3} - \frac{y}{2} = 1,$
    $\frac{3}{x} + \frac{2}{y} = \frac{3}{2}$

VARIATION

DIRECT VARIATION
When two variables are related in such a way that when one increases then so does the other and vice versa, we say they are in direct variation.

There are several ways of expressing a relationship between two quantities x and y.

Here are some examples:
x varies linearly as y.
x varies directly as y.
x is proportional to y.

These three all the same and they are written in symbols as follows:
$x \alpha y$
The 'α' sign can always be replaced by '$= k$' where k is a constant.
i.e. $x = ky$

k is referred to as constant of proportionality or constant of variation.

Note:
1. If P varies directly as the square root of Q then we write $P \alpha \sqrt{Q} \Longrightarrow P = k\sqrt{Q}$.

2. If P varies directly as the square of Q then we write $P \alpha Q^2 \Longrightarrow P = kQ^2$.

3. If P varies directly as the cube of Q then we write $P \alpha Q^3 \Longrightarrow P = kQ^3$

4. If P varies directly as the cube root of Q then we write
   $P \alpha \sqrt[3]{Q}$ or $P \alpha Q^{\frac{1}{3}}$
   $\Longrightarrow P = k\sqrt[3]{Q}$ or $P = kQ^{\frac{1}{3}}$

Example 1
If N varies directly as M and $N = 8$ when $M = 20$. Find M when $N = 7$.

Solution…
$N \alpha M \Longrightarrow N = kM$
Where k is the constant of variation

When $N = 8, M = 20$
$\Longrightarrow 8 = k(20)$
$\Longrightarrow k = \frac{8}{20} = \frac{2}{5}$
$\therefore N = \frac{2}{5}M$

When $N = 7, M = ?$
$7 = \frac{2}{5}M$
$M = \frac{35}{2} = 17\frac{1}{2}$

Example 2
Given that Z α X, copy and complete the table.

X	1	3		$5\frac{1}{2}$
Z	4		16	

Solution…
Given, Z α X $\Longrightarrow$ Z = kX
Where k is the constant of variation.

When $X = 1, Z = 4$
$\Longrightarrow 4 = k(1) \Longrightarrow k = 4$
$\Longrightarrow Z = 4X$

Now when $X = 3, Z = 4(3) = 12$

Now when $Z = 16, 16 = 4X \Longrightarrow X = 4$

Now when $X = 5\frac{1}{2}, Z = 4\left(\frac{11}{2}\right) = 22$

Therefore, the completed table.

X	1	3	4	$5\frac{1}{2}$
Z	4	12	16	22

Exercise 1 **Date:……………………**
1. If $(x + 3)$ varies directly as y and $x = 3$ when $y = 12$, what is the value of x when $y = 8$?

2. If P varies directly as r^2 and $P = 3.2$ when $r = 4$. Find the value of P when $r = 6.5$.

3. y is directly proportional to $(x - 1)^2$. When $x = 3, y = 24$. Find y when $x = 6$.

4. y varies directly as the square root of x. $y = 18$ when $x = 9$. Find y when $x = 484$.

5. m varies directly as the cube of x. $m = 200$ when $x = 2$. Find m when $x = 0.4$.

Exercise 2 Date:......................

1. V is directly proportional to the cube of $(r+1)$. When $r=1, V=24$. Work out the value of V when $r=2$.

2. y varies directly with the square root of $(x+5)$. $y=4$ when $x=-1$. Find y when $x=11$.

3. y varies as the cube root of $(x+3)$. When $x=5, y=1$. Find the value of y when $x=340$.

4. y varies directly as the square root of $(x-3)$. $y=16$ when $x=1$. Find y when $x=10$.

5. If y varies directly as the square root of $(x+1)$ and $y=6$ when $x=3$. Find x when $y=9$.

Exercise 3 Date:......................

1. If $x \alpha \sqrt{y}$ and $x=2$ when $y=16$. Find the value of y when $x=7$.

2. The periodic time, T, of a pendulum varies directly as the square of its length, L. $T=6$ when $L=9$. Find T when $L=25$.

3. The quantity y varies as the cube of $(x+2)$. $y=32$ when $x=0$. Find y when $x=1$.

4. P varies directly as the square root of q. $P=8$ when $q=25$. Find P when $q=100$.

5. S is proportional to $(v-1)^2$, if $S=8$, when $V=3$, calculate
   (a) The value of S, when $V=4$
   (b) The value of V, when $S=2$.

6. Given $w \alpha \sqrt{h}$, copy and complete the table.

h	4	9		$2\frac{1}{4}$
w	6		15	

7. If p varies directly as t^3 and $p=9.6$ when $t=4$, find t when $p=150$.

8. If P is directly proportional to q, state what happens to
   (i) P if q is doubled
   (ii) P if q is treble
   (iii) q if P doubled
   (iv) q if P is halved
   (v) P if q is increased by 20%
   (vi) q if P is decreased by 30%
   (vii) P if 3 is added to q
   (viii) if 4 is subtracted from q

INVERSE VARIATION

When two quantities are related in such a way that as one increases, the other decreases and vice versa, we say the two quantities are in inverse variation.

There are several ways of expressing an inverse relationship between two variables.

x varies inversely as y
x is inversely proportional to y (or varies indirectly)

We write $x \alpha \frac{1}{y}$ for both statements and proceed using the method outlined in the previous work.

Example 3

P varies inversely as the square of W. When $W=4, P=9$. Find the value of P when $W=9$.

Solution…

$P \alpha \frac{1}{W^2}$
$P=\frac{k}{W^2}$, where k is the constant of variation.

When $W=4, P=9, k=?$

$9=\frac{k}{4^2}$
$k=9(16)=144$

$P=\frac{144}{W^2}$

When $W=9, P=\frac{144}{81}$
$=\frac{16}{9}$
$=1\frac{7}{9}$

Example 4

Given that $Z \alpha \frac{1}{y}$, copy and complete the table below.

y	2	4		$\frac{1}{4}$
Z	8		16	

Solution…

$Z \alpha \frac{1}{y}$

$\Rightarrow Z = \frac{k}{y}$, where k is the constant of variation

When $y = 2, Z = 8, k = ?$

$8 = \frac{k}{2}$

$\Rightarrow k = 16$

$\therefore Z = \frac{16}{y}$

When $y = 4, Z = \frac{16}{4} = 4$

When $Z = 16$, $16 = \frac{16}{y} \Rightarrow y = 1$

When $y = \frac{1}{4}, Z = \frac{16}{\frac{1}{4}} = 16 \times 4 = 64$

$\therefore$ Completed table

y	2	4	1	$\frac{1}{4}$
Z	8	4	16	64

Exercise 4 Date:……………………

1. y varies inversely as $(x + 5)$. $y = 6$ when $x = 3$. Find y when $x = 7$.
2. y is inversely proportional to x^3. $y = 5$ when $x = 2$. Find y when $x = 4$.
3. t varies inversely as the square root of u. $t = 3$ when $u = 4$. Find t when $u = 49$.
4. p varies inversely as $(m + 1)$. When $p = 4, m = 8$. Find the values of p when $m = 11$.
5. p is inversely proportional to the square of $(q + 4)$. $p = 2$ when $q = 2$. Find the value of p when $q = -2$.

Exercise 5 Date:……………………

1. y is inversely proportional to x^2. When $x = 2, y = 8$. Find y in terms of x.
2. d is inversely proportional to the square of $(w + 1)$. $d = 3.2$ when $w = 4$. Find d when $w = 7$.
3. z is inversely proportional to t^2 and $z = 4$ when $t = 1$. Calculate
   (a) z when $t = 2$
   (b) t when $z = 16$
4. P varies inversely as the square of $(Q + 1)$ and P is 2 when Q is 3.
   (a) Write an expression connecting P and Q
   (b) Find the possible values of Q when $P = 8$.

Exercise 6 Date:……………………

1. The quantity P varies inversely as the square root of $(q + 2)$. $P = 5$ when $q = 3$. Find P when $q = 8$.
2. Given that P varies inversely as the square of q and $P = 4$ when $q = 20$. Find q when $p = \frac{1}{4}$.
3. If y is inversely proportional to $(x + 2)$ and $y = 48$ when $x = 10$, find x when $y = 30$.
4. R varies inversely as the cube of S. If $R = 9$, when $S = 3$. Find S when $R = \frac{243}{64}$.
5. If $y \alpha \frac{1}{x^2}$ and $y = 1\frac{1}{2}$ when $x = 4$, find the value of y when $x = \frac{1}{2}$.

Exercise 7 Date:……………………

1. If R varies inversely as the square of $(3q - 2)$ and $R = 4$ when $q = 2$, find
   (i) R when $q = 1$
   (ii) q when $R = 16$
2. $P = \frac{k}{q^2}$, when $P = 3, q = \frac{1}{3}$. Calculate
   (a) The value of k
   (b) P when $q = 0.5$
   (c) Both values of q when $P = \frac{1}{12}$

(d) Both values of q when $P = \frac{1}{3}$.

3. Given that $Z = \frac{k}{x^n}$, find k and n, hence copy and complete the table.

x	1	2	4	
Z	100	$12\,{}^{1}/_{2}$		${}^{1}/_{10}$

4. The weight of an object varies inversely as the square of its distance from the centre of the earth. A small satellite weighs 80kg on the Earth's surface. Calculate, correct to the nearest whole number, the weight of the satellite when it is 800km above the surface of the Earth.
[Take the radius of the Earth as 6,400km]

5. The length, y, of a solid is inversely proportional to the square of its height, x.
   a) Write down a general equation for x and y. Show that when $x = 5$ and $y = 4.8$ the equation becomes $x^2y = 120$
   b) Find y when $x = 2$
   c) Find x when $y = 10$
   d) Find x when $y = x$
   e) Describe exactly what happens to y when x is doubled
   f) Describe exactly what happens to x when y is increased by 36%
   g) Make x the subject of the formula $x^2y = 120$.

JOINT VARIATION

Joint variation deals with relations in which one variable depends on two or more other variables.

Note:

1. Z varies direcly as x and y
   $\Longrightarrow Z \ \alpha \ xy$

2. If R varies directly as p and inversely as Q.
   $\Longrightarrow R \ \alpha \ \frac{P}{Q}$

3. If V varies as the square of u and inversely as the cube root of w.
   $\Longrightarrow V \ \alpha \ \frac{u^2}{w^{\frac{1}{3}}}$

4. If Z varies as the square root of Y and inversely as the square of X.
   $Z \ \alpha \ \frac{\sqrt{Y}}{X^2} \Longrightarrow Z = \frac{k\sqrt{Y}}{X^2}$

Example 5

Given that $R \ \alpha \ \frac{S^2}{\sqrt{T}}$ and $R = 3$ when $T = 4$ and $S = 4$, find R when $S = 2$ and $T = 81$.

Solution…

$R \ \alpha \ \frac{S^2}{\sqrt{T}}$

$\Longrightarrow R = \frac{kS^2}{\sqrt{T}}$

where k is the constant of variation.

When $R = 3, T = 4, S = k = ?$

$3 = \frac{k(4^2)}{\sqrt{4}}$

$3 = \frac{k(16)}{2}$

$8k = 3$

$k = \frac{3}{8}$

$\therefore R = \frac{3}{8} \times \frac{S^2}{\sqrt{T}}$

When $S = 2, T = 81, R = ?$

$R = \frac{3}{8} \times \frac{2^2}{\sqrt{81}}$

$R = \frac{3}{8} \times \frac{4}{9}$

$R = \frac{1}{4}$

Example 6

x varies directly as the square root of t and inversely as s. When $x = 4, t = 9$ and $s = 18$.

(i) Express x in terms of s and t.
(ii) Find x when $t = 81$ and $s = 27$.

Solution…

$x \ \alpha \ \frac{\sqrt{t}}{s}$

$x = k\frac{\sqrt{t}}{s}$, where k is the constant of variation

When $x = 4, t = 9$ and $s = 18$

$4 = k \times \frac{\sqrt{9}}{18}$

$4 = k \times \frac{3}{18}$

$4 = \frac{k}{6}$

$k = 24$

(i) $x = 24\frac{\sqrt{t}}{s}$
(ii) When $r = 81, s = 27, x = ?$
$x = 24 \times \frac{\sqrt{81}}{27} = 8$

Exercise 8 Date:.......................

1. Given that y varies directly as x and inversely as the square of Z. If $y = 4$, when $x = 3$ and $Z = 1$. Find y when $x = 3$ and $Z = 2$.

2. Z varies directly as x and inversely as **twice** the cube root of y. If $Z = 8$, when $x = 4$ and $y = \frac{1}{8}$, find the relation for y in terms of x and Z.

3. X, Y and Z are such that X varies directly as Z and inversely as the cube root of Y. If $X = 8$, $Y = 27$ and $Z = 4$. Find
   (a) An expression for X in terms of Y and Z
   (b) The value of Y when $X = 12$ and $Z = 10$.

Exercise 9 Date:.......................

1. P varies directly as Q and inversely as the square of R. If $P = 1$ when $Q = 8$ and $R = 2$, find the value of Q when $P = 3$ and $R = 5$.

2. X varies directly as the cube of Y and inversely as the square root of Z. If $X = 108$ when $Y = 3$ and $Z = 4$, find Z when $X = 4000$ and $Y = 10$.

Exercise 10 Date:.......................

1.

W	R	Q
3	4	4
w_2	1	2
8	r_3	6

In the table, $W \propto \frac{Q}{R^2}$, where W, R and Q are positive integers. Solve for w_2 and r_3.

2. The time t, taken to buy fuel at a petrol station varies directly as the number of vehicle V on queue and jointly varies inversely as the number of pumps P available in the station. In a station, with 5 pumps, it took 10minutes to fuel 20 vehicles find
   (i) the relationship between t, P and V.
   (ii) the time it will take to fuel 50 vehicles with 20 pumps.
   (iii) the number of pumps required to fuel 40 vehicles in 20 minutes.

3. X varies jointly as the square of m and the cube of n. When $X = 9$, $m = \frac{3}{4}$ and $n = \frac{1}{2}$.
   (i) Determine the relations between X, m and n.
   (ii) Calculate, correct to three significant figures, the value of
      (a) X when $m = \frac{2}{3}$ and $n = \frac{1}{5}$
      (b) m when $X = 5$ and $n = \frac{1}{8}$

PARTIAL VARIATION

This type of variation involves relation which are connected by two or more variables and consist of parts which are sums of other variables.

Note:

1. If P is partly constant and partly varies as Q
   $\Longrightarrow P = k_1 + k_2Q$

2. If P is partly constant and partly varies inversely as Q
   $P = k_1 + \frac{k_2}{Q}$

3. If P varies partly as Q and partly varies inversely as the cube root of R.
   $P = k_1Q + \frac{k_2}{\sqrt[3]{R}}$

4. If Z is partly constant and partly varies as the product of X and Y.
   $\Longrightarrow Z = k_1 + k_2XY$

Example 7

A variable y is partly constant and partly as the square of x. When $x = 2, y = 6$ and when $x = 3, y = 10$.
(i) Find the equation connecting x and y.
(ii) Find the values of x when $y = 5$
(iii) Find the value of y when $x = 4$

Solution…

(i) $y = k_1 + k_2x^2$

Where k_1, k_2 are constants to be determined.

Given, $x = 2, y = 6$

$\Longrightarrow 6 = k_1 + k_2(2)^2$

$k_1 + 4k_2 = 6$……………………… (1)

When $x = 3, y = 10$

$10 = k_1 + k_2(3)^2$

$k_1 + 9k_2 = 10$……………………… (2)

Solving (1) and (2) simultaneously

$k_1 = \frac{14}{5}, k_2 = \frac{4}{5}$

$\therefore y = \frac{14}{5} + \frac{4}{5}x^2$

(ii) When $y = 5, x = ?$

$5 = \frac{14}{5} + \frac{4}{5}x^2$

$\frac{11}{5} = \frac{4}{5}x^2$

$x^2 = \frac{11}{4}$

$x = \pm\sqrt{\frac{11}{4}} = \pm\frac{\sqrt{11}}{2}$

(iii) When $x = 4, y = ?$

$y = \frac{14}{5} + \frac{4}{5}(4)^2$

$y = \frac{78}{5}$

Exercise 11 Date:……………………

1. The quantity y is partly constant and partly varies inversely as the square of x.
   (a) Write down the relationship between x and y.
   (b) When $x = 1, y = 11$ and when $x = 2, y = 5$. Find the values of y when $x = 4$.

2. The cost of maintaining a school is partly constant and partly varies as the number of pupils. With 50 pupils, the cost is \$15,705.00 and with 44 pupils it is \$13,305.00.
   a) Find the cost when there are 40 pupils
   b) If the fee per pupil is \$360.00, what is the least number of pupils for which the school can be run without a loss?

3. The cost C of weeding a rectangular plot of land is partly constant and partly varies jointly as the length L, and breadth B of the plot. For a plot of length $50m$ and breadth $20m$, the cost of weeding is $GH¢85{,}000.00$ and for a plot of length $40m$ and breadth $30m$ the cost of the weeding is $GH¢100{,}000.00$
   (i) Find the relationship between C, L and B
   (ii) Use your answer in (i) to calculate the cost of weeding a plot of length 50m and breadth 40m.

Exercise 12 Date:……………………

1. Z varies directly as x and inversely as twice the cube root of y. If $Z = 8$, when $x = 4$ and $y = \frac{1}{8}$, find the relation for y in terms of x and Z.

2. If $(x + 1)$ is directly proportional to the square of $(y + 3)$ given that y is 4 when x is 3. Find y when x is 8.

3. The quantity p is the sum of two terms; one of which is a constant, whilst the other varies inversely as the square of q. When $q = 1, p = -1$ and when $q = 2$, $p = 2$. Find the positive value of q when $p = 2.75$.

4. The cost (C) of producing n bricks is the sum of a fixed amount, h, and a variable amount y where y varies directly as n. If it costs GH¢950.00 to produce 600 bricks and GH¢1030.00 to produce 1000 bricks
   (i) Find the relationship between C, h and n;
   (ii) Calculate the cost of producing 500 bricks.

5. The cost C of producing a motor car in a certain factory is partly constant and partly varies inversely as the number n of cars produced per day. The cost of producing 4 cars per day is \$1,600 and

that of producing 5 cars per day is $1,420.

(i) Find the relationship between C and n.

(ii) Find the number of cars produced per day necessary to bring the cost down to $1,150

a) Explain why $C \propto n$
b) Find the proportionality constant
c) What happens to :
   (i) C if n is doubled
   (ii) n if C is increased by 50%

QUADRATIC FUNCTIONS AND EQUATIONS

A quadratic equation is an equation of the form $ax^2 + bx + c = 0$, where a , b and c are constants, $a \neq 0$.

Solving Quadratic Equations

Two values of x which make $ax^2 + bx + c$ zero are called the **solutions** or **roots** of the quadratic equation.

- **Solving by Factorization**
  Factorize the quadratic equation and use the **Null Factor Principle** which states that the product of two or more numbers equals zero if and only if at least one of the factors is equal to zero.

If $\mathbf{AB} = 0$, then $\mathbf{A} = 0$ or $\mathbf{B} = 0$

Steps in Solving Quadratic Equation by Factorizing.

Step 1: Rearrange the quadratic equation in standard form.
Step 2: Fully factorize the other side.
Step 3: Use the Null Factor Principle i.e. if AB = 0 then A = 0 or B = 0.
Step 4: Solve the resulting linear equations.

Example 1

Find the solution set of the following by factorization.

1. $(2x - 1)(1 - x) = 0$
2. $x^2 - 16 = 0$
3. $3t^2 = -4t$
4. $x^2 = 5x + 14$
5. $(x - 2)(x - 3) = 12$
6. $\frac{2}{x} = \frac{2x-1}{3}$

Solution…

1. $(2x - 1)(1 - x) = 0$
   $2x - 1 = 0$ or $1 - x = 0$
   $x = \frac{1}{2}$ or $x = 1$
   $\left\{x: x = \frac{1}{2}, 1\right\}$

2. $x^2 - 16 = 0$
   $x^2 - 4^2 = 0$
   $(x - 4)(x + 4) = 0$
   $x - 4 = 0$ or $x + 4 = 0$
   $x = 4$ or $x = -4$
   $\{x: x = -4, 4\}$

3. $3t^2 = -4t$
   $3t^2 + 4t = 0$
   $t(3t + 4) = 0$
   $t = 0$ or $3t + 4 = 0$
   $t = 0$ or $t = -\frac{4}{3}$
   $\left\{t: t = -\frac{4}{3}, 0\right\}$

4. $x^2 = 5x + 14$
   $x^2 - 5x - 14 = 0$
   $(x + 2)(x - 7) = 0$
   $x + 2 = 0$ or $x - 7 = 0$
   $x = -2$ or $x = 7$
   $\{x: x = -2, 7\}$

5. $(x - 2)(x - 3) = 12$
   $x(x - 3) - 2(x - 3) = 12$
   $x^2 - 3x - 2x + 6 = 12$
   $x^2 - 5x - 6 = 0$
   $(x - 6)(x + 1) = 0$
   $x - 6 = 0$ or $x + 1 = 0$
   $x = 6$ or $x = -1$
   $\{x: x = -1, 6\}$

6. $\frac{2}{x} = \frac{2x-1}{3}$
   $2 \times 3 = x(2x - 1)$
   $6 = 2x^2 - x$
   $2x^2 - x - 6 = 0$
   $(x - 2)(2x + 3) = 0$
   $x - 2 = 0$ or $2x + 3 = 0$
   $x = 2$ or $x = -\frac{3}{2}$
   $\left\{x: x = -\frac{3}{2}, 2\right\}$

Exercise 1 **Date:……………………..**

Find the truthset of the following

1. $(x - 2)(x + 3) = 0$
2. $x^2 + 3x = 0$
3. $x^2 - 100 = 0$
4. $3x^2 + 5x - 2 = 0$
5. $2x^2 + 5x - 12 = 0$
6. $4y^2 + 5y = 21$

Exercise 2 Date:..........................

Find the solution set by factorization.

1. $6x^2 - 7x = 5$
2. $x^2 + 2x + 1 = 25$
3. $4x^2 - 16x + 15 = 0$
4. $3(2x + 9) = x^2$
5. $8x^2 - 5 = -6x$
6. $x^2 - \frac{13}{2}x + \frac{15}{2} = 0$

Exercise 3 Date:..........................

Find the solution set by factorization

1. $(4x^2 - 2)^2 = 100$
2. $\frac{x}{x+1} = \frac{2}{3x+7}$
3. $\frac{a-1}{a} = \frac{a}{a-2}$
4. $x + \frac{12}{x} = 7$
5. $3x + \frac{8}{x} = 10$

Exercise 4 Date:..........................

Find the truthset of the following equations

1. $5x^2 = (x + 2)(x + 3)$
2. $\frac{y-5}{y} = \frac{5}{3y} - \frac{1}{5}$
3. $\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2}$
4. $\frac{2x+3}{x-1} = \frac{5x-3}{3-x}$
5. $\frac{2}{x+3} + \frac{1}{12} = \frac{3}{2x-1}$

Exercise 5 Date:..........................

Solve the following equations

1. $x = \sqrt{2x + 3}$
2. $x = \sqrt{7x - 6}$
3. $x = \sqrt{\frac{13x-5}{6}}$
4. $2x^{-2} = \frac{8}{9}$
5. $\frac{1}{x+1} + \frac{2}{x+2} = 1$
6. $6\left(\sqrt{4x^2 + 1}\right) = 13x$

Exercise 6 Date:..........................

Solve by factorization.

1. $(x - 2)(x + 3) = (x - 2)(4 - x)$
2. $2(x - 5)(x + 5) = 21x$
3. $2x - 1 = \frac{x+1}{2x}$
4. $(x + 1)^2 = 2x^2 - 5x + 11$

Exercise 7 Date:..........................

1. If $10x^2 - 9xy + 2y^2 = 0$, find the ratio $x : y$.

2. Factorize $6y^2 - 149y - 102$, hence solve the equation $6y^2 - 149y - 102 = 0$.

Forming Quadratic Equation When The Roots Are Given

If α and β are the roots of the general quadratic equation
$ax^2 + bx + c = 0$ then
$x = \alpha$ or $x = \beta$

$(x - \alpha)(x - \beta) = 0$
$x(x - \beta) - \alpha(x - \beta) = 0$
$x^2 - \beta x - \alpha x + \alpha\beta = 0$
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

The equation may be written as:
$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

Example 2

Find the quadratic equation whose roots are 1 and 3.

Solution…

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

sum of roots $= 1 + 3 = 4$
product of roots $= 1 \times 3 = 3$

$\therefore$ The equation is $x^2 - 4x + 3 = 0$.

Example 3

Find the quadratic equation whose roots are $-\frac{1}{3}$ and 3.

Solution…

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$
sum of roots $= -\frac{1}{3} + 3 = \frac{8}{3}$

product of roots $= -\frac{1}{3} \times 3 = -1$

$\therefore$ The equation is $x^2 - \left(\frac{8}{3}\right)x + (-1) = 0$

Example 4
Find the equation whose roots are 3 and $-m$.

Solution…
$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

sum of roots $= 3 + (-m) = 3 - m$

product of roots $= 3 \times (-m) = -3m$

$\therefore$ The equation is
$x^2 - (3 - m)x + (-3m) = 0$
$x^2 - (3 - m)x - 3m = 0$

Example 5
Find the equation whose roots are $\left(3 - \sqrt{3}\right)$ and $\left(3 + \sqrt{3}\right)$.

Solution…
$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

sum of roots
$= \left(3 - \sqrt{3}\right) + \left(3 + \sqrt{3}\right)$
$= 3 + 3 - \sqrt{3} + \sqrt{3} = 6$

product of roots
$= \left(3 - \sqrt{3}\right)\left(3 + \sqrt{3}\right)$
$= (3)^2 - \left(\sqrt{3}\right)^2 = 9 - 3 = 6$

$\therefore$ The equation is $x^2 - 6x + 6 = 0$.

Exercise 8 Date:……………………..
Find the quadratic equation whose roots are
(i) 2 and 3
(ii) -4 and 1
(iii) -2 and -1
(iv) -1 and -3

Exercise 9 Date:……………………..
Find the quadratic equation whose roots are
(i) $-\frac{1}{2}$ and 2
(ii) $-\frac{1}{4}$ and 3
(iii) $-\frac{2}{3}$ and $-\frac{1}{4}$
(iv) $\frac{1}{2}$ and $-\frac{2}{3}$
(v) $\frac{3}{4}$ and -4

Exercise 10 Date:……………………..
Find the quadratic equation whose roots are
(i) 2 and $-n$
(ii) $3\frac{1}{2}$ and -2
(iii) $\left(\sqrt{2} + 1\right)$ and $\left(\sqrt{2} - 1\right)$
(iv) 1 and $\frac{1}{\alpha}$
(v) $\frac{-3 \pm \sqrt{5}}{2}$
(vi) $1 - \sqrt{13}$ and $1 + \sqrt{13}$
(vii) $-m$ and $2n$

Example 6
If $(m + 1)$ and $(m - 3)$ are factors of $m^2 - km + c$, find the values of k and c.

Solution…
$$\begin{aligned} m^2 - km + c &= (m + 1)(m - 3) \\ &= m(m - 3) + 1(m - 3) \\ &= m^2 - 3m + m - 3 \\ &= m^2 - 2m - 3 \end{aligned}$$
$m^2 - km + c = m^2 - 2m - 3$

Comparing coefficients:
$k = 2$ and $c = -3$.

Exercise 11 Date:……………………..
1. Find the value of k if $(x - 1)$ and $(x + 2)$ are the factors of $x^2 + x + k$.
2. If $(x - 5)$ and $(x + 2)$ are the factors of $x^2 + kx - 10$, find the value of k.
3. If $(x - 2)$ and $(x + 1)$ are factors of $x^2 + bx + c$, find the values of $b + c$.
4. If $(x - 3)$ and $(2x + 3)$ are factors of $2x^2 + mx + n$, find the value of $(m + n)$.
5. Given that $x^2 + bx + 18$ is factorized as $(x + 2)(x + c)$. Find the values of c and b.

Exercise 12 Date:……………………..
1. If $x^2 + mx + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2$, find the value of m.
2. Find the value of k if $a^2 + 6a + k = (a + 3)^2$.
3. Given that $x^2 + 4x = (x + 2)^2 + k$. Find the value of k.

4. Given that $x^2 + 4x + k = (x + r)^2 + 1$, find the values of k and r.

5. When k is added to the expression $y^2 - 12y$, the expression becomes $(y + p)^2$. Find the values of p and k.

Exercise 13 Date:........................

1. The equation $px^2 + 16x + 4 = 0$ is satisfied by $x = -\frac{2}{3}$, find
   (i) The value of p
   (ii) The other value of x which makes the equation true.

2. The roots of the equation $2x^2 + (p + 1)x + q = 0$, are 1 and 3, where p and q are constants. Find the values of p and q.

3. If 3 is a root of the quadratic equation $x^2 + bx - 15 = 0$, determine the value of b. Find the other root.

4. The graph of the equation $y = Ax^2 + Bx + C$ passes through the points $(0, 0)$, $(1, 4)$ and $(2, 10)$. Find the
   (i) Value of C
   (ii) Values of A and B.
   (iii) Coordinates of the point where the graph cuts the $x -$ axis.

Solving by Completing the Square

$x^2 + kx$ can be put into a perfect square if we add to it the square of one – half the coefficient of x.

Let's recall:

$x^2 + 2ax + a^2 \equiv (x + a)^2$

$x^2 - 2ax + a^2 \equiv (x - a)^2$

i.e. Since the coefficient of x in $x^2 + kx$ is k, the square of one – half of the coefficient of x is $\left(\frac{k}{2}\right)^2$ or $\frac{k^2}{4}$.

Adding $\left(\frac{k}{2}\right)^2$ to $x^2 + kx$, we have:

$$x^2 + kx + \left(\frac{k}{2}\right)^2 = \left(x + \frac{k}{2}\right)^2$$

Note:

- The quadratic equation $ax^2 + bx + c = 0$ can be put in the form $(x + a)^2 = b$, from which the solution is easy to obtain.
- If $x^2 = p$, then $x = \pm\sqrt{p}$.

Example 7

Solve $x^2 + 5x - 3 = 0$ by completing the square correct to two decimal places.

Solution…

$x^2 + 5x - 3 = 0$

Put the constant on the RHS

$x^2 + 5x + 3$

Completing the squares

$$x^2 + 5x + \left(\frac{1}{2} \times 5\right)^2 = 3 + \left(\frac{1}{2} \times 5\right)^2$$

Factorizing the LHS

$$\Rightarrow \left(x + \frac{5}{2}\right)^2 = 3 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{37}{4}$$

$$= \pm\frac{\sqrt{37}}{2}$$

$$x = -\frac{5}{2} \pm \frac{\sqrt{37}}{2}$$

$$x = \frac{-5+\sqrt{37}}{2} \text{ or } x = \frac{-5-\sqrt{37}}{2}$$

$x = 0.54$ or $x = -5.54$

Exercise 14 Date:........................

Using completing the square method, solve correct to two decimal places, the following equations.

1. $x^2 - 6x + 7 = 0$
2. $x^2 - 3x - 1 = 0$
3. $3y^2 - 5y + 2 = 0$
4. $2x^2 - 5x + 3 = 0$
5. $2x^2 + 7x + 2 = 0$

Exercise 15 Date:........................

Using completing the square method, solve correct to two decimal places, the following equations.

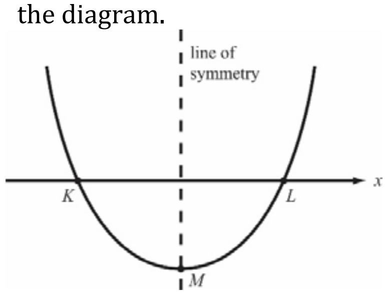
1. $\frac{x-2}{4} = \frac{x+2}{2x}$
2. $7 - 5x - 2x^2 = 0$
3. $2x^2 - x - 3 = 0$
4. $12x^2 - 6x + 1 = 0$
5. $5 - 2x - 4x^2 = 0$

Exercise 16 Date:……………………

1. Find the truthset of the equation $(x-2)^2 + 3 = (x+1)^2 - 6$.

2. Find the quadratic equation whose roots are $\frac{1}{2}$ and $-\frac{3}{2}$.

3. Solve the equation: $\frac{1}{5x} + \frac{1}{x} = 3$
4. The product of two consecutive positive odd numbers is 195. By constructing a quadratic equation and solving it, find the two numbers.
5. A sketch of the graph of the quadratic function $y = px^2 + qx + r$ is shown in the diagram.



The graph cuts the $x-$axis at K and L. The point M lies on the graph and M the line of symmetry.
(a) When $p = 1, q = -2, r = -3$, find
(i) The $y-$coordinate of the point where $x = 4$.
(ii) The coordinate of K and L.
(iii) The coordinates of M.

(b) Describe how the above sketch of the graph would change in each of the following cases.
(i) p is negative.
(ii) $p = 1, q = r = 0$.

(c) Another quadratic function is $y = ax^2 + bx + c$.
(i) The graph passes through the origin. Write down the value of c.
(ii) The graph also passes through the points $(3, 0)$ and $(4, 8)$. Find the values of a and b.

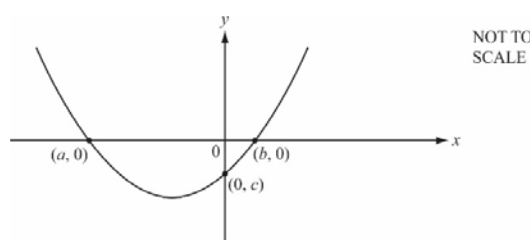
6. If $\sqrt{\frac{4x^2}{3} + \frac{1}{2}} = \frac{7}{6}x$, when $x > 0$, find the value of x.
7. Solve $\sqrt{2x^2 - 9} = x$

8. $\sqrt{\left(x^2 + \frac{1}{4}\right)} = \frac{13x}{12}$

9. Solve $3x^{1/2} + 5 - 2x^{-1/2} = 0$

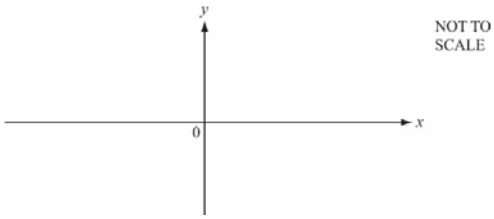
10. Solve the following
(i) $\sqrt{\frac{4+x}{6}} = 2$
(ii) $\frac{\sqrt{x+4}}{4} = 1$
(iii) $8 = 4 + \sqrt{7 + \frac{x}{4}}$
(iv) $\frac{40-2x^2}{2} = 4$
(v) $\frac{37-3\sqrt{x}}{5} = 2$
(vi) $\sqrt{3 + \frac{(4+\sqrt{x+3})^2}{6}} = 3$

11.
(i) Factorise $x^2 + 3x - 10$
(ii) The graph of $y = x^2 + 3x - 10$ is sketched below.



Write down the values of a, b and c.
(iii) Write down the equation of the line of symmetry of the graph of $y = x^2 + 3x - 10$.

12. Sketch the graph of $y = 18 + 7x - x^2$ on the axes below. Indicate clearly the values of x where the graph crosses the x and y axes.



13. (i) $x^2 + 12x - 7 = (x + p)^2 - q$.
    Find the value of p and the value of q.
    (ii) Write down the minimum value of y for the graph $y = x^2 + 12x - 7$.

14. Find the solution set of the following
    (i) $\frac{1}{x+1} + \frac{2}{3} = \frac{1}{x-1}$
    (ii) $\frac{3x-1}{2x+1} - \frac{x-4}{x-1} = \frac{x+1}{x-1}$

INTERPRETATION OF LINEAR AND QUADRATIC GRAPH

Exercise 1 Date:…………………………………

(a) Copy and complete the following table or values for the relation $y = x^2 - 2x - 5$ for $-3 \leq x \leq 5$.

x	-3	-2	-1	0	1	2	3	4	5
y			-2				-2		10

(b) Using a scale of 2cm to 1 unit on the x −axis and 2cm to 2 unit on the y −axis, draw the graph of the relation $y = x^2 - 2x - 5$ for $-3 \leq x \leq 5$.

(c) On the same axes, draw the graph of $y = 2x - 3$.

(d) Using the graphs, find the coordinates of the points of intersection.

Exercise 2 Date:…………………………………

(a) Copy and complete the table of values for the relation $y = x^2 - 5x + 5$ for $-1 \leq x \leq 6$.

x	-1	0	1	2	3	4	5	6
y		5	1				5	

(b) Using scales of 2cm to represent 1 unit on the x −axis and 2cm to represent 2 unit on the y −axis, draw the graph of $y = x^2 - 5x + 5$ for $-1 \leq x \leq 6$.

(c) Use the graph to find the:

(i) minimum value of y (ii) roots of $x^2 - 5x + 5 = 0$. (iii) solution of $x^2 + 2x + 5 = 7x + 2$.

Exercise 3 Date:…………………………………

An object is thrown vertically upwards from the top of a cliff and its height, y metres, above sea level after t seconds is given by $y = -16t^2 + 64t + 5$

(a) Copy and complete the table of values for $y = -16t^2 + 64t + 5$; $0 \leq t \leq 4.0$.

x	0.0	0.5	1.0	1.5	2.0	2.5	3.0	4.0
y	5			65			53	

(b) Using scales of 2cm to 0.5 seconds on the t −axis and 2cm to 10m on the y −axis, draw the graph of $y = -16t^2 + 64t + 5$ for $0 \leq t \leq 4.0$.

(c) Use the graph to find the:

(i) Height reached when $t = 1.75$ seconds.

(ii) Times the object was at a height of 50m.

(iii) Maximum height reached.

Exercise 4 Date:…………………………………

(a) Copy and complete the following table of values for the relation $y = 2x^2 - 7x - 3$.

x	-2	-1	0	1	2	3	4	5
y	19		-3		-9			

(b) Using scales of 2cm to 1 unit on the x −axis and 2cm to 5 units on the y −axis, draw the graph of $y = 2x^2 - 7x - 3$ for $-2 \leq x < -5$.

(c) From the graph, find the

(i) minimum value of y.

(ii) gradient of curve at $x = 1$, correct to the nearest whole number.

(iii) values of x for which $2x^2 - 5x + 1 = 2x + 4$.

Exercise 5 Date:……………………………….

(a) Copy and complete the following table for the relation $y = 8x^2 - 18x - 35$ for $-2 \leq x \leq 4$.

x	-2.0	-1.5	-1.0	0	0.5	1.0	1.5	2.0	3.0	4.0
y	33		-9.0	-35					-17	

(b) Using scales of 2cm to 1 unit on the x —axis and 2cm to 10 units on the y —axis, draw the graph of the relation $y = 8x^2 - 18x - 35$ in the given interval.

(c) Use your graph to solve

(i) $8x^2 = 18x + 35$ (ii) $8x^2 - 18x = 15$.

Exercise 6 Date:……………………………….

(a) Copy and complete the table of values for the relation $y = -x^2 + x + 2$ for $-3 \leq x \leq 3$.

x	-3	-2	-1	0	1	2	3
y		-4		2			-4

(b) Using scale of 2cm to 1 unit on the x —axis and 2cm to 2unit on the y —axis, draw a graph of the relation $y = x^2 + x + 2$.

(c) From the graph, find the

(i) maximum value of y.

(ii) roots of the equation $x^2 - x - 2 = 0$.

(iii) gradient of the curve at $x = -0.5$.

Exercise 7 Date:……………………………….

(a) Copy and complete the following table following values for the relation $y = (x - 4)(x + 2)$ for $-3 \leq x \leq 5$.

x	-3	-2	-1	0	1	2	3	4	5
y				-8					

(b) Using scales of 2 cm to 1 unit on the x —axis and 2 cm to 2 units on the y —axis, draw the graph of $y = (x - 4)(x + 2)$ for $-3 \leq x \leq 5$.

(c) Using the graph, find the

(i) values of x for which y is decreasing. (ii) gradient of the curve at $x = 0$.

Exercise 8 Date:……………………………….

(a) Copy and complete the table below for the relation $y = 7 - 5x - 2x^2$ for $-4 \leq x \leq 2$.

x	-4	$-3\frac{1}{2}$	-3	-2	-1	0	1	$1\frac{1}{2}$	2
y	-5		4			7			

(b) Using a scale of 2cm to 1 unit on the x — axis and 2cm to 2 units on the y —axis, draw the graph of the relation in (a).

(c) Using the graph, find the:

(i) equation of the axis of symmetry of the curve;

(ii) maximum value of y;

(iii) roots of the equation $5 - 5x - 2x^2 = 0$.

(iv) range of the values of x for which $7 - 5x - 2x^2 > 0$.

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 9 Date:..

(a) Copy and complete the following of values of the equation $y = 4 + 5x - 2x^2$ for $-3 \leq x \leq 5$.

x	-3	-2	-1	0	1	2	3	4	5
y		-14		4	7				-21

(b) Using 2cm to 1 unit on the x –axis and 2cm to 5 units on the y –axis, draw the graph of $4 + 5x - 2x^2$ for $-3 \leq x \leq 5$.

(c) From your graph, find the
   (i) value of x for which y is maximum.
   (ii) gradient at $x = 0$.
   (iii) values of x for which $1 + 5x - 2x^2 = 0$.

Exercise 10 Date:..

(a) Copy and complete the table of values for the relation $y = \frac{1}{2}(x - 3)(x + 1)$ for intervals $-3 \leq x \leq 5$.

x	-3	-2	-1	0	1	2	3	4	5
y	6			-1.5	-2		0		

(b) Draw on the same graph sheet and using the same axes the graphs of the relations $y = \frac{1}{2}(x - 3)(x + 1)$ and $y = -\frac{1}{2}x$ for the given interval.

(c) Use your graph to solve
   (i) $(x - 3)(x + 1) = 2$.
   (ii) $(x - 3)(x + 1) = -x$.

Exercise 11 Date:..

(a) Copy and complete the table of values for the function $y = (x + 1)(3 - x)$ for the interval $-2 \leq x \leq 4$.

x	-2.0	-1.0	0	0.5	1.0	1.5	2.5	3.0	3.5	4.0
y	-5		3				1.75		-2.25	-5

(b) Draw the graph of $y = (x + 1)(3 - x)$ using a scale of 2cm to 1 unit on both axes for the interval $-2 \leq x \leq 4$.

(c) Using your graph, find the greatest value of $(x + 1)(3 - x)$ and the value of x at which it occurs.

Exercise 12 Date:..

(a) Copy and complete the table of values for the relation $y = \frac{10}{x+3} + 3x - 3$ for the interval $-2.25 \leq x \leq 1.5$

x	-2.25	-2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
y	3.58			-1.0	-0.5		1.36	2.5	

(b) Using a scale of 4cm to 1 unit on the x –axis and 2cm to 1 unit in the y –axis, draw a graph for the relation.

(c) Estimate from your graph;
   (i) the solution of the equation $(3x - 3)(x + 3) + 10 = 0$.
   (ii) the values of x for which $y = 1$.

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 13 Date:……………………………………

(a) Copy and complete the table for the relation $y = 7 + 4x - 3x^2$, for the interval $-3 \leq x \leq 4$.

x	-3	-2	-1	0	0.5	1	1.5	2	2.5	3	3.5	4
y	-32			7	8.25			3		-8	-15.75	

(b) Taking 2cm as 1 unit on the x –axis and 2cm as 5 units on the y –axis, draw the graph of $y = 7 + 4x - 3x^2$ for the given interval.

(c) Draw on the same graph sheet the graph of the relation $y + 2x + 2 = 0$.

(d) Using your graphs
   (i) solve the equation $9 + 6x - 3x^2 = 0$.
   (ii) find the value of x for which $17 + 4x - 3x^2 = 0$.

Exercise 14 Date:……………………………………

The table is for the relation $y = px^2 - 5x + q$.

x	-3	-2	-1	0	1	2	3	4	5
y	21	6		-12				0	13

(a) i. Use the table to find the values of p and q.
   ii. Copy and complete the table.

(b) Using scales of 2cm to 1 unit on the x –axis and 2cm to 5 units on the y –axis, draw the graph of the relation for $-3 \leq x \leq 5$.

(c) Use the graph to find
   i. y when $x = 1.8$;
   ii. x when $y = -8$.

Exercise 15 Date:……………………………………

The relation for the volume of y of a tray of depth $x$cm is given as $y = x(12 - x)(8 - x)\text{cm}^3$.
Copy and complete the table of values for $(12 - x)(8 - x)$.

x	0	1	2	3	4	5	6	7	8
$12 - x$	12	11		9	8		6		4
$8 - x$	8	7	6	5		3		1	0
$x(12 - x)(8 - x)$	0	77		135					0

(a) Use the table of values to draw the graph of $y = x(12 - x)(8 - x)$ from $x = 0$ to $x = 8$, taking 2cm for 1 unit on the x –axis and 2cm for 20 units on the y –axis.

(b) Find the values of x if the volume of the tray is 100cm^3.

(c) What value of x gives the maximum volume of the tray?

Exercise 16 Date:……………………………………

(a) Copy and complete the following table of values for the relation $y = x^2 - 2x - 5$ for $-3 \leq x \leq 5$.

x	-3	-2	-1	0	1	2	3	4	5
y			-2		-6		-2	3	10

(b) Draw the graph of the relation $y = x^2 - 2x - 5$; using a scale of 2cm to 1 unit on the x –axis and 2cm to 2 units on the y –axis.

(c) Using the same axes, draw the graph of $y = 2x - 3$.

(d) Obtain in the form $ax^2 + bx + c = 0$ where a, b and c are integers, the equation which is satisfied by the x –cordinate of the points of intersection of the two graphs.

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 17 Date:…………………………………

When a stone is thrown vertically upwards, its distance d metres after t seconds is given by the formula $d = 60t - 10t^2$. Draw the graph of $d = 60t10t^2$ for values of t from 0 to 6 seconds using 2cm to 1 unit on the t axis and 2cm to 20 units on the d axis.

(a) Using your graph
- (i) using long it take to reach height of 70 metres?
- (ii) determine the height of the stone after 5 seconds.
- (iii) after how many seconds does it reach its maximum height?

(b) Determine the slope of the graph when $t = 4$ seconds.

Exercise 17 Date:…………………………………

(a) Copy and complete the table of values of the relation $y = 3x^2 - 5x - 7$.

x	-3	-2	-1	0	1	2	3	4
y	35			-7	-9		5	

(b) Using a scale of 2cm to 1 unit on the x –axis and 2ccm to 5 units on the y –axis, draw the graph of $y = 3x^2 - 5x - 7$ for $-3 \leq x \leq 4$.

(c) From your graph;
- (i) find the roots of the equation $3x^2 - 5x - 7 = 0$.
- (ii) estimate the minimum value of y;
- (iii) calculate the gradient of the curve at the point $x = 2$.

Exercise 18 Date:…………………………………

A bag of food aid is released from an aeroplane when it is 1000m above a military camp. The height, h metres, of the bag above the camp at time t seconds is given by the relation $h = 1000 - 5t^2$.

(a) Copy and complete the following table for the relation $h = 1000 - 5t^2$.

$t(s)$	0	1	3	5	7	9	11	13	15
$h(m)$				875			395		-125

(b) Using a scale of 2cm to 2 seconds on the t –axis and 2cm to 100m on the h –axis, draw a graph of the relation $h = 1000 - 5t^2$ for $0 \leq t \leq 15$.

(c) Use the graph to find, correct to **one** decimal place, the
- (i) time the bag takes to reach the ground;
- (ii) time the bag takes to drop through the first 650m;
- (iii) height of the bag above the camp after falling for 7.5 seconds.

Exercise 19 Date:…………………………………

The table shows the values of the relation $y = 11 - 2x - 2x^2$ for $-4 \leq x \leq 3$.

x	-4	3	-2	-1	0	1	2	3
y	-13				11			

(a) Copy and complete the table.

(b) Using a scale of 2cm to 1 unit on the x –axis and 2cm to 5 units on the y –axis, draw the graph of $y = 11 - 2x - 2x^2$.

(c) Use your graph to find:
- (i) the roots of the equation $11 - 2x - 2x^2 = 0$.
- (ii) the values of x for which $3 - 2x - 2x^2$.
- (iii) the gradient of the curve at $x = 1$.

Exercise 20 Date:..

(a) Copy and complete the following tables of values for the relation $y = 2x^2 - 7x - 3$.

x	-2	-1	0	1	2	3	4	5
y	19		-3		-9			

(b) Using 2cm to 1 unit on the x —axis and 2cm to 5 units on the y —axis, draw the graph of $y = 2x^2 - 7x - 3$ for $-2 \le x \le 5$.

(c) From your graph, find the
  (i) minimum value of y.   (ii) gradient of the curve at $x = 1$.

(d) By drawing a suitable straight line, find the values of x for which $2x^2 - 7x - 5 = x + 4$.

Exercise 21 Date:..

(a) Copy and complete the table of values for the relation $y = -x^2 + x + 2$ for $-3 \le x \le 3$.

x	-3	-2	-1	0	1	2	3
y		-4		2			-4

(b) Using scales of 2cm to 1 unit on the x —axis and 2cm to 2 units on the y —axis, draw a graph of the relation $y = -x^2 + x + 2$.

(c) From the graph, find the
  (i) minimum value of y
  (ii) roots of the equation $x^2 - x - 2 = 0$.
  (iii) gradient of the curve at $x = -0.5$.

Exercise 22 Date:..

(i) Copy and complete the following table for the relation: $y = 2(x + 2)^2 - 3$ for $-5 \le x \le 2$.

x	-5	-4	-3	-2	-1	0	1	2
y			-1	-3		5		

(ii) Using scales of 2cm to 1 unit on the x — axis and 2cm to 5 units on the y —axis, draw the graph of the relation $y = 2(x + 2)^2 - 3$ for $-5 \le x \le 2$.

(iii) Use the graph to find the solution of:
  (i) $2(x + 2)^2 = 3$;   (ii) $2(x + 2)^2 = 5$.

(iv) For what values of x, from the graph, is y increasing in the interval?

Exercise 23 Date:..

(a) Copy and complete the table of values for $y = x^2 - 1, -4 \le x \le 4$

x	-4	-3	-2	-1	0	1	2	3	4
y		8			-1				

(b) Using a scale of 2 cm to 1 unit on the x — axis and 2 cm to 2 units on the y — axis, draw the graph for $y = x^2 - 1$.

(c) Using the graph:
  (i) solve $x^2 = 8$
  (ii) find the range of values of x for which y decreases as x increases.

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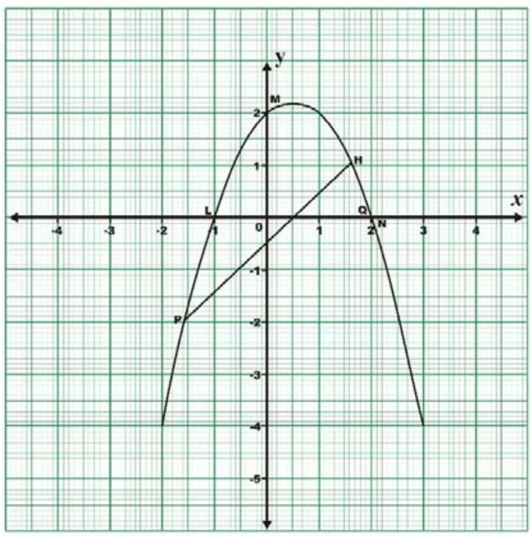
Exercise 23 Date:................................

The diagram shows the graph of $y = ax^2 + bx + c$ and $y = mx + k$, where a, b, c, m and k are constants. Use the graph to:

(i) Find the roots of the equation $ax^2 + bx + c = mx + k$.

(ii) Determine the values of a, b and c using the coordinates of points L, M and N and hence write down the equation of the curve;

(iii) Determine the line of symmetry of the curve $y = ax^2 + bx + c$.



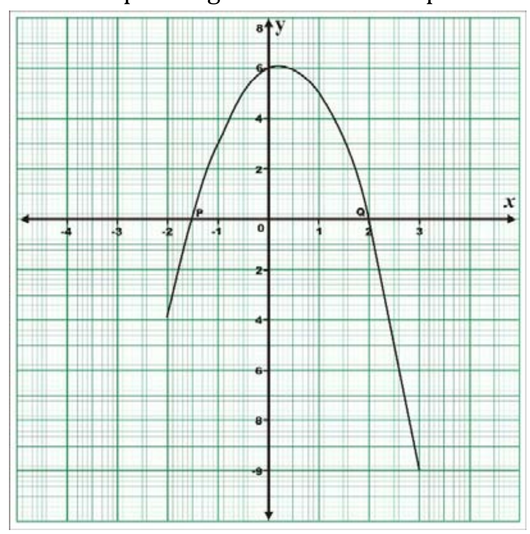
Exercise 24 Date:................................

Below is the graph of the relation $y = ax^2 + bx + c$, when a, b, c are constants. Use the graph to

(i) Find the roots of the quadratic equation $ax^2 + bx + c = 0$.

(ii) Determine the values of constants a, b and c in the relation using the values of the coordinates P, Q and hence write down the relation illustrated in the graph;

(iii) Find the maximum value of y and the corresponding value of x at this point.



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Exercise 25 Date:…………………………..

(a) Copy and complete the following table of values for the relation $y = 3 - 2x - x^2$ for $-5 \leq x \leq 3$.

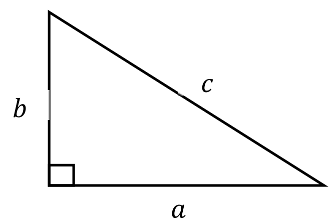
x	-5	-4	-3	-2	-1	0	1	2	3
y				2		3			-12

(b) Using scales of 23 cm to 1 unit in $x -$ axis and 2 cm to 2 units on the $y -$ axis, draw the graph of $y = 3 - 2x - x^2$ for $-5 \leq x \leq 3$.

(c) For your graph, find the
- (i) equation of the axis of symmetry;
- (ii) values of x from which y decreases;
- (iii) values of x for which $x^2 + 3x - 3 = 0$.

TRIGONOMETRY I

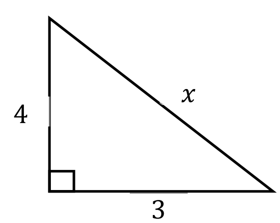
Pythagoras' Theorem
Pythagoras' theorem says that for any right – angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides.



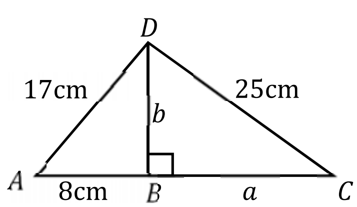
$$\boxed{a^2 + b^2 = c^2}$$

Example 1
Calculate the lengths indicated by the letters in each of the following diagrams.

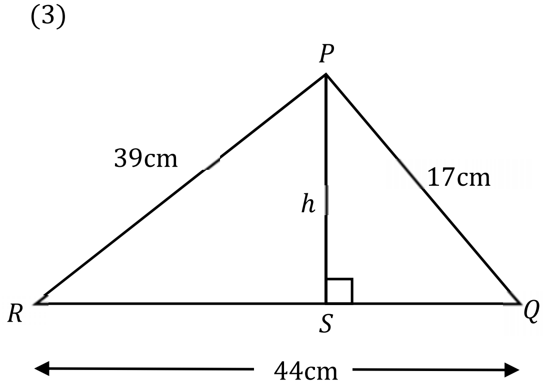
(1)



(2)



(3)



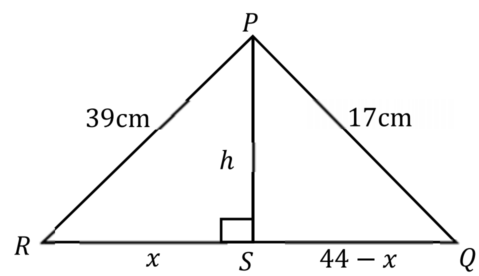
Solution...

1. $4^2 + 3^2 = x^2$
$$x = \sqrt{4^2 + 3^2}$$
$$x = 5\text{cm}$$

2. $|AB|^2 + |BD|^2 = |AD|^2$
$$8^2 + b^2 = 17^2$$
$$b^2 = 17^2 - 8^2$$
$$b = \sqrt{17^2 - 8^2}$$
$$b = 15\text{cm}$$

$$|DB|^2 + |BC|^2 = |DC|^2$$
$$b^2 + a^2 = 25^2$$
$$15^2 + a^2 = 25^2$$
$$a^2 = 25^2 - 15^2$$
$$a = \sqrt{25^2 - 15^2}$$
$$a = 20\text{cm}$$

3.



ΔRPS,
$|RS|^2 + |PS|^2 = |RP|^2$
$x^2 + h^2 = 39^2$ ………………………….. (1)

ΔPSQ,
$|PS|^2 + |SQ|^2 = |PQ|^2$
$h^2 + (44 - x)^2 = 17^2$ ………………….. (2)

From (1)
$h^2 = 39^2 - x^2$ ………………………….. (3)

Put (3) into (2)
$$39^2 - x^2 + (44 - x)^2 = 17^2$$
$$39^2 - x^2 + 44^2 - 2(44)(x) + x^2 = 17^2$$
$$88x = 3168$$
$$x = 36$$

Put $x = 36$ into (3)
$h^2 = 39^2 - 36^2$
$h^2 = 225$
$h = \sqrt{225} = 15\text{cm}$

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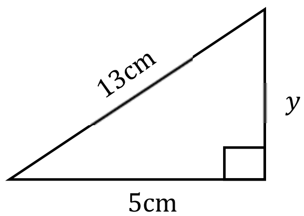
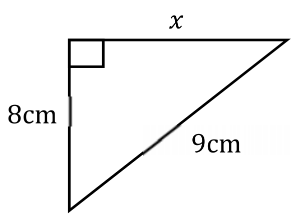
Exercise 1 Date :…………………….…

Calculate the lengths by the letters in each of the following diagrams.

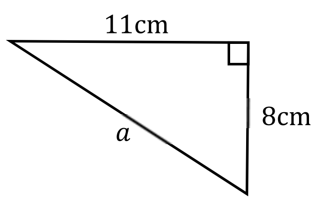
(1)



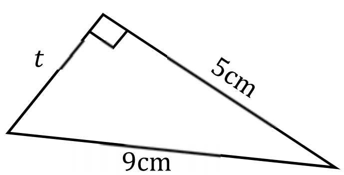
(2)



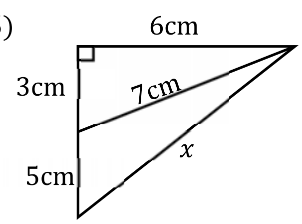
(3)



(4)



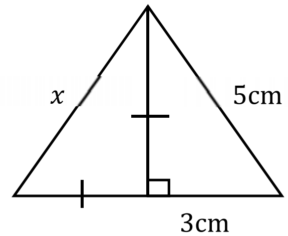
(5)



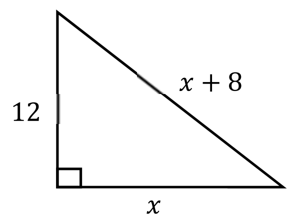
Exercise 2 Date :…………………….…

Calculate the lengths by the letters in each of the following diagrams.

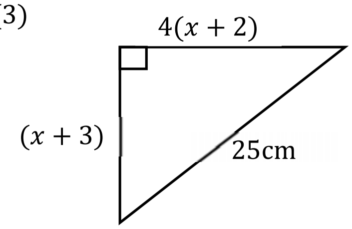
(1)



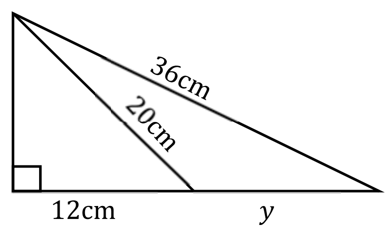
(2)

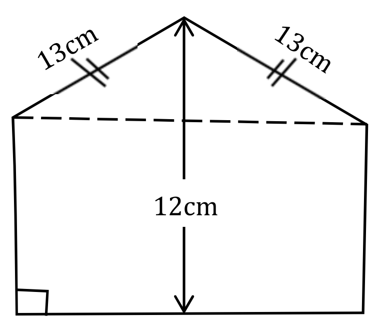


(3)



(4)



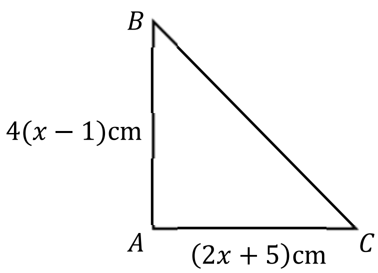


Exercise 3 Date :……………………

1. The sides of a rectangular floor are x m and $(x+7)$m. The diagonal is $(x+8)$m. Calculate, in meters:
   (a) the value of x
   (b) the area of the floor

2.
   (a) The diagonal of a rectangle exceeds the length by 2cm. If the width of the rectangle is 10cm, find the length.

   (b)



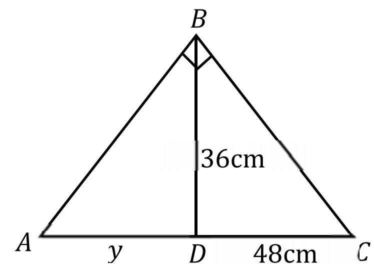
   The area of this triangle is 30cm^2. Find
   (i) the values of x
   (ii) $|BC|$

3. A rope 60cm long is made to form a rectangle. If the length is 4 times its breadth, calculate, correct to one decimal place, the
   (i) length
   (ii) diagonal
   of the rectangle.

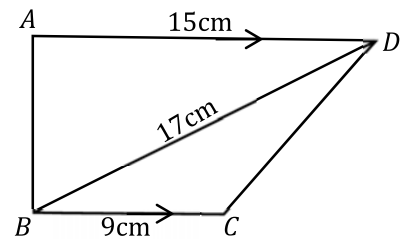
4. A rectangular lawn of length $(x+5)$ metres is $(x-2)$ metres wide. If the diagonal is $(x+6)$ metres, find
   (i) the value of x
   (ii) the area of the lawn.

Exercise 4 Date :……………………

1. In the diagram below, ADC is a straight line. $|CD| = 48$cm, $|BD| = 36$cm and $|AD| = y$ cm. Find the value of y.

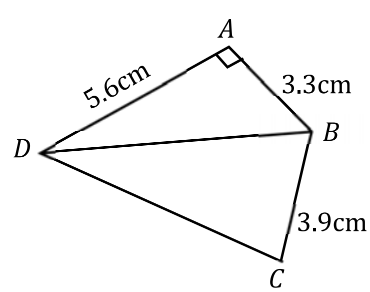


2. In the diagram, $ABCD$ is a trapezium in which $AD//BC$ and $\angle ABC$ is a right angle. If $|AD| = 15$cm, $|BD| = 17$cm and $|BC| = 9$cm.

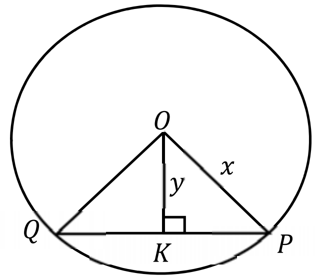


   Calculate
   (a) $|AB|$
   (b) The area of the triangle BCD
   (c) $|CD|$
   (d) The perimeter of the trapezium.

3. The diagram shows a quadrilateral $ABCD$ in which $\angle DAB$ is a right angle. $|AB| = 3.3$cm, $|BC| = 3.9$cm, $|CD| = 5.2$cm and $|DA| = 5.6$cm.
   (i) Find the length of BD
   (ii) Show that $\angle BCD = 90°$



4.



In the diagram, O is the centre of the circle radius x.
$|PQ| = z$, $|OK| = y$ and $\angle OKP = 90°$.
Find the value of z in terms of x and y.

Exercise 5 Date :……………………

Two points A and C, are on the same level ground as the foot of the vertical pole, B. The distance between A and C is 70m and A and C are on the opposite sides of the vertical pole. The distance from the top of the pole, D, to A and C are 45m and 59m respectively. Find

(i) distance between the foot of the pole, B and the point A
(ii) the height BD of the pole.

Exercise 6 Date :……………………

A man standing 40m away from a tower notices that the distances from the top and the bottom of a flag staff on top of the tower are 50m and 454m respectively. Find the

(i) height of the flag staff.
(ii) height of the tower.

Exercise 7 Date :……………………

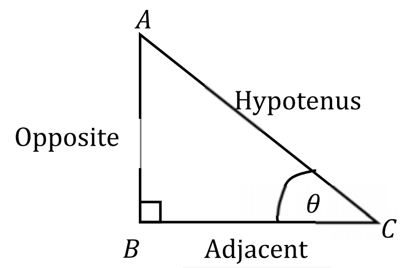
Two point A and C, are on the same level ground as the pole, B. The distance between A and C is 40m and A and C are on the same sides of the vertical pole. The distances from the top of the pole, D to A and C are 53m and 85m respectively. Find correct to one decimal place

(i) the distance between the foot of the pole, B and the point A.
(ii) the height BD, of the pole.

TRIGONOMETRY RATIOS

There are three trigonometric ratios: sine, cosine and tangent. Each of these relates an angle of a right – angled triangle to a ratio of the lengths of two of its sides.

The sides of the triangle have names, two of which are dependent on their position in relation to a specific angle. The longest side (always opposite the right angle) is called **hypotenuse.** The side opposite the angle is called the **opposite side** and the side next to the angle is called the **adjacent side.**



$$\boxed{\textbf{SOH CAH TOA}}$$

$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{|AB|}{|BC|}$

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{|AB|}{|AC|}$

$\cos\theta = \frac{A}{H}$

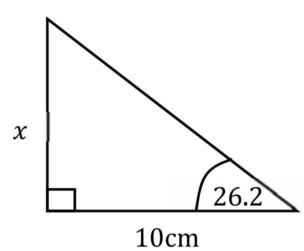
$\cos\theta = \frac{|BC|}{|AC|}$

Finding The Length of A Side

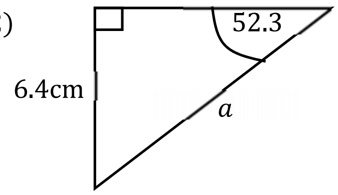
Example 2

Find the side marked with letters.

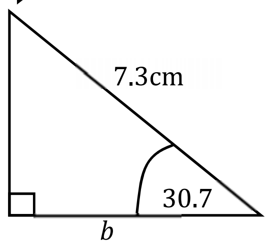
(1)



(2)



(3)



Solution…

1. $\tan 26.2 = \frac{x}{10}$

   $x = 10 \tan 26.2 = 4.92$

2. $\sin 52.3 = \frac{6.4}{a}$

   $a = \frac{6.4}{\sin 52.3} = 8.09$

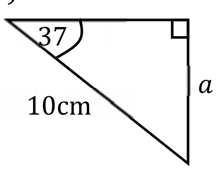
3. $\cos 30.7 = \frac{b}{7.3}$

   $b = 7.3 \cos 30.7 = 6.28$

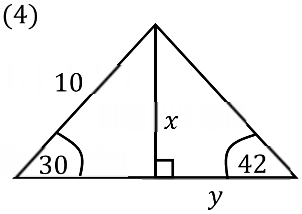
Exercise 8 **Date :……………………..**

Find the sides marked with letters.

(1)



(4)



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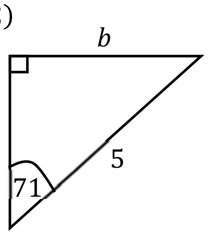

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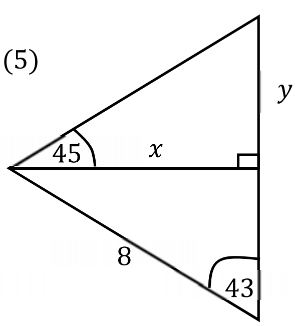

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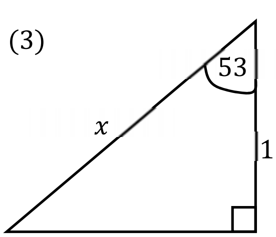
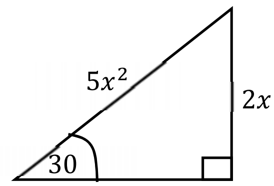


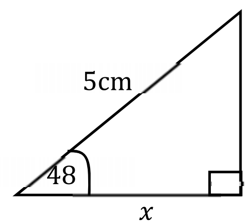
Exercise 9 **Date :……………………..**

Calculate the length of the side marked x in each of the following.

(1)



(2)



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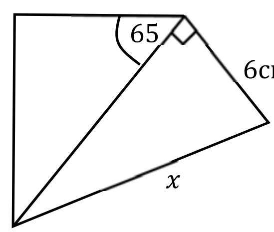
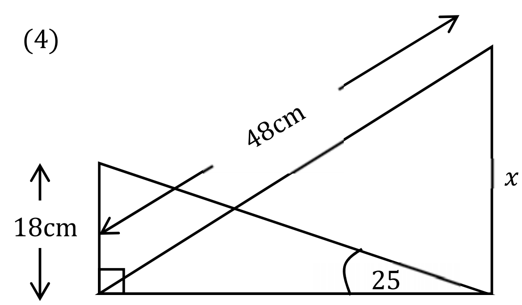


(4)

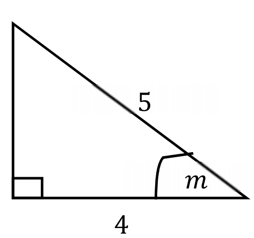


Example

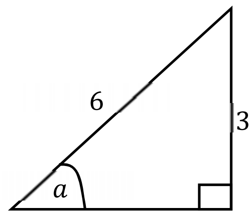
Finding An Unknown Angle

Find the angle marked with letters.

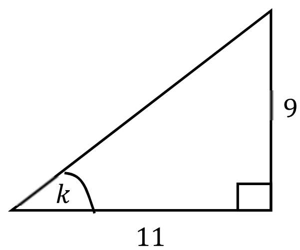
(1)



(2)



(3)



Solution…

1. $\cos m = \frac{4}{5}$

   $m = \cos^{-1}\left(\frac{4}{5}\right)$

   $m = 36.9°$

2. $\sin a = \frac{3}{6}$

   $a = \sin^{-1}\left(\frac{3}{6}\right)$

   $a = 30°$

3. $\tan k = \frac{9}{11}$

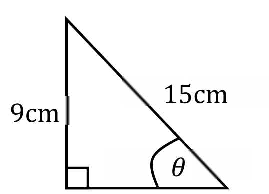
   $k = \tan^{-1}\left(\frac{9}{11}\right)$

   $k = 39.3°$

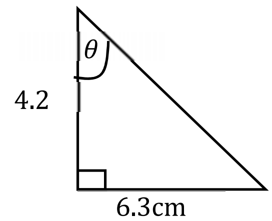
Exercise 10 **Date :……………………..**

Find the unknown quantities represented by letters in each of the following figures.

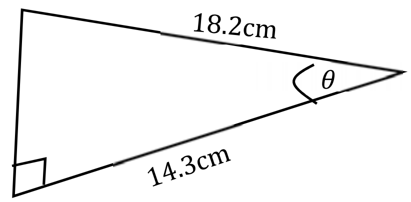
(1)



(2)



(3)



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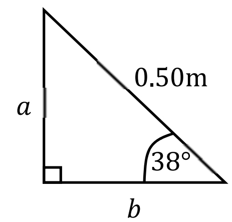
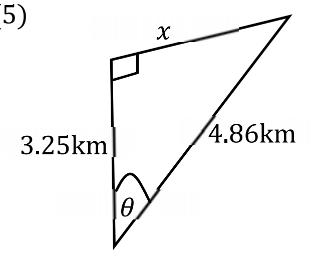
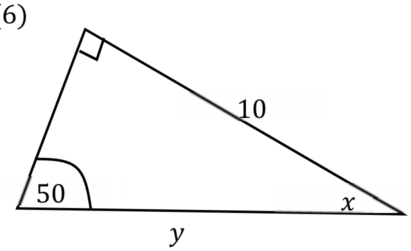


(5)



(6)



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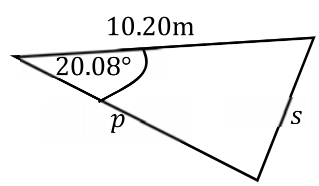


Example 4

1. Solve $\frac{\tan^2 x}{3} - 1 = 0$, where $0° \leq x \leq 90°$.

Solution…

$\frac{\tan^2 x}{3} - 1 = 0$

$\frac{\tan^2 x}{3} = 1$

$\tan^2 x = 3$

$\tan x = \sqrt{3}$

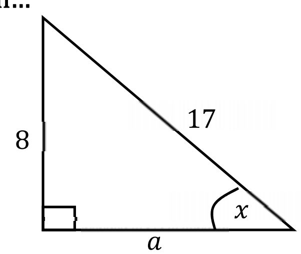
$x = \tan^{-1}(\sqrt{3})$

$x = 60°$

Example 5

If $\sin x = \frac{8}{17}$, find the value of $\frac{\tan x}{1+2\tan x}$

Solution…



$8^2 + a^2 = 17^2$

$a = \sqrt{17^2 - 8^2}$

$a = 15$

$\therefore \tan x = \frac{8}{15}$

$\therefore \frac{\tan x}{1+2\tan x} = \left[\frac{8}{15}\right] \div \left[1 + 2 \times \frac{8}{15}\right]$

$= \frac{8}{15} \div \left[1 + \frac{16}{15}\right]$

$= \frac{8}{15} \div \frac{15+16}{15}$

$= \frac{8}{15} \div \frac{31}{15}$

$= \frac{8}{15} \times \frac{15}{31}$

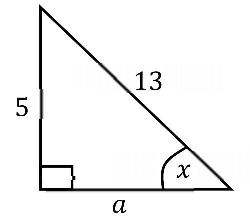
$= \frac{8}{31}$

Example 6

If $\sin x = \frac{5}{13}$, $0° \leq x \leq 90°$, evaluate, without tables or calculator, $\frac{\cos x - 2\sin x}{2\tan x}$.

Solution…

Given, $\sin x = \frac{5}{13}$



$5^2 + a^2 = 13^2$

$a^2 = 13^2 - 5^2$

$a = \sqrt{13^2 - 5^2}$

$a = 12$

$\therefore \cos x = \frac{12}{13}, \tan x = \frac{5}{12}$

$\therefore \frac{\cos x - 2\sin x}{2\tan x} = \left[\frac{12}{13} - 2 \times \frac{5}{13}\right] \div 2 \times \left[\frac{5}{12}\right]$

$= \left[\frac{12}{13} - \frac{10}{13}\right] \div \frac{10}{12}$

$= \frac{12-10}{13} \times \frac{12}{10}$

$= \frac{2}{13} \times \frac{12}{10} = \frac{12}{65}$

Example 7

Find θ, if $\cos(\theta + 60°) = 0.0872$, where $0° \leq \theta \leq 90°$.

Solution…

$\cos(\theta + 60°) = 0.0872$

$\theta + 60° = \cos^{-1}(0.0872)$

$\theta + 60° = 85°$

$\theta = 85° - 60°$

$\theta = 25°$

Exercise 11 Date :…………………..…

1. Given that $\tan(x + 25°) = 5.145$, where $0° \leq x \leq 90°$, find, correct to one decimal place, the value of x.

2. Given that $5\cos(x + 8.5)° - 1 = 0$, $0° \leq x \leq 90°$, calculate, correct to the nearest degree, the value of x.

3. If $4\tan x = 3$, where $0° \leq x \leq 90°$
   (i) Find x
   (ii) Evaluate $\frac{1+\cos x}{2-\cos x}$

Exercise 12 Date :…………………..…

1. If $\sin x = \frac{1}{2}$, where $0° \leq x \leq 90°$, evaluate $\frac{\sin x \cos x}{\cos x + \tan x}$.

2. Given that $\sin x = 0.6$ and $0° \leq x \leq 90°$, find $1 - \tan x$, leaving your answer in the form $\frac{a}{b}$.

3. Given that $\tan y = \sqrt{2}$, where x and y are acute angles, find:
   (a) $\cos y$
   (b) the value of x if $\sin x = 1 - \cos y$.

4. If $\cos t = \frac{4}{5}, 0 \leq t \leq 90°$, find without using mathematical tables or calculators the value of $\frac{1}{1-\sin t} - \frac{1}{1+\sin t}$.

5. Given that $\cos x = \frac{3}{5}$, $0° \leq x \leq 90°$, calculate, without using mathematical tables or calculator, calculate, $\frac{3\tan x}{2\sin x + 3\cos x}$.

6. If $8\sin x + 2 = 5$, find x correct to the nearest degree.

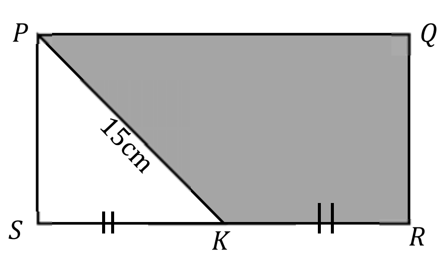
Exercise 13 **Date :……………………..**

1. Without Mathematical tables or calculators, simplify:
   (i) $\frac{2\tan 60° + \cos 30°}{\sin 60°}$
   (ii) $\frac{\sin 45° + \tan 30°}{\tan 45° - \cos 60°}$
   (iii) $3\cos 315° - 2\sin 210°$

2. The height hm, of a dock above sea level is given by:
   $h = 6 + 4\cos(15p)°, 0 < p < 6$.
   Find
   (i) the value of h when $p = 4$
   (ii) correct to two significant figures, the value of p when $h = 9$m.

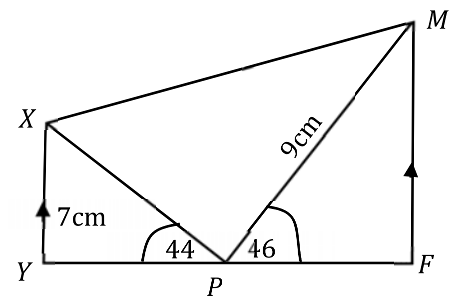
Exercise 14 **Date :……………………..**

1.



In the diagram $PQRS$ is a rectangle $|PK| = 15$cm, $|SK| = |KR|$ and $< PKS = 37°$. Calculate correct to three significant figures
   (i) $|PS|$
   (ii) $|SK|$
   (iii) the area of the shaded portion.

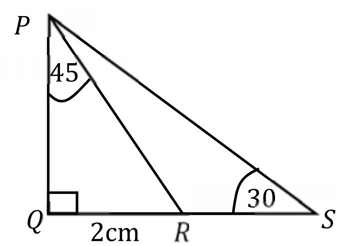
2.



In the diagram, YPF is a straight line, $< XPY = 44°, < MPF = 46°$, $< XYP =< MFP = 90°$, $|XY| = 7$cm and $|MP| = 9$cm.
   (i) Calculate, correct to **3 significant figures** $|XM|$ and $|YF|$
   (ii) $< XMP$.

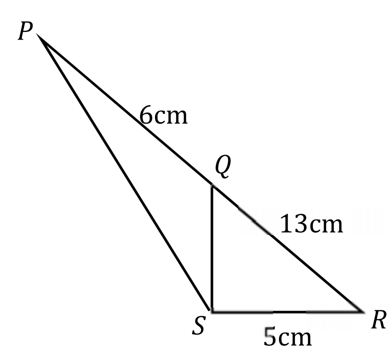
Exercise 15 **Date :……………………..**

1.



In the diagram, $|QR| = 2$cm, $< PQR = 90°$, $< RSP = 30°$ and $< QPR = 45°$. Find
   (i) $|PR|$
   (ii) $|RS|$ in surd form (radicals)

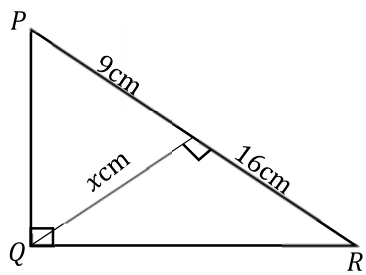
2.



In the diagram, $|PQ| = 6$cm, $|QR| = 13$cm, $|RS| = 5$cm and $< RSQ$ is a right angle. Calculate, correct to one decimal place, $|PS|$.

3. A small stone is tied to a point P vertically above it by an elastic string 102m long. If the string is moved such that it is inclined at an angle of $50°$ to the vertical, how high does the stone rise?

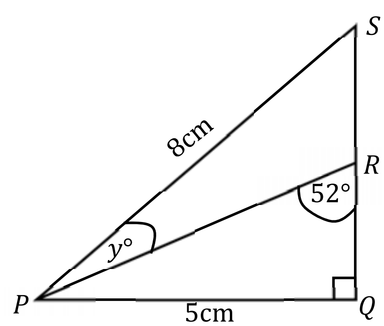
4.



In the diagram, $< PQR =< PSQ = 90°$, $|PS| = 9$cm, $|SR| = 16$cm and $|SQ| = x$. Find:
(a) The value of x
(b) $< QRS$, correct to the nearest degree,
(c) $|PQ|$

Exercise 16 Date :……………………..

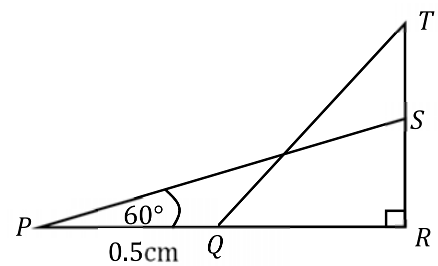
1.



In the diagram, ΔPQS is a right – angled triangle at Q. $|PQ| = 5$cm, $< SPR = y°$, $|PS| = 8$cm and $< PRQ = 52°$. Calculate correct, the value of angle $y°$.

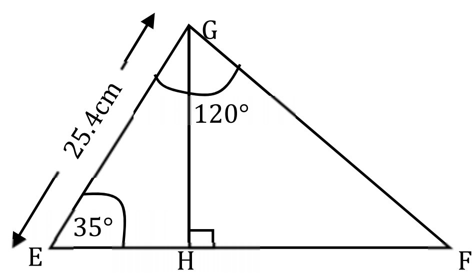
2. The length of the sides of a triangle are in the ratio $7: 9: 9$, calculate, correct, to the nearest degree, the angle between the equal sides.

3. In the diagram, PS and QT are two ladders 10m and 12m long respectively, placed against a vertical wall TR. PS makes an angle of $60°$ with the horizontal and $|PQ| = 0.5$cm. Calculate, correct to two significant figures
   (i) the angle which QT makes with the horizontal
   (ii) the length of the point T above the horizontal.



Exercise 17 Date :……………………..

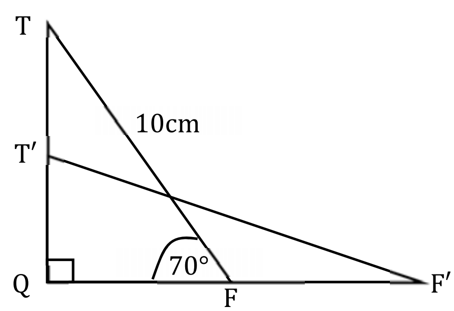
1.



In the figure, angle $GEH = 35°$, angle $EGF = 120°$, $|\overline{EG}| = 25.4$cm and $\overline{GH}$ is perpendicular to $\overline{EF}$. Calculate $|\overline{EF}|$, correct to three significant figures.

2. ABC is an isosceles triangle with $|AB| = |AC|$ and $< ABC = 30°$. The perpendicular from A to BC is $\sqrt{3}$ metres. Without using Mathematical tables, calculate the area of the triangle ABC, leaving your answer in surd form.

3.



In the diagram, a ladder TF, 10m long is placed against a wall at an angle of 70° to the horizontal.

(a) How high up the wall correct to the nearest metre, does the ladder reach?

(b) If the foot (F) of the ladder is pulled from the wall to F' by 1 metre,

(i) how far, correct to two significant figures, does the top T slide down the wall to T'?

(ii) calculate, correct to the nearest degree, angle $QF'T'$.

4.

(a) If $\sin\theta = \frac{3}{5}$, find the value of $\tan\theta$.

(b) A ladder 5m long, leans against a vertical wall at an angle of 70° to the ground. The ladder slips down the wall 2m. find, correct to two significant figures,

(i) the new angle which the ladder makes with the ground;

(ii) the distance the ladder slipped back on the ground from its original position.

Exercise 18 Date :……………………..

1. if $\cos x = P$, find $\sin x$ in terms of P.

2. If $\sin(2x - 12)° = \frac{1}{2}$ and $0° \leq x \leq 90°$, find x.

3. If $\cos x = \sin 39$, $0° < x < 90°$, find x.

4. If $\sin(x - 10)° = \cos(x + 10)°$, calculate the value of x.

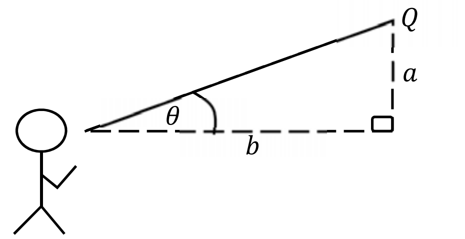
5. Given that $\sin(5x - 28)° = \cos(3x - 50)°$, $0° < x < 90°$, find the value of x.

6. Given that $\sin(A + B) = \sin A\cos B + \cos A\sin B$, without using mathematical tables or calculator, evaluate $\sin 105°$, leaving your answer in surd for. [You may use $105° = 60° + 45°$]

7. Given that $x = \cos 30°$ and $y = \sin 30°$, evaluate without using a mathematical table or calculator. $\frac{x^2+y^2}{y^2-x^2}$.

ANGLE OF ELEVATION AND DEPRESSION

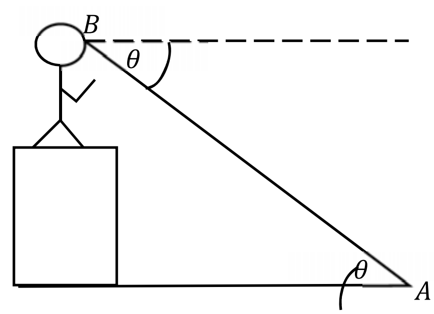
The angle of elevation of an object Q from an observer at P who is below the level of Q is the angle which PQ makes with the horizontal.



$\tan\theta = \frac{a}{b}$

$\theta = \tan^{-1}\left(\frac{a}{b}\right)$

The angle of depression of an object at B from an observer at A, who is above the level of B, is the angle which AB makes with the horizontal.



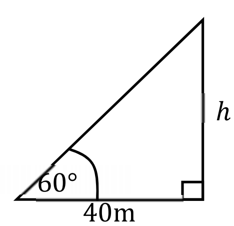
Example 8

From a point 40m from the foot of a vertical pole, along level ground, the angle of elevation of the top of the pole is 60°. What is the

(i) length of the pole?

(ii) height of the pole?

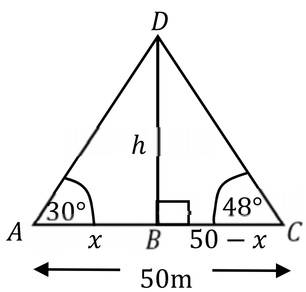
Solution…



$\tan 60 = \frac{h}{40}$
$h = 40 \tan 60$
$h = 40\sqrt{3}$
$h = 69.28$
$h \cong 69.3\text{m}$

Example 9
Two points A and C, on opposite sides of a vertical pole, are on the same level ground as the foot of the pole, B. The angle of elevation of the top of the pole D from A and C are 30° and 48° respectively. If the distance between A and C is 50m, find $|BD|$, the height of the pole.

Solution…



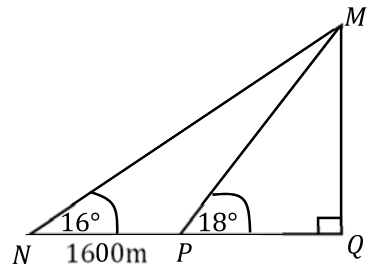
$\tan(30) = \frac{h}{x}$
$h = x\tan(30)$……………………………….. (1)

$\tan(48) = \frac{h}{50-x}$
$(50 - x)\tan 48 = h$…………………… (2)

$(1) = (2)$
$x\tan(30) = (50 - x)\tan(48)$
$x\tan(30) = 50\tan(48) - x\tan(48)$
$x\tan(30) + x\tan(48) = 50\tan(48)$
$x = \frac{50\tan 48}{\tan 30 + \tan 48}$
$x = 32.898$

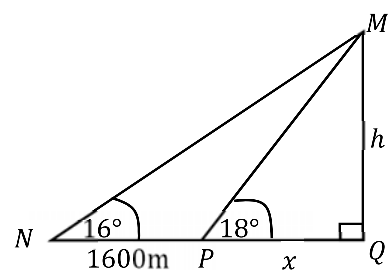
Put $x = 32.898$ into (1)
$h = 32.898 \tan 30$
$h = 18.994$
$h \cong 19.00\text{m}$

Example 10



A surveyor at sea level observed that the angle of elevation of the top of a mountain M from two points N and P due West of it are 16° and 18° respectively as show in the diagram.
If $|NP| = 1600\text{m}$ and the base of the mountain Q is vertically below M, calculate the height of the mountain.

Solution…



From the diagram,
$\tan(16) = \frac{|MQ|}{|NQ|}$
$\tan(16) = \frac{h}{1600+x}$
$h = (1600 + x)\tan 16$ …………………… (1)

Also,
$\tan(18°) = \frac{|MQ|}{|PQ|}$
$\tan(18°) = \frac{h}{x}$
$h = x\tan(18°)$…………………….. (2)

$(1) = (2)$
$(1600 + x)\tan(16) = x\tan(18°)$
$1600\tan(16) + x\tan(16) = x\tan(18°)$
$1600\tan(16) = x(\tan 18 - \tan 16)$
$x = \frac{1600\tan 16}{\tan 18 - \tan 16}$
$x = 12018.36\text{m}$

Put $x = 12018.36$ into (2)
$h = 12018.36 \tan 18$
$h = 3905.002$

$h \cong 3905.00\text{m}$

$\therefore$ The height of the mountain is 3905m.

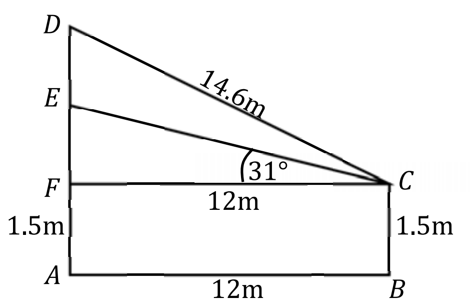
Example 11 Date :……………………

A boy 1.5m tall is standing 12m away from a church building which has a tower on top of its roof. The top of the cross on the tower is 14.6m away from the boy's head (eyes). If the boy has to raise his eyes through an angle of 31° in order to see the top of the roof, calculate:

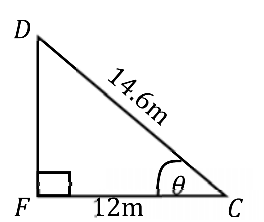
(a) correct to the nearest degree, the angle through which the boy must raise his eyes to see the top of the cross on the tower.
(b) correct to 1 d.p, the height of the cross from the ground
(c) correct to one decimal place, the height of the church building.

Solution…

(a)



Let θ be the angle through which the boy must raise his eyes to see the top of the top of the cross on the tower.



$\cos\theta = \frac{12}{14.6}$

$\theta = \cos^{-1}\left(\frac{12}{14.6}\right)$

$\theta = 34.722$

$\theta \cong 35°$

(b) $|DF|^2 + |FC|^2 = |DC|^2$

$|DF|^2 + 12^2 = 14.6^2$

$|DF|^2 = 14.6^2 - 12^2$

$|DF| = \sqrt{14.6^2 - 12^2}$

$|DF| = 8.316\text{m}$

$|AD| = |AF| + |FD|$

$|AD| = 1.5 + 8.316$

$|AD| = 9.816$

$|AD| \cong 9.8\text{m}$

$\therefore$ Correct to one decimal place, the height of the cross from the ground is 9.8m.

(c) $\tan 31 = \frac{|FE|}{12}$

$|FE| = 12\tan 31$

$|FE| = 7.21$

Height of the building $|AE|$

$|AE| = |AF| + |FE|$

$|AE| = 1.5 + 7{,}21$

$|AE| = 8.7\text{m}$

Exercise 19 Date :……………………

1. A vertical pole AB is erected on a level ground. A man 1.7m tall stands at C, 24m away from the foot, B, of the pole. The angle of elevation of the top A of the pole from the man is 54°. Calculate, correct to 1 dp, the height of the pole.

2. The angle of elevation of the top, X, of a vertical pole from a point, W, on the same horizontal ground as the foot, Z, of the pole is 60°. If W is 15km away from X and 12km from a point Y on the pole,
   (a) illustrate this information with a diagram;
   (b) calculate, correct to two decimal places, the:
      (i) angle of elevation of Y from W;
      (ii) length, XY.

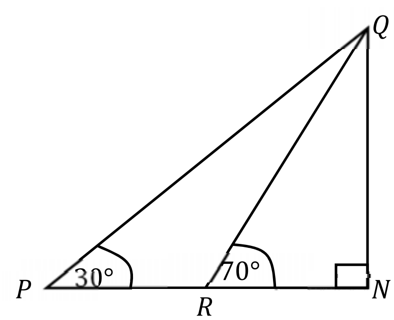
Exercise 20 Date :……………………

1. From two points P and Q, 15m apart and on the same horizontal line as the foot of a tower, the angles of elevation of the top of the tower are 35° and 45°, respectively. If P and Q are on the same side of the tower,
   (a) represent the information in a diagram;
   (b) find, correct to the nearest metre, the height of the tower.

2. A simple measuring device is used at points X and Y on the same horizontal level to measure the angle of elevation of the peak P of a certain mountain. If X is known to be 5,200m above sea level, $|XY| = 4{,}000$m and the measurements of the angles of elevation of P at X and Y are $15°$ and $35°$ respectively, fins the height of the mountain.
[Take $\tan 15° = 0.3$ and $\tan 35° = 0.7$]

Exercise 21 Date :……………………

1.



The diagram above represents the vertical cross – section of a mountain with height NQ standing on a horizontal ground PRN. If the angles of elevation of the top Q of the mountain from P And R are $30°$ and $70°$ respectively, and $|PR| = 500$m, calculate correct to 3 significant figures
(i) $|QP|$;
(ii) the height of the mountain.

2. A tower and a building stand on the same horizontal level. From the point P at the bottom of the building, the angle of elevation of the top T, of the tower is $65°$. From the top Q of the building, the angle of elevation of the point T is $25°$. If the building is 20m high, calculate
(a) the distance PT
(b) hence or otherwise, calculate the height of the tower.
(Give your answer correct to 3 significant figures).

Exercise 22 Date :……………………

1.
(a) A surveyor walks 100m up a hill which slopes at an angle of $24°$ to the horizontal. Calculate, correct to the nearest metre, the height through which he rises.

(b) Two boats, 70 metres apart and on opposite sides of a light house, are in a straight line with the light – house. The angles of elevation of the top of the light – house from the two boats are $71.6°$ and $45°$. Find the height of the lighthouse.
[Take $\tan 71.6 = 3$]

2. A boy 1.2m tall, stands 6m away from the foot of a vertical lamp pole 4.2m long. If the lamp is at the tip of the pole,
(a) represent this information in a diagram.
(b) Calculate the:
(i) length of the shadow of the boy cast by the lamp;
(ii) angle of elevation of the lamp from the boy, correct to the nearest degree.

3. From two points on opposite sides of a pole 33m high, the angles of elevation of the top of the pole are $53°$ and $67°$. If the two points and the base of the pole are on the same horizontal level, calculate, correct to three significant figures, the distance between the two points.

4. From a point T on a horizontal ground, the angle of elevation of the top R of a tower RS, 38m high, is $63°$. Calculate, correct to the nearest metre, the distance between T and S.

5. The angle of elevation of the top of a flagpole is $42°$ from a point P is 180 metres from the foot B of the flagpole. Q is a point on the same horizontal line BP such that $|BQ| = 45$m. Calculate, correct to one decimal place, the angle of elevation of the flagpole from Q.

6. Two towers A and B are 48m and 30m respectively. Tower A lies to the West and B to the East of a man 1.5m tall. From the man's eye level, the angles of elevation of the top of A and B are $66°$ and $28°$ respectively. Calculate, correct to three significant figures, the distance between A and B.

7. From a point P on a level ground and directly West of a pole, the angel of elevation of the top of the pole is 45° and from point Q East of the pole, the angle of elevation of the top of the pole is 58°. If $|PQ| = 10\text{m}$, calculate, correct to 2 significant figures, the:
   (i) distance from P to the pole;
   (ii) height of the pole.

8. It was observed that the shadow of a vertical pole was 6m longer when the angle of elevation of the sum was 30° than it was 60°. By means of a sketched diagram, calculate correct to two decimal places, the height of the pole.

9. A tank 2m tall stands on top of a concrete pillar. From a point (P) on the same horizontal ground as the foot of the pillar, the angles of elevation of the top (T) and bottom (B) of the tank are 49° and 42° respectively.
   (i) draw a diagram to represent this information.
   (ii) calculate, correct to **one** decimal place, the height of the pillar.
   (iii) calculate, correct to **one** decimal place, $|PB|$.

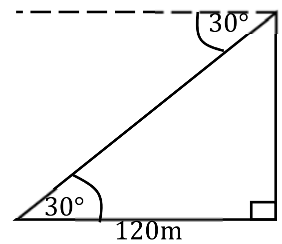
10. A point X is between two towers TP and QW and are all on the same horizontal ground. The angles of elevation of the tops T and Q from X are 62° and 48° respectively. $|TP| = 100\text{m}$ and $|PW| = 80\text{m}$.
   (i) Illustrate the information in a diagram.
   (ii) Calculate, correct to the **nearest** metre, $|QW|$.

ANGLE OF DEPRESSION

Example 12

A boat is in the horizontal level as the foot of a cliff, and the angle of depression of the boat from the top of the cliff is 30°. If the boat is 120m away from the foot of the cliff, find the height of the cliff, correct to three significant figures.

Solution…

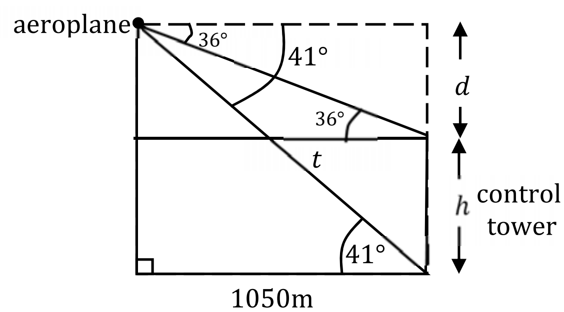


$\tan 30 = \frac{h}{120}$
$h = 120 \tan 30$
$h = 69.3\text{m}$

Example 13

From an aeroplane in the air and at a horizontal distance of 1050m the angles of depression of the top and base of a control tower at an instant are 36° and 41° respectively. Calculate, correct to the nearest metre, the:
(i) height of the control tower;
(ii) shortest distance between the aeroplane and the base of the control tower.

Solution…



(i) Let h be the height of the control tower

$\tan 36° = \frac{d}{1050}$
$d = 1050 \tan 36°$
$d = 762.825\text{m}$
$\tan 41° = \frac{d+h}{1050}$

$$d + h = 1050 \tan 41°$$
$$d + h = 912.765$$
$$h = 912.765 - d$$
$$h = 912.765 - 762.825$$
$$h = 149.94\text{m}$$

$\therefore$ Height of the control tower is 150m (to the nearest metre).

(ii) Let t be the shortest distance

$$\cos 41° = \frac{1050}{t}$$
$$t = \frac{1050}{\cos 41°}$$
$$t = 1391.28$$

The shortest distance is 1391m (to the nearest metre)

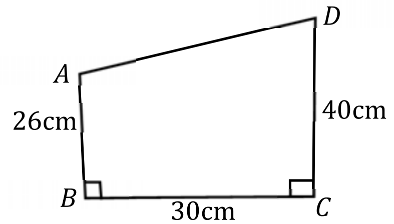
Exercise 23 Date :……………………

1. A hawk on top of a tree 20 metres high, views a chick on the ground at an angle of depression of 39°. Find correct to 2 significant figures the distance of the chick from the bottom of the tree.

2. From a horizontal distance of 8,5km , a pilot observes that the angles of depression of the top and base of a control tower are 30° and 33° respectively. Calculate, correct to three significant figures:
   (i) the shortest distance between the pilot and the base of the control tower.
   (ii) the height of the control tower.

Exercise 24 Date :……………………

1. In the diagram below, points B and C are on a horizontal plane and $|BD| =$ 30cm, A and D are points vertically above B and C respectively. $|DC| =$ 40cm and $|AB| = 26$cm. Calculate the angles of depression.
   (i) B from D;
   (ii) A from D.



2. A point H is 20m away from the foot of a tower on the same horizontal ground. From the point H. The angle of elevation of the point (P) on the tower and the top (T) of the tower are 30° and 50° respectively. Calculate, correct to 3 significant figures.
   (a) $|PT|$
   (b) the distance between H and the top of the tower.
   (c) the position of H if the angle of depression of H from the top of the tower is to be 40°.

Exercise 25 Date :……………………

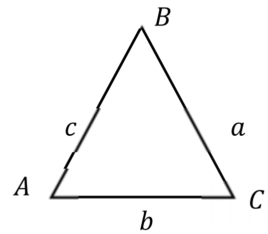
1. The angle of depression of a boat from mid – point of a vertical cliff is 35°. If the boat is 120m from the foot of the cliff, calculate the height of the cliff.

2. The angles of depression of the top and bottom of building are 51° and 62° respectively from the top of a tower 72m high. The base of the building is on the same horizontal level as the foot of the tower. Calculate the height of the building correct to **two** significant figures.
3. A woman looking out from the window of a building at a height of 30m, observed that the angle of depression of the top of a flagpole was 44°. If the foot of the pole is 25m from the foot of the building and on the same horizontal ground, find, correct to the nearest whole number, the
   (a) angle of depression of the foot of the pole from the woman;
   (b) height of the flagpole

FURTHER TRIGONOMETRY

The Sine Rule

The **sine rule** is a relationship which can be used with non-right – angled triangles.



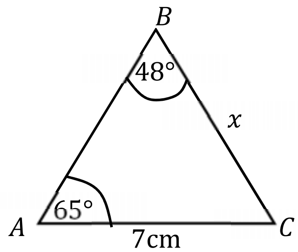
The sine rule states that:

$$\boxed{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}}$$

Or alternatively,

$$\boxed{\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}}$$

Example 14



Calculate the length of BC.

Solution…

Using the sine rule;

$\frac{\sin 48}{7}=\frac{\sin 65}{x}$

$\Longrightarrow x\sin 48 = 7\sin 65$

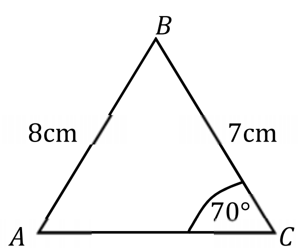
$\Longrightarrow x=\frac{7\sin 65}{\sin 48}$

$\Longrightarrow x = 8.5369$

$\Longrightarrow x = 8.54$

$\therefore |BC| = 8.54\text{cm}$

Example 15



Calculate the size of angle A.

Solution…

Using the sine rule;

$\frac{\sin 70}{8}=\frac{\sin A}{9}$

$\frac{\sin 70}{8}=\frac{\sin A}{7}$

$\sin A=\frac{7\sin 70}{8}$

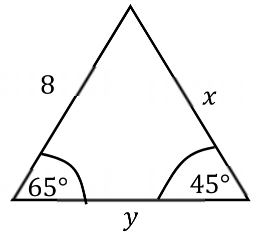
$A=\sin^{-1}\left(\frac{7\sin 70}{8}\right)$

$< A = 55.3°.$

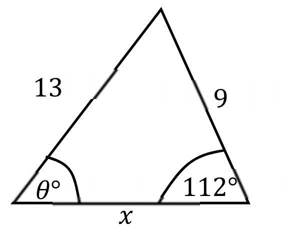
Exercise 26 **Date :………………..…..**

Calculate the length of the side or size of angles.

1



2



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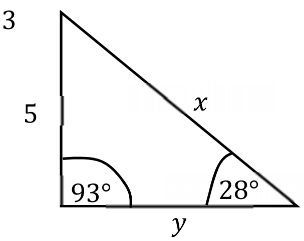
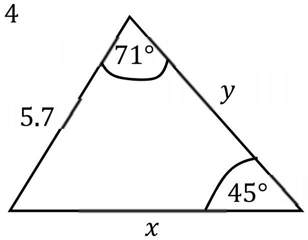
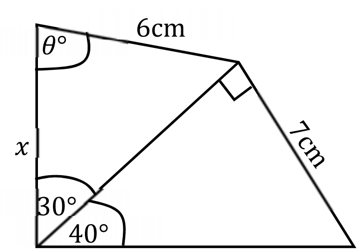


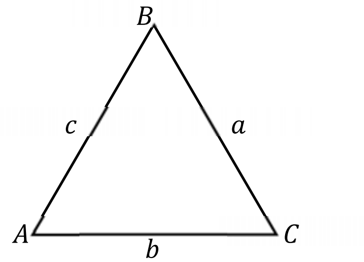
4



5



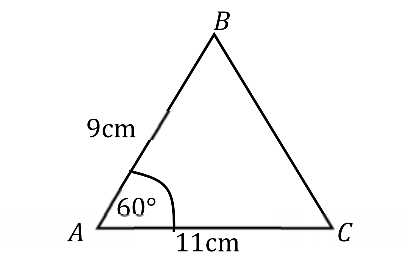
The Cosine Rule
The cosine rule is another relationship which can be used with non – right – angled triangles.



The cosine rule states that:

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

Example 16

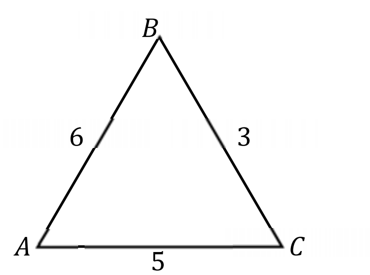


Find $|BC|$

Solution…
Using the sine rule:

$|BC|^2 = |AB|^2 + |AC|^2 - 2|AB||AC|\cos A$
$|BC|^2 = 9^2 + 11^2 - 2 \times 9 \times 11\cos 60$
$|BC|^2 = 202 - 198\cos 60$
$|BC| = \sqrt{202 - 198\cos 60}$
$|BC| = 10.14889$
$|BC| \cong 10.15\text{cm}$

Example 17



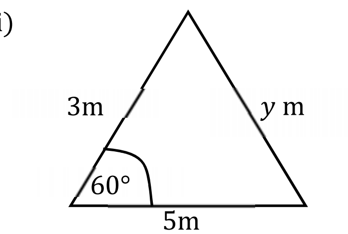
Find $< BAC$.

Solution…
$|BC|^2 = |AB|^2 + |AC|^2 - 2|AB||AC|\cos A$
$3^2 = 6^2 + 5^2 - 2 \times 6 \times 5\cos\theta$
$9 = 36 + 25 - 60\cos\theta$
$9 - 61 = -60\cos\theta$
$-52 = -60\cos\theta$
$\cos\theta = \frac{-52}{-60}$
$\cos\theta = \frac{13}{15}$
$\theta = \cos^{-1}\left(\frac{13}{15}\right)$
$\theta = 29.93°$
$\theta = 29.9°$

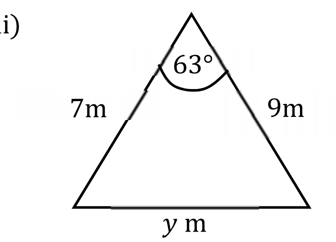
Exercise 27 **Date :……………………..**

(a) Calculate the length of the side marked y in each case.

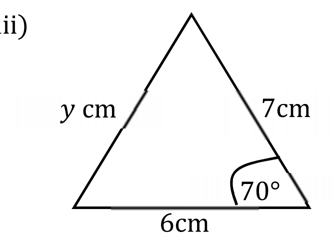
(i)



(ii)

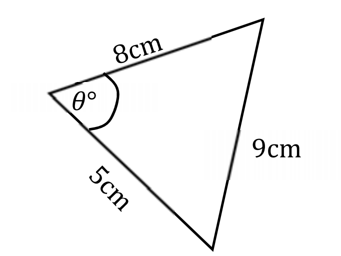


(iii)

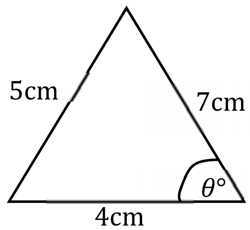


(b) Calculate the angle marked θ in each of the following.

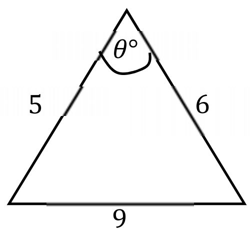
(i)



(ii)



(iii)



Exercise 28 Date :……………………..

1. $ABCD$ is a parallelogram in which $|AB| = 5.0\text{cm}$, $|AC| = 7.0\text{cm}$ and $< BAC = 42°$. Calculate, correct to 3 significant figures the length of BC.

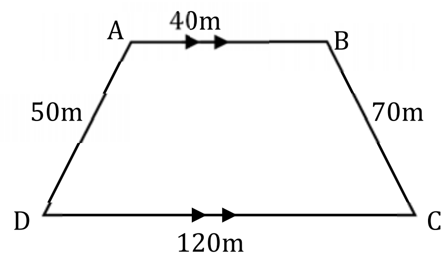
2. The triangle ABC has sides $|AB| = 17\text{m}$, $|BC| = 12\text{m}$ and $|AC| = 10\text{m}$. Calculate the:
   (i) largest angle of the triangle
   (ii) area of the triangle.

3. The length of the sides of a triangle are in the ratio $14 : 18 : 18$. Calculate correct to the nearest degree, the angle between the equal sides.

4. The sides of an isosceles triangle are in the ratio $7 : 5 : 7$. Calculate, correct to the nearest degree, the angle included between the equal sides.

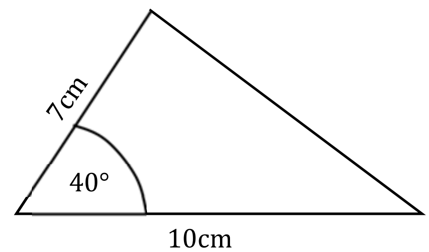
Exercise 29 Date :……………………..

1. The diagram represents a playing field in the form of a trapezium. If $|AB| = 40\text{m}$, $|BC| = 70\text{m}$, $|DC| = 120\text{m}$, $|AD| = 50\text{m}$ and $AB//DC$.



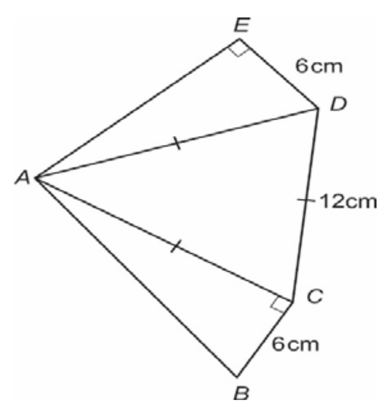
Calculate:
(a) angle ADC in degrees.
(b) the length of AC, correct to **three significant figures.**

2.



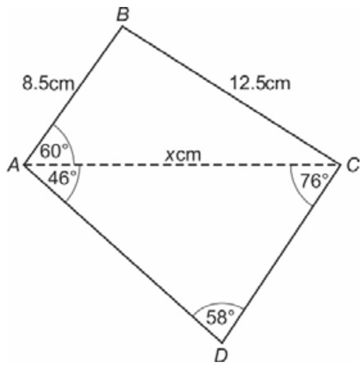
(i) Calculate the area of the triangle
(ii) Calculate the perimeter of the triangle correct to **one decimal place.**

3.



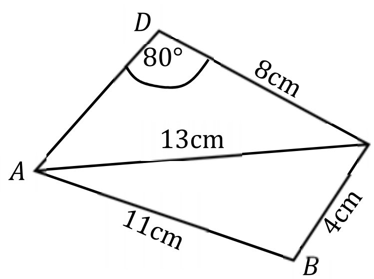
In the pentagon $ABCDE$, angle $ACB =$ angle $AED = 90°$. Triangle ACD is equilateral with side length 12cm. $|DE| = |BC|$6cm.
(i) Calculate $< BAE$
(ii) Calculate $|AB|$
(iii) Calculate $|AE|$
(iv) Calculate the area of the pentagon

4.



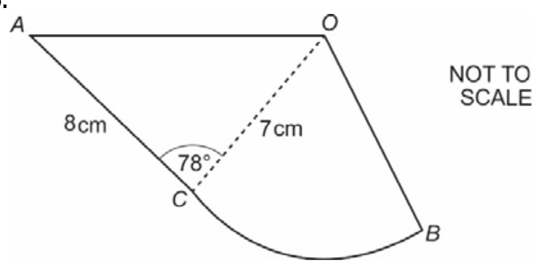
(a) Solve for the values of x correct to 2 decimal places
(b) Find $|CD|$
(c) Calculate the area of the quadrilateral $ABCD$.

5.



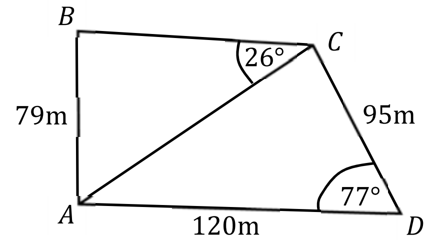
(a) Calculate angle ACB
(b) Calculate $< ACD$
(c) Calculate the area of the quadrilateral $ABCD$

6.



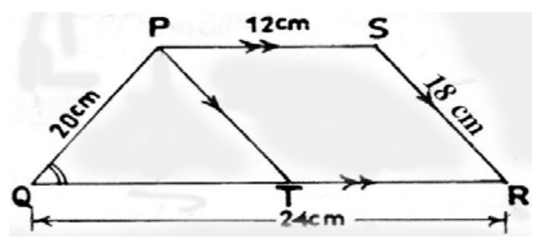
The diagram shows a design made from a triangle AOC joined to a sector OCB. $|AC| = 8\text{cm}, |OB| = |OC| = 7\text{cm}$ and angle $ACO = 78°$.
(a) Calculate $|OA|$ correct to 2 decimal place
(b) Calculate $< OAC$
(c) The perimeter of the design is 29.5cm. Find $< COB$, correct to 1 decimal place
(d) Calculate the total area of the design.

7.



The quadrilateral $ABCD$ represents an area of land. There is a straight road form A to C. $|AB| = 79\text{m}, |AD| = 120\text{m}$ and $|CD| = 95\text{m}$. $< BCA = 26°$ and $< CDA = 77°$.
(a) Calculate $|AC|$, correct to the nearest metre.
(b) Calculate $< ABC$
(c) A straight path is to be built on the road, $|AC|$. Calculate the length of this path.
(d) Houses are to be built on the land in triangle ACD. Each house needs at least 180m^2 of land. Calculate the maximum number of houses which can be built. Show all of your working.

8.

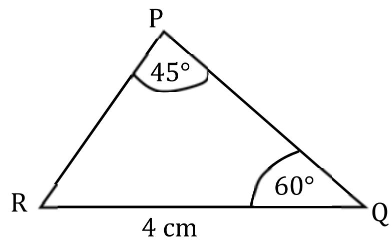


In the diagram, $PS//QR$, $PT//SR$, $|QR| = 24\text{cm}, |SR| = 18\text{cm}$, $|PQ| = 20\text{cm}$ and $|PS| = 12\text{cm}$. Calculate, correct to 3 significant figures;
(a) $< PQT$
(b) the height of triangle PQT
(c) the area of $PTRS$ as a percentage of the area of $PQRS$.

THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

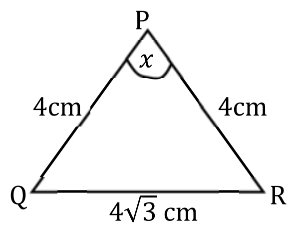
Exercise 30 Date :……………………

1.



Find $|RP|$.

2.



Find the value of x.

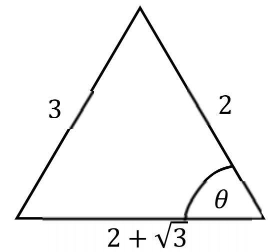
3. PQR is a triangle. $|PQ| = 2$cm, $|QR| = 4$cm and $|PR| = 3$cm. Calculate correct to three decimal places, the cosine of the smallest angle.

4. ABC is a triangle, $|AB| = 8$cm, $|BC| = 7$cm and angle $ABC = 36°$. Find $|AC|$, correct to one decimal place.

5. In a triangle XYZ, $|XY| = 12$cm , $|XZ| = 8$cm and angle $YXZ = 42°$. Calculate
   (i) correct to two places $|YZ|$
   (ii) $< XZY$

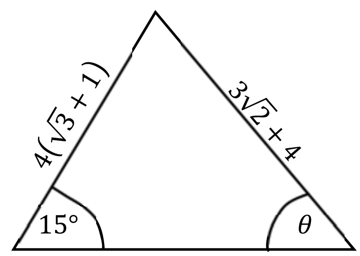
6. The top edges of two boards are inclined to each other at 30°. If the boards are 6m and 5m long respectively, find
   (a) the distance between the bottom edges of the boards, correct to three significant figures.
   (b) the angle the longer board makes with the floor.
   (c) the shortest distance between the edges of the boards and the floor, correct to two significant figures.

7.
   (α)



Without using a calculator, find the value of $\cos\theta$, giving your answer in the form $\frac{a+b\sqrt{3}}{c}$, where a, b and c are integers.

(β)



Using $\sin 15° = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$ and without using a calculator, find the value of $\sin\theta$ in the form $a + b\sqrt{2}$, where a and b are integers.

RIGID MOTION

Formulae under Rigid Motion

1. Reflection in the x −axis
   (i.e. the line $y = 0$)
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ -y \end{pmatrix}$ or $(x, y) \to (x, -y)$

2. Reflection in the y −axis
   (i.e. the line $x = 0$)
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -x \\ y \end{pmatrix}$ or $(x, y) \to (-x, y)$

3. Reflection in the line $x = k$ or $x - k = 0$
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2k - x \\ y \end{pmatrix}$ or $(x, y) \to (2k - x, y)$

4. Reflection in the line $y = k$ or $y - k = 0$
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ 2k - y \end{pmatrix}$ or $(x, y) \to (x, 2k - y)$

5. Reflection in the line $y = kx$
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} \frac{1}{k}y \\ kx \end{pmatrix}$ or $(x, y) \to \left(\frac{1}{k}y, kx\right)$

6. Reflection in the line $y = x$ or $y - x = 0$
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ x \end{pmatrix}$ or $(x, y) \to (y, x)$

7. Reflection in the line $y = -x$ or $x + y = 0$
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ -x \end{pmatrix}$ or $(x, y) \to (-y, -x)$

8. Anticlockwise rotation of 90° about O
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix}$ or $(x, y) \to (-y, x)$

9. Anticlockwise rotation of 180° about O
   $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -x \\ -y \end{pmatrix}$ or $(x, y) \to (-x, -y)$

10. Anticlockwise rotation of 270° about O
    $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ -x \end{pmatrix}$ or $(x, y) \to (y, -x)$

11. Translation by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$
    $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x + a \\ y + b \end{pmatrix}$

12. Enlargement from O with scale factor k
    $\begin{pmatrix} x \\ y \end{pmatrix} \to k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$ or $(x, y) \to (kx, ky)$

13. Enlargement from any point (a, b) other than the origin $(0, 0)$
    $$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} k(x - a) + a \\ k(y - b) + b \end{pmatrix}$$

Exercise 1 **Date:……………………**

1. Find the image of $(-2, 4)$ under the mapping $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2y \\ 2 - 3x \end{pmatrix}$.
2. A translation T takes the point $P(1, 2)$ to $P'(5, 3)$. What is the image of $Q(3, 4)$ under T?
3. Find the image of the position vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ under the translation $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.
4. If $A(2, 3)$ is reflected in the x − axis, find the image A' of A.
5. Find the image of $(3, -7)$ under the transformation $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2x + y \\ y - 3x \end{pmatrix}$.

Exercise 2 **Date:……………………**

A triangle has vertices $A(1, 1)$, $B(2, 4)$ and $C(5, 8)$.

(a) Calculate the coordinates of the vertices of the image $A'B'C'$ of triangle ABC under a translation by the vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(b) Calculate the vertices of the image triangle $A''B''C''$ of triangle ABC under an enlargement with scale factor 2 from the origin.

(c) The triangle ABC undergoes transformation involving a rotation in anticlockwise direction through 90° about the origin followed by a translation, such that $A \to A'''$, $B \to B'''$ and $C \to C'''$. If A''' is $(2, -1)$, find:
   (i) the translation vector;
   (ii) the coordinates of B''' and C'''.

Exercise 3 **Date:……………………**

(a) Using a scale of 2cm to 2 units on both axes, draw on a sheet of graph paper two perpendicular axes $0x$ and $0y$ for the intervals $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

(b) Draw on the same graph sheet, indicating clearly the coordinates of all vertices,

   i. The quadrilateral ABCD with A $(2, 4)$, B $(4, 7)$, C $(8, 8)$, and D $(6, 3)$.
   ii. the image $A_1B_1C_1D_1$ of quadrilateral ABCD under an anti-clockwise

rotation of 90° about the origin, where $A \longrightarrow A_1, B \longrightarrow B_1, C \longrightarrow C_1$ and $D \longrightarrow D_1$.

iii. the image $A_2B_2C_2D_2$ of quadrilateral ABCD under translation by the vector $\binom{-10}{-9}$, where $A \longrightarrow A_2, B \longrightarrow B_2$, $C \longrightarrow C_2$ and $D \longrightarrow D_2$.

iv. the image $A_3B_3C_3D_3$ of quadrilateral ABCD under a reflection in the line $y = 1$ where $A \longrightarrow A_3, B \longrightarrow B_3$, $C \longrightarrow C_3$ and $D \longrightarrow D_3$.

(c) Find $\overrightarrow{C_2C_3}$.

Exercise 4 Date:......................

(a) Using a scale of 2cm to 1 units on both axes, draw on a graph sheet, two perpendicular axes $0x$ and $0y$ for intervals $-5 \leq x \leq 5$ and $-6 \leq y \leq 6$.

(b) Draw clearly, indicating the coordinates of all vertices
   i. rectangle ABC with vertices A $(-2, -1)$ B $(1, -1)$ C $(1, 1)$ and D $(-2, 1)$;
   ii. the image $A_1B_1C_1D_1$ of ABCD under an enlargement from the origin O with scale factor 2 where $A \longrightarrow A_1$, $B \longrightarrow B_1, C \longrightarrow C_1, D \longrightarrow D_1$.
   iii. the image $A_2B_2C_2D_2$ of ABC under a reflection in the line $y = 2$ where $A \longrightarrow A_2, B \longrightarrow B_2, C \longrightarrow C_2$, $D \longrightarrow D_2$
   iv. Determine the equation of the line AC.

Exercise 5 Date:......................

Using a scale of 1cm to 1 unit on each axes, draw two perpendicular axes $0x$ and $0y$ for $-5 \leq x \leq 5$ and $-6 \leq y \leq 6$ on a graph sheet.

(a) Draw on the same graph sheet, labeling all vertices clearly together with their coordinates.
   i. Triangle PQR with vertices P $(-3, 3)$, Q $(-1, -2)$ and R $(3, -1)$;
   ii. Triangle UVW with vertices U $(-2, 6)$, V $(0, 1)$ and W $(4, 2)$.

(b) Deduce the transformation that maps triangle PQR onto UVW.

(c) Draw, labeling all vertices together with their coordinates.
   i. The triangle $U_1V_1W_1$ of triangle UVW under a rotation of 180° about the origin where $U \longrightarrow U_1, V \longrightarrow V_1$ and $W \longrightarrow W_1$.

The image $U_2V_2W_2$ of triangle $U_1V_1W_1$ under the reflection in the line $x = 0$, where $U_1 \longrightarrow U_2, V_1 \longrightarrow V_2$ and $W_1 \longrightarrow W_2$.

Exercise 6 Date:......................

(a) Using a scale of 2cm to 2 units on each axes, draw on a sheet of graph paper two perpendicular axes $0x$ and $0y$ for the intervals $-10 \leq x \leq 10$ and $-12 \leq y \leq 12$.

(b) Draw on this graph sheet, indicating the coordinates of all vertices.
   i. The quadrilateral ABCD with vertices A $(-5, -4)$, B $(2, -1)$, C $(0, 3)$, and D $(-8, 4)$.
   ii. the image $A_1B_1C_1D_1$ of ABCD under a translation by the vector $\binom{3}{-8}$, where $A \longrightarrow A_1, B \longrightarrow B_1, C \longrightarrow C_1$ and $D \longrightarrow D_1$.
   iii. the image $A_2B_2C_2D_2$ of ABCD under an enlargement from the origin with a scale factor $-{}^1/_2$ where $A \longrightarrow A_2$, $B \longrightarrow B_2, C \longrightarrow C_2$ and $D \longrightarrow D_2$.

(c) Find the equation of $\overrightarrow{A_1D}$.

Exercise 7 Date:......................

a) Using a scale of 1cm to 1 units on both axes, draw on a graph sheet, two perpendicular axes $0x$ and $0y$ for intervals $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

b) Draw on the same graph sheet, indicating clearly all vertices and their coordinates.

   i. triangle ABC with vertices A (4, 2) B (0, 2) C (0,8);

   ii. the image triangle $A_1B_1C_1$ of ABC under the mapping $\binom{x}{y} \longrightarrow \binom{x}{y+{}^1/_2x}$ where $A \longrightarrow A_1, B \longrightarrow B_1, C \longrightarrow C_1$;

   iii. the image triangle $A_2B_2C_2$ of ABC under a clockwise rotation of 90°

about the origin where $A \longrightarrow A_2$, $B \longrightarrow B_2$ and $C \longrightarrow C_2$

iv. the image triangle $A_3B_3C_3$ of ABC under an enlargement with the scale factor -1 where $A \longrightarrow A_3$, $B \longrightarrow B_3$ and $C \longrightarrow C_3$.

Exercise 8 Date:……………………

(a) Given the point $A(2,3)$ and the vectors $\overrightarrow{AB} = \begin{pmatrix}1\\2\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}-4\\1\end{pmatrix}$, draw on the same graph sheet, indicating clearly all vertices and their coordinates

(i) Triangle ABC

(ii) The image $\Delta A_1B_1C_1$ of ΔABC under a reflection in the line $x - 4 = 0$ where $A \rightarrow A_1$, $B \rightarrow B_1$ and $C \rightarrow C_1$

(b) Using the graph, calculate $|\overrightarrow{A_1C_1}|$, leaving the answer in the form $p\sqrt{q}$ where p and q are positive integers.

Exercise 9 Date:……………………

(a) Using scales of 2cm to 2 units on both axes, draw on a sheet of graph paper two perpendicular axes $0x$ and $0y$, for $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

(b) Given points $E(3,2)$, $F(-1,5)$ are the vectors $\overrightarrow{FG} = \begin{pmatrix}1\\3\end{pmatrix}$ and $\overrightarrow{GH} = \begin{pmatrix}3\\-1\end{pmatrix}$, find the coordinates of the points G and H.

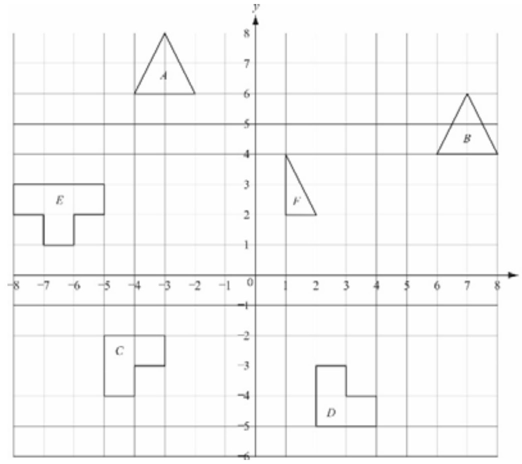
(c) Draw, on the same graph, indicating clearly the vertices and their coordinates, the:

(i) Quadrilateral $EFGH$;

(ii) Image $E_1F_1G_1H_1$ of the quadrilateral $EFGH$ under an anticlockwise rotation of 90° about the origin where $E \rightarrow E_1$, $F \rightarrow F_1$, $G \rightarrow G_1$ and $H \rightarrow H_1$.

(d) The side E_1F_1 of the quadrilateral $E_1F_1G_1H_1$ cuts the x – axis at the point P. Calculate, correct to **one** decimal place, the area $E_1H_1G_1P$.

Exercise 10 Date:……………………



(a) Describe fully the single transformation that maps

(i) Shape A onto shape B

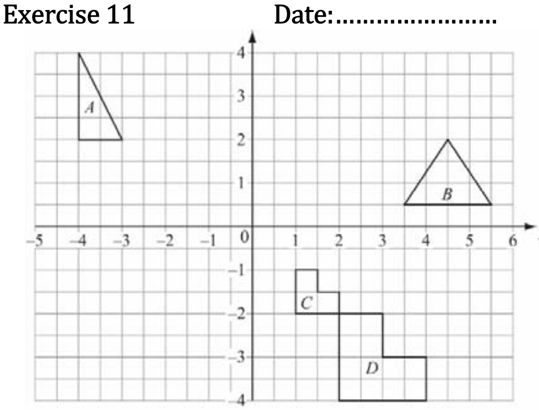
(ii) Shape C onto shape D

(b) On the grid above, draw

(i) The reflection of shape E in the y – axis.

(ii) The enlargement of shape F, with scale factor 2 and centre $(0,0)$.

Exercise 11 Date:……………………



(a) A translation is given by $\begin{pmatrix}6\\3\end{pmatrix} + \begin{pmatrix}-3\\-4\end{pmatrix}$.

(i) Write this transformation as a single column vector.

(ii) On the grid, draw the translation of triangle A using this vector.

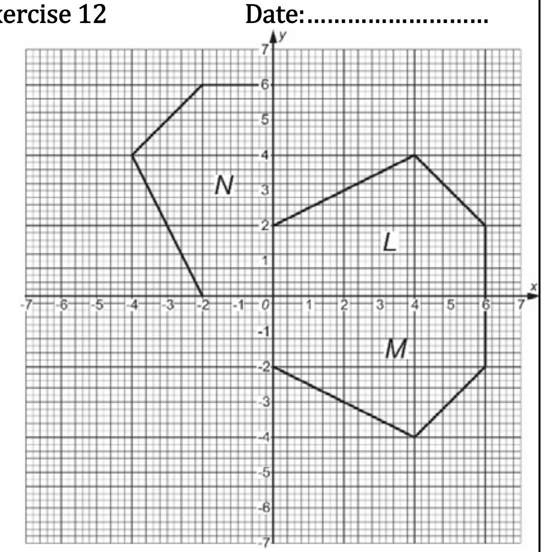
(b) Another translation is given by $-2\begin{pmatrix}1\\-1\end{pmatrix}$.

(i) Write this translation as a single column vector.

(ii) On the grid, draw the translation of triangle B using this vector.

(c) Describe fully the transformation that maps shape C onto shape D.

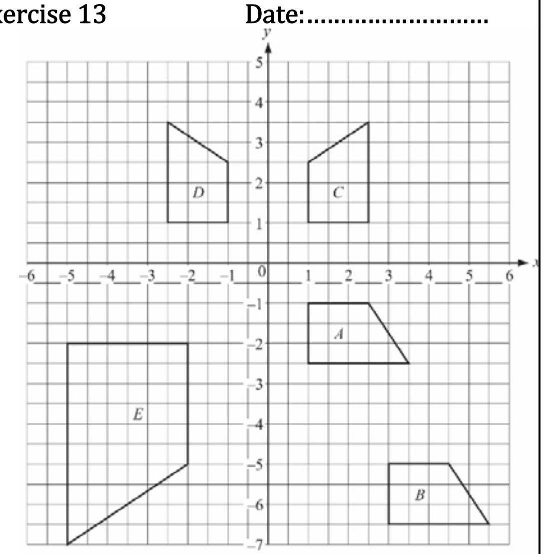
Exercise 12 Date:.........................



(a) Describe fully the transformation which maps
   (i) Shape L onto shape M
   (ii) Shape L onto shape N

(b) (i) Translate shape L using the vector $\begin{pmatrix}-7\\-4\end{pmatrix}$.
   (ii) Enlarge shape L with centre of enlargement O, scale factor $\frac{1}{2}$.

Exercise 13 Date:.........................



Describe fully the **single** transformation which maps

(a) A onto B
(b) C onto D
(c) A onto C
(d) C onto E

Exercise 14 Date:.........................

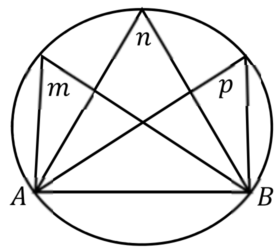
(a) Using a scale of 2cm to 1 unit on both axes, draw on a sheet of graph paper, two perpendicular axes $0x$ and $0y$ for $-5 \le x \le 5$ and $-5 \le y \le 5$.

(b) Draw on the same graph sheet, indicating clearly all vertices and their coordinates:
   (i) ΔABC with vertices $A(2,1), B(1,4)$ and $C(-1,2)$;
   (ii) the image $\Delta A_1B_1C_1$ of ΔABC under a reflection in the line $y = 0$, where $A \to A_1, B \to B_1$ and $C \to C_1$.
   (iii) the image $\Delta A_2B_2C_2$ of ΔABC under a translation by the vector $\begin{pmatrix}-2\\1\end{pmatrix}$ where $A \to A_2, B \to B_2$ and $C \to C_2$.
   (iv) the image $\Delta A_3B_3C_3$ under an anticlockwise rotation of 90° about the origin, where $A \to A_3, B \to B_3$ and $C \to C_3$.

(c) What single transformation maps $\Delta A_1B_1C_1$ onto $\Delta A_3B_3C_3$, where $A_1 \to A_3$, $B_1 \to B_3$ and $C_1 \to C_3$.

ANGLE PROPERTIES IN A CIRCLE

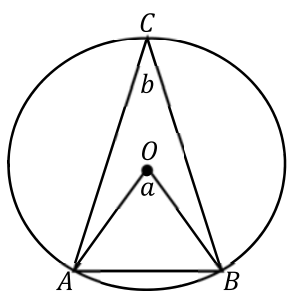
PROPERTY 1
The angles a chord or arc subtends at the circumference in the same segment of a circle are equal.



$m = n = p$

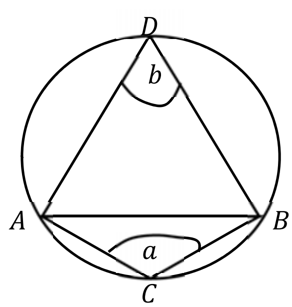
Where AB is a chord.

PROPERTY 2
The angle a chord subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle.



O is the centre of the circle.
i.e. $< AOB = 2 < ACB$
$a = 2b$

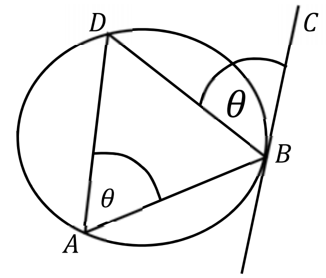
PROPERTY 3
The sum of the angles a chord or an arc subtends at the circumference of opposite segments of a circle is equal to 180°.



$$\boxed{< ACB + < ADB = 180^\circ}$$

i.e. $a + b = 180^\circ$

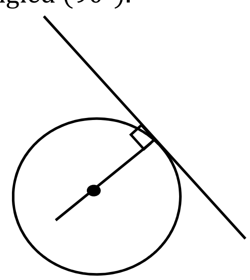
PROPERTY 4
The angle a chord and a tangent makes is equal to the angle the chord makes in the alternate segment.



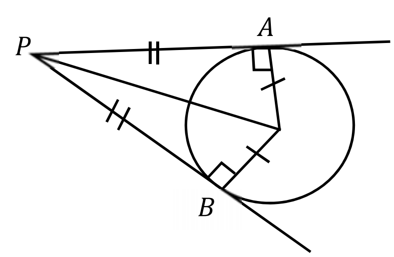
CB is a tangent at B.

$< CDB = < DAB$.

PROPERTY 5
The angle made by a tangent and the radius is right – angled (90°).



PROPERTY 6
From any point outside the circle, you can only draw two tangents to the circle and these tangents will be equal in length.

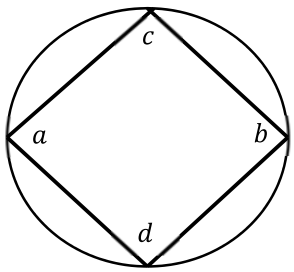


$|PA| = |PB|$

NOTE: more often than not, this theorem leads to some isosceles triangles. So be on the lookout.

PROPERTY 7

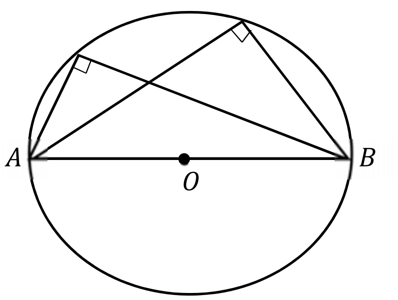
The opposite angles of a cyclic quadrilateral are supplementary. Thus, they add up to 180°.



$a + b = 180°$
$c + d = 180°$

PROPERTY 8

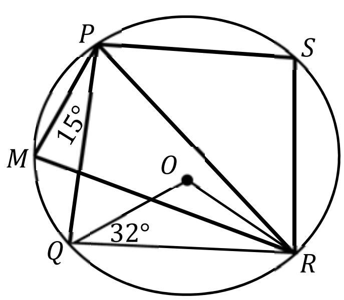
The angle the diameter of a circle subtends at the circumference is 90°.



AB − diameter of the circle.

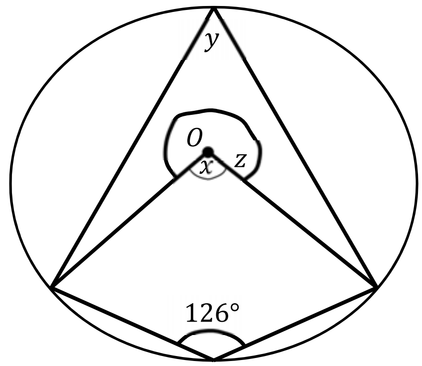
Exercise 1 Date:……………………

1.



In the diagram, O is the centre of the circle, $< OQR = 32°$ and $< MPQ = 15°$. Calculate
(i) $< QPR$ (ii) $< MQO$

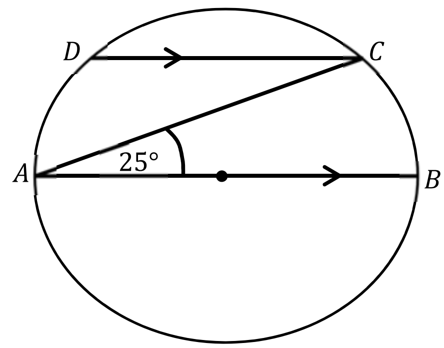
2.



Where O is the centre.
Find the angles x, y and z in the diagram.

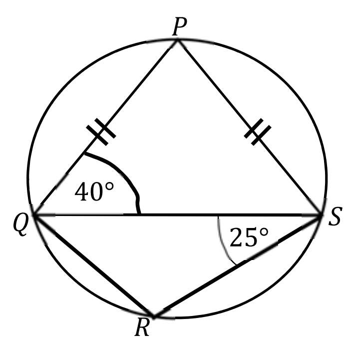
Exercise 2 Date:……………………

1.



In the diagram, (not drawn to scale), AB is a diameter of the circle $ABCD$. DC is parallel to AB and $< BAC = 25°$. Calculate
(i) $< ADC$ (ii) $< CAD$.

2.

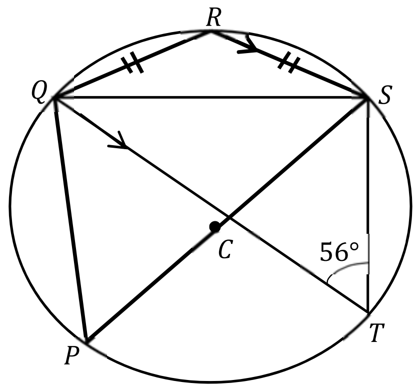


In the diagram, P, Q, R and S are points on a circle. $|PQ| = |PS|$, $< PQS = 40°$ and $< QSR = 25°$. Calculate the value of
(i) $< QPS$ (ii) $< QRS$ (iii) $< RQS$

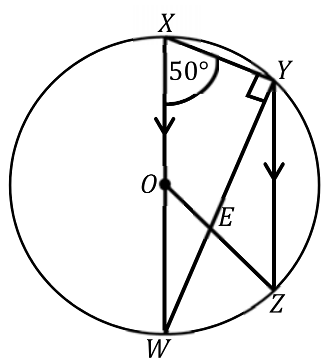
THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 3 Date:……………………

1. $PQRST$ is a circle with centre C. PCS is a straight line. $RS//QT$. $|QR| = |RS|$ and $< QTS = 56°$. Find
   (i) $< SQT$ (ii) $< PQT$



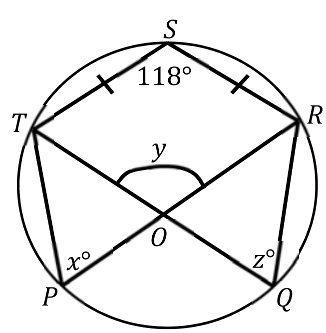
2.



In the diagram, O is the centre of the circle. WX is parallel to YZ and $< WXY = 50°$, find the value of
   (i) $< WYZ$ (ii) $< YEZ$.

Exercise 4 Date:……………………

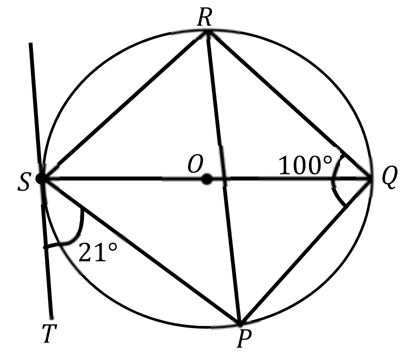
1.



In the diagram, O is the centre of the circle, $|TS| = |SR|$, $T\hat{P}R = x°$, $T\hat{Q}R = z°$.
   (i) Find the relationship between x, y and z.

   (ii) Find $S\hat{T}P$

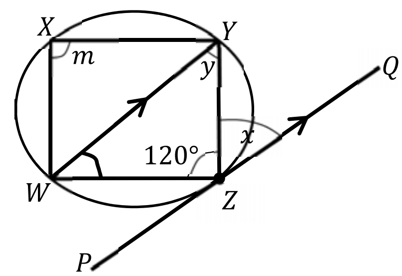
2.



In the diagram, $\overline{TS}$ is a tangent to the circle at S. If O is the centre of the circle, $< TSP = 21°$ and $R\hat{Q}P = 100°$, find with reasons:
   (i) $< SPR$ (ii) $< QSR$

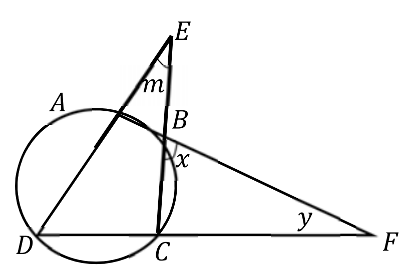
Exercise 5 Date:……………………

1.



In the diagram, PQ is a tangent to the circle at Z. If $PQ//WY$, $< WZY = 120°$, $< WXZ = m$, $< WYZ = y$ and $< YZQ = x$, find the value of
   (i) m; (ii) x.

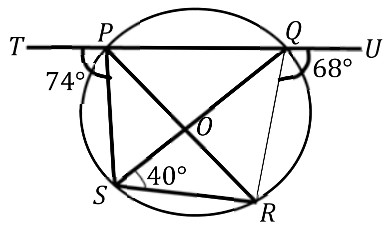
2. In the diagram above, $ABCD$ is a circle. DAE, CBE, ABF and DCF are straight lines. If $y + m = 90°$, find the value of x.



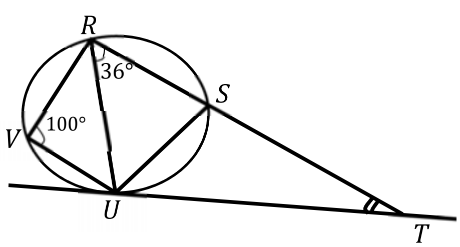
THE ONLY WAY OF LEARNING MATHEMATICS IS BY SOILING YOUR HANDS SOLVING MATHEMATICAL QUESTIONS.

Exercise 6   Date:......................

1. The diagram below shows a circle $PQRS$ with centre O, $< UQR = 68°$, $< TPS = 74°$ and $< QSR = 40°$. Calculate the value of $< PRS$.

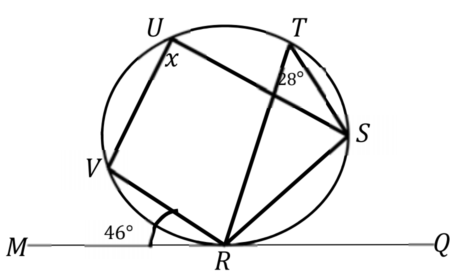


2. In the diagram, $\overline{TU}$ is a tangent to the circle. $< RVU = 100°$ and $< URS = 36°$. Calculate the value of angle STU.

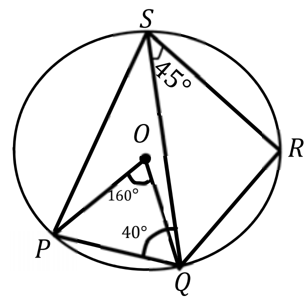


Exercise 7   Date:......................

1. In the diagram, $< RTS = 28°$, $< VRM = 46°$, MQ is a tangent to the circle $VRSTU$ at the point R. Find $< VUS$.

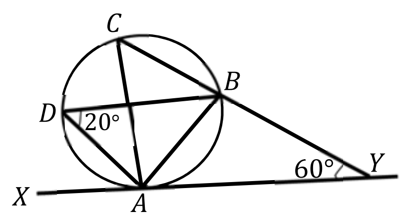


2. In the diagram, P, Q, R and S are points on the circumference of the circle centre O. $P\hat{O}Q = 160°$, $O\hat{S}R = 45°$ and $P\hat{Q}S = 40°$. Calculate
(i) $Q\hat{P}S$ (ii) $R\hat{Q}S$

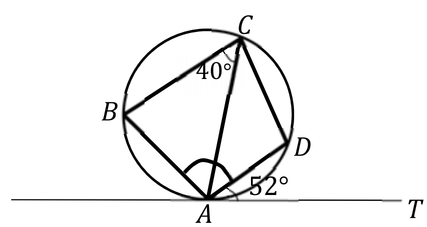


Exercise 8   Date:......................

1. In the diagram, A, B, C and D are points on the circumference of a circle. XY is a tangent at A. Find
(i) $< ABC$
(ii) $< CAX$
(iii) $< ABY$

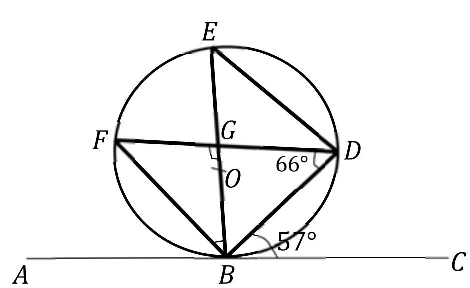


2.



In the diagram, TA is a tangent to the circle at A. If $B\hat{C}A = 40°$ and $DT = 52°$, find $B\hat{A}D$.

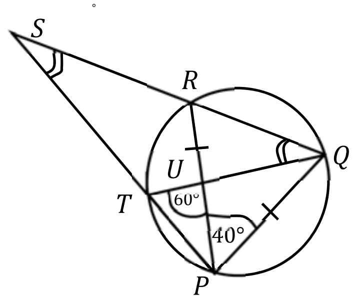
3.



In the diagram, O is the centre of the circle and ABC is a tangent at B. If $B\hat{D}F = 66°$ and $D\hat{B}C = 57°$, calculate,
(i) $E\hat{B}F$ (ii) $B\hat{G}F$

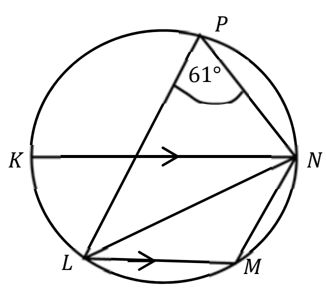
Exercise 9 **Date:……………………**

1.



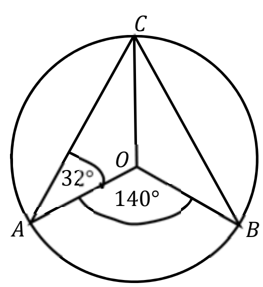
In the figure, $PQRT$ is a circle. $|PQ| = |PR|, < QPR = 40°$ and $< PUT = 60°$. Find
(i) $< TQR$; (ii) $< PSQ$

2.



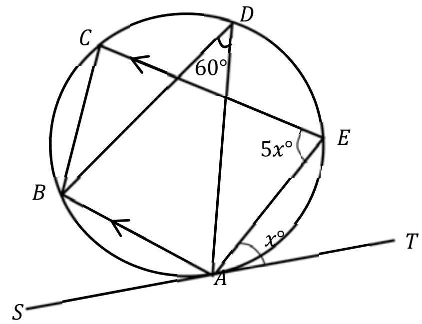
In the figure above LM is a chord parallel ot the diameter KN of the circle $KLMNP$. If $< NPL = 61°$, calculate $< MLN$.

3.



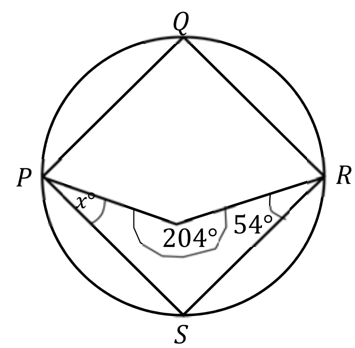
In the diagram, O is the centre of the circle. The points $A < B$ and C are on the circumference of the circle. Angles CAO and AOB are $32°$ and $140°$ respectively. Calculate
(i) angle OBC; (ii) angle COB

4. In the diagram, TS is a tangent to the circle at A. $|AB|//|CE|$, $AEC = 5x°$, $< ADB = 60°$ and $< TAE = x$. Find the value of x.

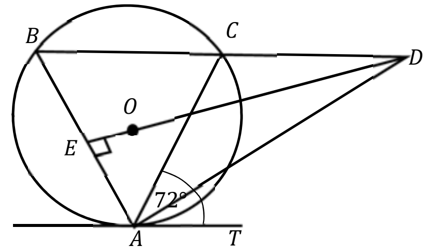


Exercise 10

1. The diagram shows a circle $PQRS$ with centre O. The reflex angle at O is $204°$, angle $ORS = 54°$ and angle $OPS = x°$. Find x.



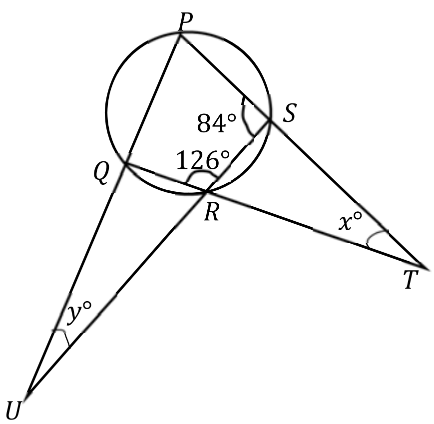
2.



In the diagram, A, B and C are points on a circle with centre O. AT is the tangent to the circle at A. The line DOE is perpendicular to AB, $|AB| = |AC|$ and $< TAC = 72°$.
(a) Calculate
(i) $< BCA$ (iii) $< CDA$
(ii) $< CAD$

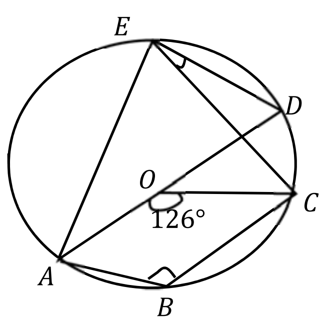
(b) Use your results in (a) to show that
  (i) AD bisects angle TAC
  (ii) $|CD| = |CA|$

3. In the diagram, $PQRS$ is a cyclic quadrilateral. PS and QR are produced to meet at T. SR and PQ are produced to meet at U. If $< PSR = 84°$ and $< SRQ = 126°$. Find x and y.



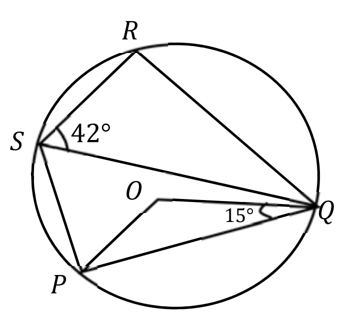
Exercise 11

1.



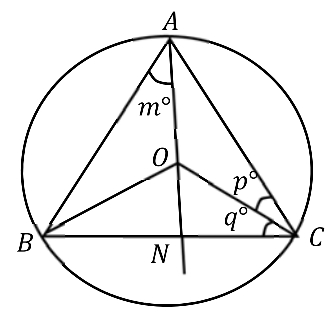
In the diagram, $ABCDE$ is a circle with centre O. AOD is a straight line and $< AOC = 126°$. Find
(i) $< ABC$; (ii) $< CED$.

2.



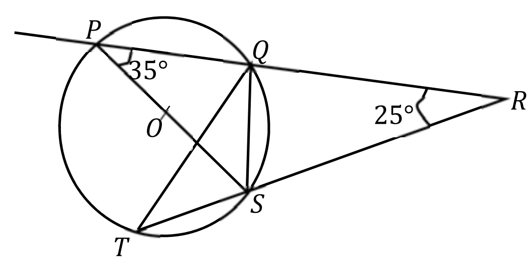
In the diagram, O is the centre of the circle, $< PQO = 15°$ and $< QSR = 42°$. Calculate
(a) $< QSP$; (b) $< RQO$.

3.



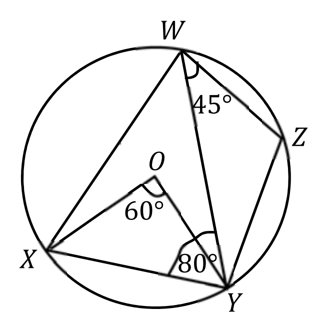
In the figure above, A, B and C are three points on a circle with centre O. If $< OAB = m°$, $< OCA = p°$ and $< OCB = q°$, show that $p + q + m = 90°$.

4.



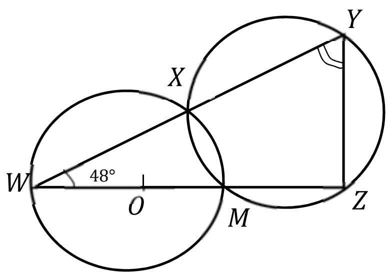
In the diagram, PQR and TSR are straight lines, $< QRS = 25°$ and $< QPS = 35°$. Find $< SQT$.

5.

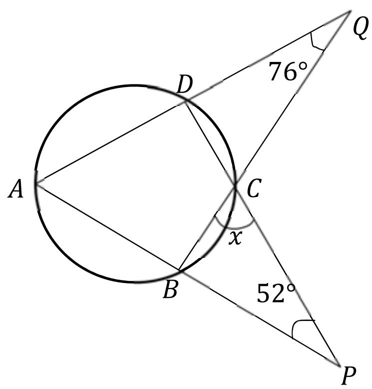


In the diagram, $WXYZ$ are points on the circumference of a circle centre O, $< XOY = 60°$, $< YWZ = 45°$ and $< XYW = 80°$. Calculate $< ZYW$.

6. In the diagram, WZ and WY are straight lines, O is the centre of circle XWM and $< XWM = 48°$. Calculate the value of $< WYZ$.

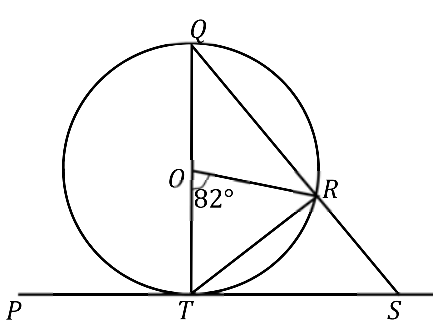


7. In the diagram, $ABCD$ is cyclic quadrilateral. AB and DC are produced to meet at P, AD and BC are produced to meet at Q. If $< DQC = 76°$, $< BPC = x$, calculate the value of x.



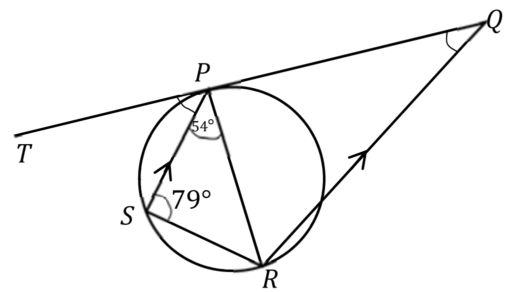
Exercise 12

1.



In the diagram, PS is a tangent to the circle of centre O. If QS is a straight line and $< TOR = 82°$, find $< RST$.

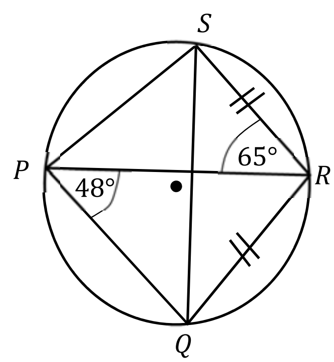
2.



In the diagram, $PS//QR$, $< PSR = 79°$, $< SPR = 54°$ and TQ is a tangent to the circle at P. Calculate:

(i) $< TPS$ (ii) $< PQR$

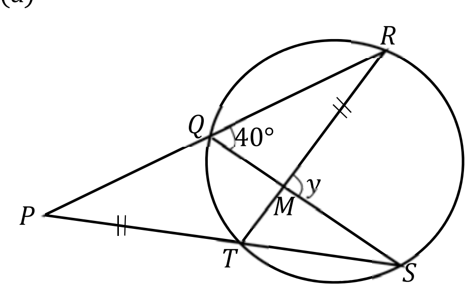
3.



In the diagram, $PQRS$ is a cyclic quadrilateral. $|SR| = |RQ|$, $< SRP = 65°$ and $< RPQ = 48°$. Find $< PRQ$.

4.

(a)



In the diagram, $< RQS = 40°$, $|RT| = |PT|$ and $< RMS = y$. Find the value of y.

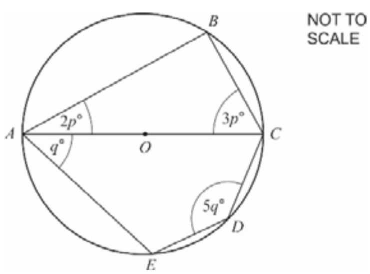
(b) XY is a tangent to a circle LMN at the point M. XLN is a straight line, $< NXM = 34°$ and $< NMY = 65°$.
- (i) Illustrate the information in a diagram.
- (ii) Find the value of:
  - (α) $< MLX$;
  - (β) $< LNM$.

REVIEW EXERCISE

2. Prove that the angle which an arc of a circle subtends at the center is twice that which it subtends at any point on the remaining part of the circumference.

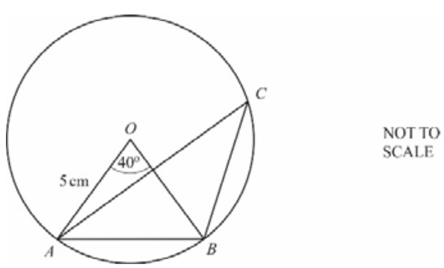
3. Prove that angles in the same segment of a circle are equal.

4.



A, B, C, D and E lie on a circle, centre O. AOC is a diameter. Find the value of
(a) p (b) q

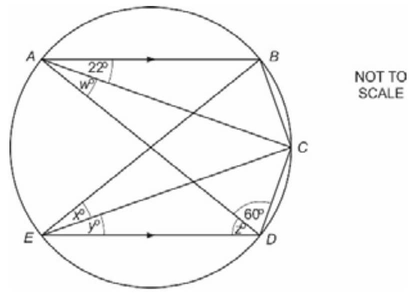
5.



A, B and C are points on a circle, centre O. Angle $AOB = 40°$
(a)
- (i) Write down the size of angle ACB.
- (ii) Find the size of angle OAB.

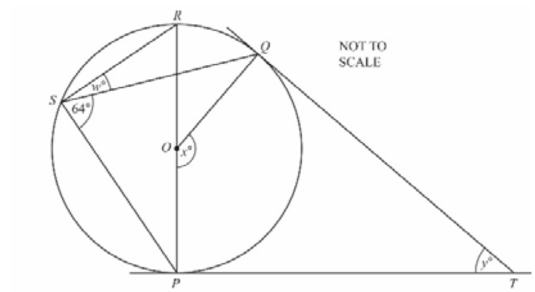
(b) The radius of the circle is 5cm.
- (i) Calculate the length of the minor arc AB
- (ii) Calculate the area of the minor sector OAB

6.



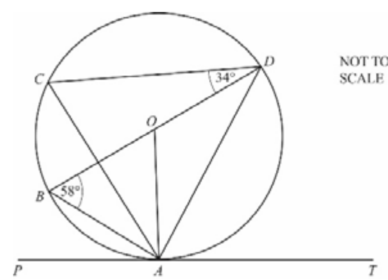
AD is a diameter of the circle $ABCDE$. Angle $BAC = 22°$ and angle $ADC = 60°$. AB and ED are parallel lines. Find the values of w, x, y and z.

7.



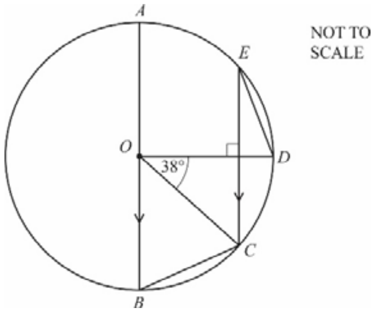
P, Q, R and S lie on a circle, centre O. TP and TQ are tangents to the circle. PR is a diameter and angle $PSQ = 64°$.
(a) Work out the values of w and x.
(b) Showing all your workings, find the value of g.

8.



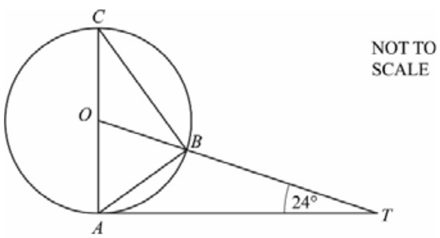
A, B, C and D lie on the circle, centre O. BD is a diameter and PAT is the tangent at A. Angle $ABD = 58°$ and angle $CDB = 34°$. Find
(a) $< ACD$ (c) $< DAT$
(b) $< ADB$ (d) $< CAO$

9.



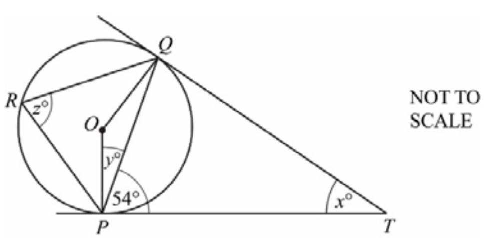
AB is the diameter of a circle, centre O. C, D and E lie on the circle. EC is parallel to AB and perpendicular to OD. Angle DOC is 38°. Work out
(a) $< BOC$
(b) $< CBO$
(c) $< EDO$

10.



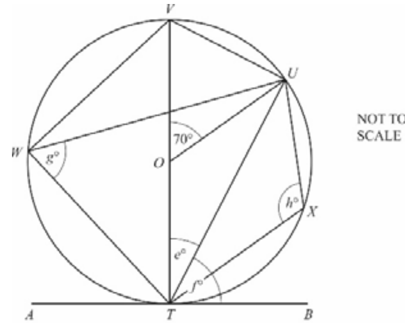
A, B and C are points on a circle, centre O. TA is a tangent to the circle at A and OBT is a straight line. AC is a diameter and $< OTA = 24°$. Calculate
(a) $< AOT$
(b) $< ACB$
(c) $< ABT$

11.



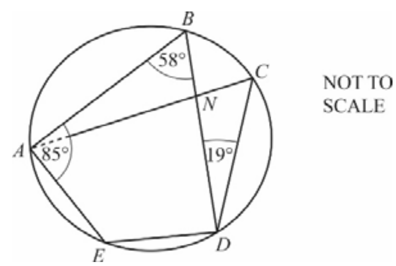
The points P, Q and R lie on a circle, centre O. TP and TQ are tangents to the circle. $< TPQ = 54°$. Calculate the value of
(a) x (b) y (c) z

12.



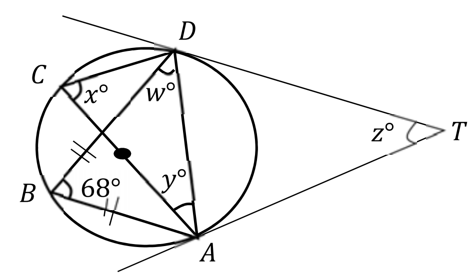
The diagram shows a circle, centre O. VT is a diameter and ATB is a tangent to the circle at T. U, V, W and X lie on the circle and $< VOU = 70°$. Calculate the value of
(a) e (c) g
(b) f (d) h

13.



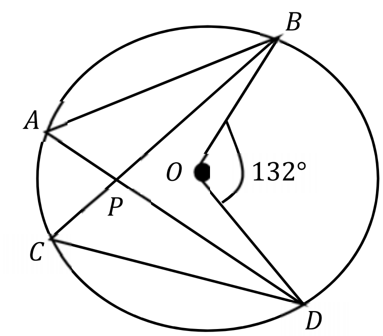
A, B, C, D and E are points on a circle. $< ABD = 58°$, $< BAE = 85°$ and $< BDC = 19°$. BD and CA intersects at N. Calculate
(i) $< BDE$ (ii) $< AND$

14.



AOC is the diameter of circle $ABCD$. AT and DT are tangents, $BD = BA$ and angle $DBA = 68°$. Find the angles marked w, x, y and z.

15.

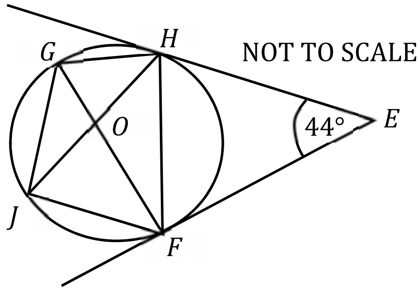


O is the centre of the circle. Angle $BOD = 132°$. The chords AD and BC meet at P.

(a)
- (i) Calculate angles BAD and BCD.
- (ii) Explain why triangles ABP and CDP are similar.
- (iii) $AP = 6\text{cm}, PD = 8\text{cm}, CP = 3\text{cm}$, and $AB = 17.5\text{cm}$. Calculate the lengths of PB and CD.
- (iv) If the area of triangle ABP is $n\text{cm}^2$, write down, in terms of n, the area of triangle CPD.

(b)
- (i) The tangents at B and D meet at T. Calculate angle BTD.
- (ii) Use $OB = 9.5\text{cm}$ to calculate the diameter of the circle which passes through O, B, T and D, giving your answer to the nearest centimeter.

16. The diagram, below, not drawn to scale, shows a circle, centre O. EH and EF are tangents to the circle. FOG and JOH are straight lines.
The measure of $< FEH = 44°$.

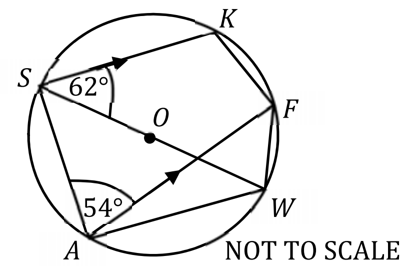


Calculate, giving reasons for your answer, the measure of:

(i) $< EHF$ (iii) $< JHE$

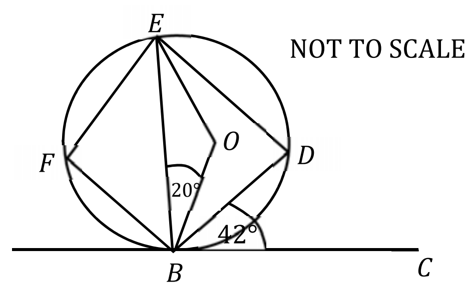
(ii) $< FGH$ (iv) $< JGH$

17. In the diagram below, not drawn to scale, O is the centre of the circle. The lines SK and AF are parallel.
$< KSW = 62°$ and $< SAF = 54°$.



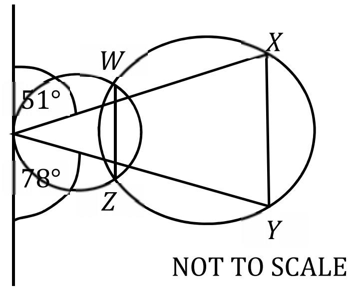
Calculate, giving reasons for your answer, the measure of
(i) $< FAW$
(ii) $< SKF$
(iii) $< ASW$

18. The diagram below, not drawn to scale, shows a circle, centre O. The line BC is a tangent to the circle at B.
$< CBD = 42°$ and $< OBE = 20°$



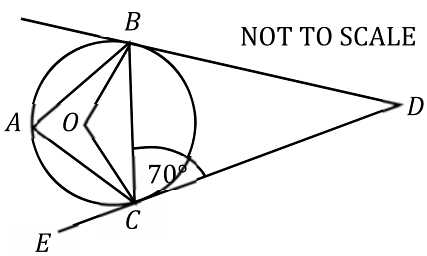
Calculate, giving a reason for EACH step of your answer, the measure of:
(i) $< BOE$
(ii) $< OED$
(iii) $< BFE$

19. In the diagram below, VWZ and $WXYZ$ are two circles intersecting at W and Z. SXT is a tangent to the circle at V, VWX and VZY are straight lines, $< TVY = 78°$ and $< SVX = 51°$.



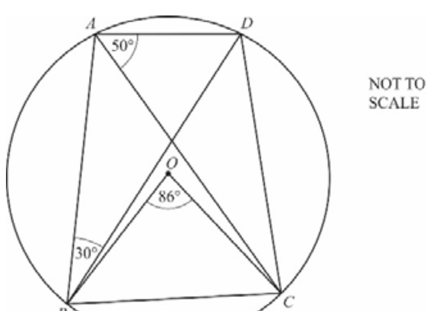
Calculate
(a) $< VZW$ (b) $< XYZ$

20. The diagram below, not drawn to scale, shows a circle, centre O. the lines BD and DCE are tangents to the circle, and angle $BCD = 70°$. Calculate, giving your reasons for each step of your answer,



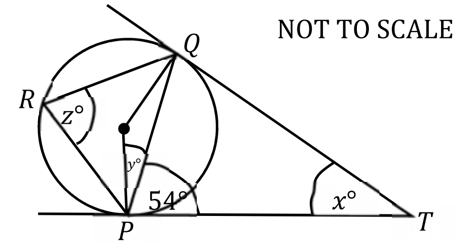
(i) $< OCE$ (iii) $< BOC$
(ii) $< BAC$ (iv) $< BDC$

21.



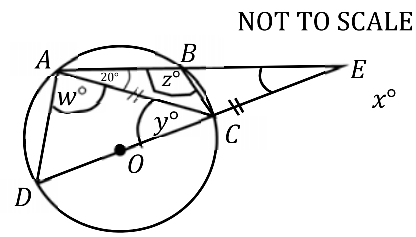
The points A, B, C and D lie on the circumference of the circle, centre O. Angle $ABD = 30°$, angle $CAD = 50°$ and angle $BOC = 86°$.
(a) Give the reason why angle $DBC = 50°$.
(b) Find
  (i) Angle ADC
  (ii) Angle BDC
  (iii) Angle OBD

22.



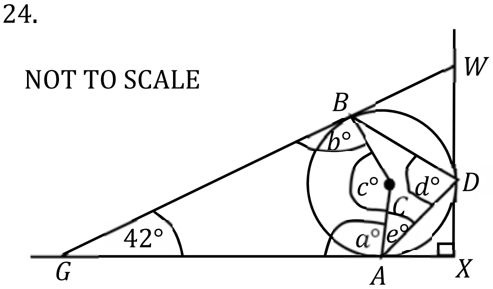
The points P, Q and R lie on a circle, centre O. TP and TQ are tangents to the circle. Angle $TPQ = 54°$.
Calculate the value of
(a) x
(b) y
(c) z

23.



The centre of the circle $ABCD$ is O. ABE and $DOCE$ are straight lines. $AC = CE$ and angle $BAC = 20°$. Find the values of w, x, y and x.

24.

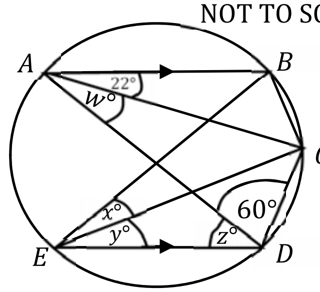


A sphere, centre C, rests on a horizontal ground at A and touches a vertical wall at D. A straight plank of wood, GBW, touches the sphere at B, rests on the ground at G and against the wall at W. The wall and the ground meet at X. Angle $WGX = 42°$.
(a) Find the values of a, b, c, d and e marked on the diagram.
(b) Write down one word which completes the following sentence.

Angle CGA is $21°$ because triangle GBC and triangle GAC are ______________________________.

25.

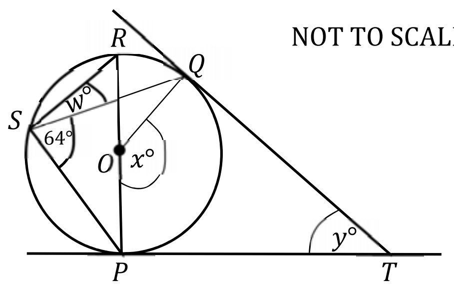
NOT TO SCALE



AD is a diameter of the circle $ABCDE$. Angle $BAC = 22°$ and angle $ADC = 60°$. AB and ED are parallel lines. Find the values of w, x, y and z.

26.

NOT TO SCALE



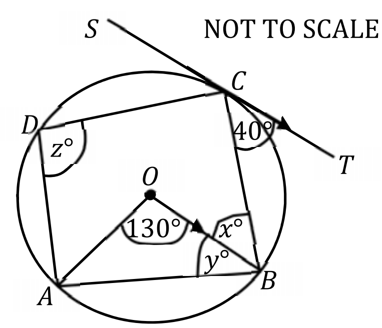
P, Q, R and S lie on a circle, centre O. TP and TQ are tangents to the circle. PR is a diameter and angle $PSQ = 64°$.

(a) Work out the values of w and x.

(b) **Showing all your working**, find the value of y.

27.

NOT TO SCALE



A, B, C and D lie on a circle, centre O. SCT is the tangent at C and is parallel to OB. Angle $AOB = 130°$, and angle $BCT = 40°$. Angle $OBC = x°$, angle $OBA = y°$ and angle $ADC = z°$.

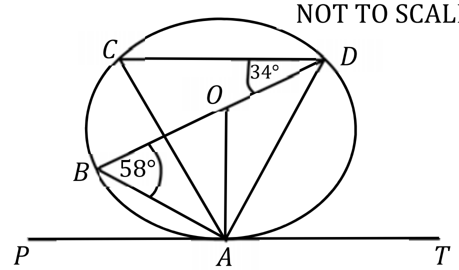
(i) Find the values of x, y and z

(ii) Write down the value of angle OCT

(iii) Find the value of the **reflex** angle AOC.

28.

NOT TO SCALE



A, B, C and C lie on the circle, centre O. BD is a diameter and PAT is the tangent at A. Angle $ABD = 58°$ and angle $CDB = 34°$. Find

(a) Angle ACD,

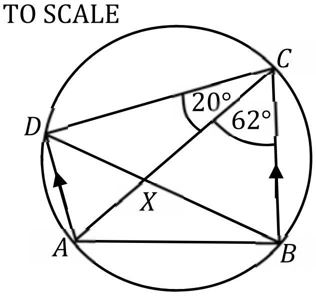
(b) Angle ADB,

(c) Angle DAT,

(d) Angle CAO

29.

NOT TO SCALE



$ABCD$ is a cyclic quadrilateral. AD is parallel to BC. The diagonals DB and AC meet at X. Angle $ACB = 62°$ and angle $ADC = 20°$. Calculate

(a) Angle DBA

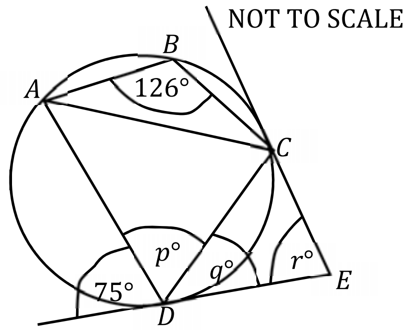
(b) Angle DAB

(c) Angle DAC

(d) Angle AXB

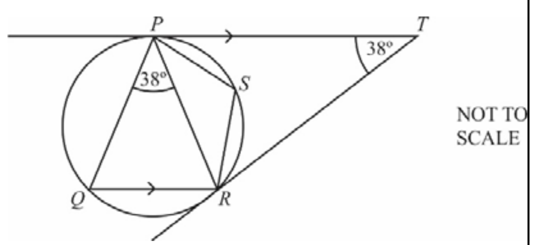
(e) Angle CDB

30.



$ABCD$ is a cyclic quadrilateral.
The tangents at C and D meet at E.
Calculate the values of p, q and r.

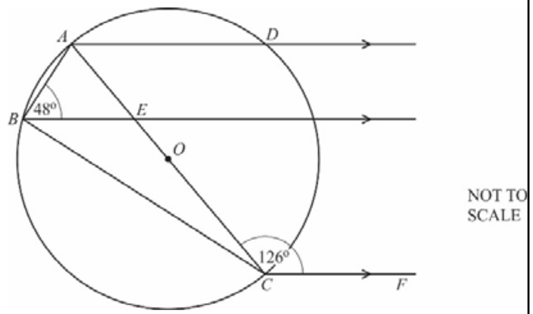
31.



In the diagram PT and QR are parallel.
TP and TR are tangents to the circle $PQRS$.
Angle $PTR =$ angle $RPQ = 38°$.

(a) What is the special name of triangle TPR. Give a reason for your answer.
(b) Calculate
  (i) Angle PQR,
  (ii) Angle PSQ

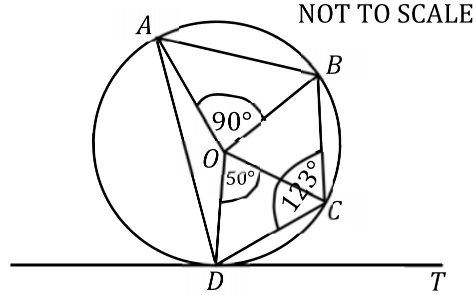
32.



A, B, C and D lie on a circle centre O. AC is a diameter of the circle. AD, BE and CFG are parallel lines. Angle $ABE = 48°$ and angle $ACF = 126°$. Find

(a) Angle DAE,
(b) Angle EBC,
(c) Angle BAE,

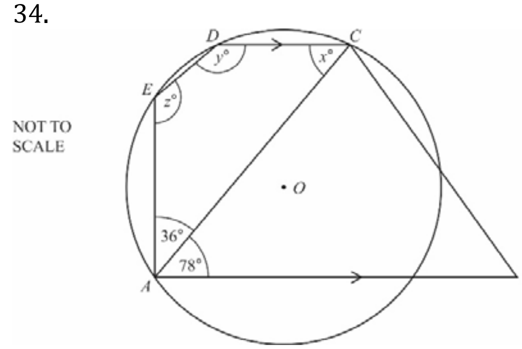
33.



The points A, B, C and D lie on a circle centre O. Angle $AOB = 90°$, angle $COD = 50°$ and angle $BCD = 123°$. The line DT is a tangent to the circle at D.
Find

(a) Angle OCD
(b) Angle TDC
(c) Angle ABC
(d) Reflex angle AOC

34.



$ABCDE$ is a pentagon.
A circle, centre O, passes through the points A, C, D and E.
Angle $EAC = 36°$, angle $CAB = 78°$ and AB is parallel to DC.

(a) Find the values of x, y and z, giving a reason for each
(b) Explain why ED is **not** parallel to AC
(c) Find the value of angle EOC

(d) $AB = AC$.
Find the value of angle ABC.

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