SC4012
WASSCE 2022
FURTHER MATHEMATICS/
MATHEMATICS (ELECTIVE) 2
2½ hours

2

CANDIDATE'S NAME	
INDEX NUMBER	SIGNATURE
DATE:	

THE WEST AFRICAN EXAMINATIONS COUNCIL

West African Senior School Certificate Examination
for School Candidates

SC 2022 FURTHER MATHEMATICS/MATHEMATICS (ELECTIVE) 2 2½ hours
[100 marks]

INSTRUCTIONS TO CANDIDATES

1. *In the spaces provided above, insert your* **name, full index number,** *normal* **signature** *and the* **date of examination.**
2. *This booklet consists of* **two** *sections*: **A** *and* **B**. *Answer* **all** *the questions in Section* **A (compulsory)** *and* **four** *questions from Section* **B.**
3. *In each question, all necessary details of working, including rough work,* **must** *be shown with the answer.*
4. *Give answers as accurately as data and tables allow.*
5. *Graph paper is provided for your use on page* **20.**
6. *The use of non-programmable, silent and cordless calculator is allowed.*
7. *Write your* **name, index number** *and the* **number of each question you answer, at the top of each page.**
8. *Write on both sides of the paper unless otherwise instructed on the question paper.*
9. *Begin* **each** *answer to a question on a fresh page. Leave two lines between answers where there are sub-sections to the same question.*
10. *On* **no account** *should you tear off any part of the booklet. It is an examination malpractice to do so. The answer booklet will be collected at the end of the examination.*
11. *Write in the space provided below, the* **question number of the questions you have answered,** in the order in which you have answered them.

For Examiner's Use Only	
Question Number	**Mark**
TOTAL	

Do not write in this margin.

SECTION A

[48 marks]

Answer **all** *the questions in this section.*

All *questions carry* **equal** *marks.*

1. Solve $3^x + 243\left(3^{-x}\right) - 36 = 0$.

2. A binary operation ∇ is defined on a set of real numbers, R, by $p \nabla q = \frac{p^2 - q^2}{4}$, where $p, q \in R$.

   (a) Evaluate $2 \nabla \left(\frac{3}{5}\right)$.

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(b) If $x \nabla 6 = 3\frac{1}{4}$, find the values of x.

Given that $\frac{2\tan 15°}{1+\tan^2 15°} = \sin 30°$, find $\tan 15°$, leaving the answer in surd form(radicals).

Given that $f: x \rightarrow \frac{x+m}{x-3}$, $x \neq 3$, where m is a constant,

(a) find the value of m if $f(5) = 1\frac{1}{2}$;

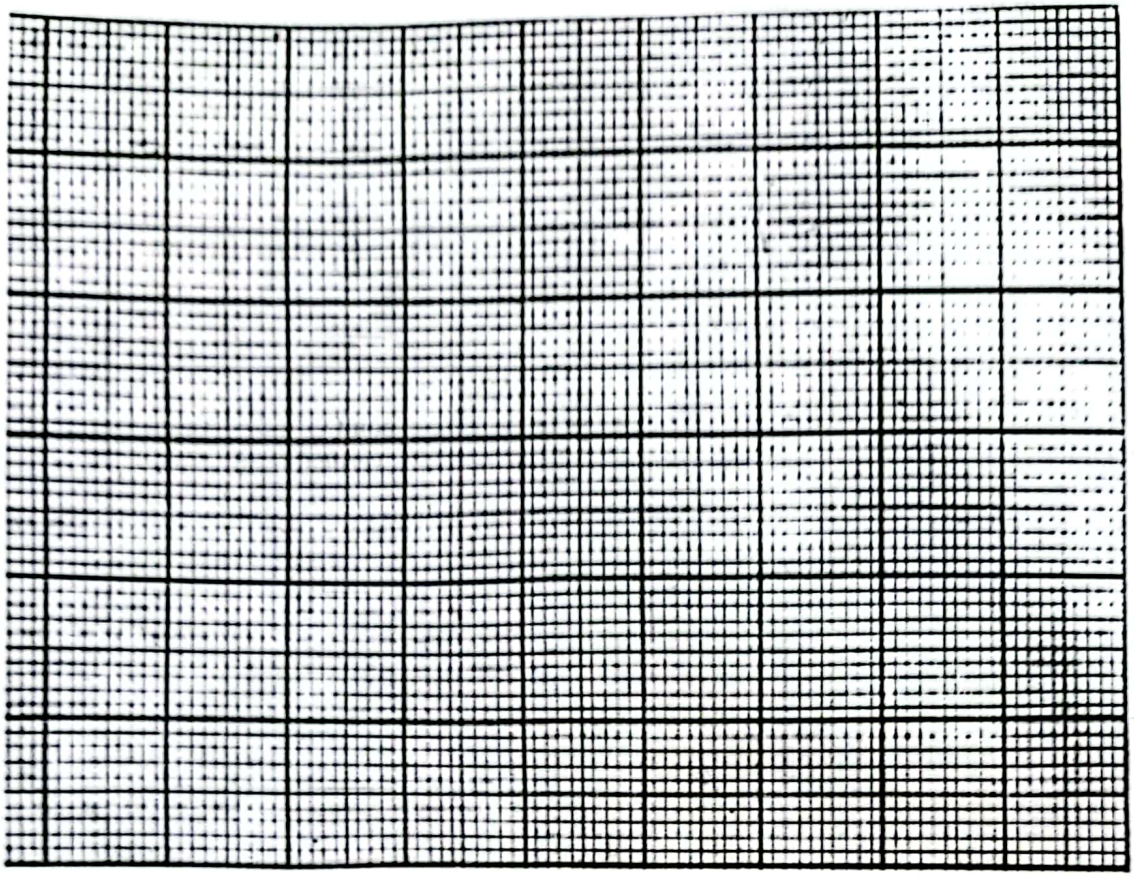
(b) find f^{-1};

(c) state the value of x for which f^{-1} is undefined.

5. The table shows the distribution of marks obtained by students in a test.

Marks	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49
Frequency	4	5	6	12	10	8	3	2

(a) Draw a histogram for the distribution.



(b) Use the histogram to estimate the modal mark.

6. Two independent events X and Y are such that $P(X) = m$, $P(Y) = m + \frac{1}{5}$ and $P(X \cap Y) = \frac{3}{20}$.

(a) Find the value of m.

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(b) Use the value of m in **6***(a)* to find $P(X \cup Y)$.

7. A body of mass 5 *kg* is suspended by an inextensible string from a point O and is pulled by a horizontal force F *N* until the angle of inclination of the string to the vertical is 30°. Calculate, correct to **one** decimal place, the:

(a) tension in the string;

(b) value of F. [Take $g = 10\ ms^{-2}$]

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8. The position vectors P, Q, R and S are $(-2i + 3j)$, $(3i + 4j)$, $(4i - 5j)$ and $(3i)$ respectively. Calculate, correct to the **nearest** degree, $\angle PSQ$.

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SECTION B
[52 marks]

Answer **four** *questions* **only** *from this section* **with at least one** *question from* **each** *part.*

All *questions carry* **equal** *marks.*

PART I

PURE MATHEMATICS

9. *(a)* The 2nd term of a Geometric Progression (*G. P.*) is equal to the 8th term of an Arithmetic Progression (*A. P.*). The first terms, common difference and common ratio are all equal and non-zero. Find the sum of the **first five** terms of the Geometric Progression (*G. P.*).

(b) The sum of the first eleven terms of a linear sequence (*A. P.*) is 165. If the third term is 9, find the value of the:
(i) common difference;
(ii) first term.

10. *(a)* Expand $\left(\frac{1}{2}-x\right)^6$ in ascending powers of x.

(b) Using the result in **10***(a)*, find the value of:

(i) q for which the coefficient of x^2 in the expansion of $(1+qx)\left(\frac{1}{2}-x\right)^6$ is zero;

(ii) $(0.45)^6$, correct to **four** significant figures.

11. *(a)* The volume of a spherical balloon increases at the rate of $8\ cm^3s^{-1}$. Find the rate at which the surface area increases when the radius is 4 *cm*.

(b) Find the gradient of the curve $3x^2 - 4xy + 3y^2 = 15$ at (2, 3).

PART II

STATISTICS AND PROBABILITY

12. The table shows the marks a group of students obtained in Mathematics (X) and English (Y) tests.

Mathematics (X)	77	50	71	72	81	94	96	90	67
English (Y)	82	66	78	34	47	85	89	99	69

(a) Draw a scatter diagram to represent the information.

(b) Calculate $\bar{X}$ and $\bar{Y}$, the mean of X and Y respectively.

(c) Draw a line of best fit to pass through the point $(\bar{X}, \bar{Y})$.

(d) Determine the equation of the line of best fit.

(e) If a student scored 92 in Mathematics test, what would be his likely score in English test?

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13. A panel of 6 members is to be formed from 5 doctors and 10 nurses to include **each** profession. Find, correct to **two** decimal places, the probability of selecting the members to include:
   (a) **equal** doctors and nurses;
   (b) **at most 2** nurses;
   (c) **more** doctors than nurses.

PART III

VECTORS AND MECHANICS

14. The position vectors of the points W, X, Y and Z relative to the origin O are given by $\overrightarrow{OW} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $\overrightarrow{OX} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\overrightarrow{OY} = \begin{pmatrix} 4 \\ n \end{pmatrix}$ and $\overrightarrow{OZ} = \begin{pmatrix} m \\ -1 \end{pmatrix}$ where m and n are constants. Find the:
   (a) unit vector in the direction of $\overrightarrow{WX}$;
   (b) value of n for which angle $WOY = 90°$;
   (c) values of m for which the length $\overrightarrow{WZ}$ is $\sqrt{37}$ *units*.

15. *(a)* A stone is thrown vertically upwards from the top of a cliff, 58 m high, with a velocity of 25 ms^{-1}. Calculate the maximum height attained. [Take $g = 10\ ms^{-2}$]

   (b) A particle of mass 8 kg moving with a velocity $\begin{pmatrix} 5 \\ 4 \end{pmatrix} ms^{-1}$ collides with another particle of mass 4 kg moving with a velocity $\begin{pmatrix} 1 \\ 14 \end{pmatrix} ms^{-1}$ in the opposite direction. If they moved together after collision, find the:

   (i) magnitude of their common velocity;
   (ii) direction of their common velocity.

END OF PAPER